Student Name:_____

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

2007 Trial Examination

Reading Time: 15 minutes
Writing Time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, a protractor, set-squares, aids for curve sketching, one bound reference and an approved **graphics** calculator and if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 26 pages.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** or **best answers** the question.
A correct answer scores 1, an incorrect answer scores 0.
No mark will be given if more than one answer is completed for any question.
Marks will **not** be deducted for incorrect answers

Question 1

The range of the function $f: [-1, 5) \to R, f(x) = 3 - x$ is

A. [-2, 4]
B. (-2, 4)
C. R
D. (- 2, 4]
E. (-2, ∞)

Question 2

If $f(x) = cos\, x$ and $g(x) = \sqrt{x}$, then the domain of $f(g(x))$ is

A. $(-\infty, \infty)$
B. $[0, \infty)$
C. R
D. $(0, \infty)$
E. $[-1,1]$

Question 3

The function f: $[1, \infty) \to R, f(x) = 3e^{x-1} + 2$ has an inverse function $f^{-1}(x)$.

The function $f^{-1}(x)$ is given by

A. $f^{-1}: [5, \infty) \to R, f^{-1}(x) = 3e^{x-1} + 2$

B. $f^{-1}: [1, \infty) \to R, f^{-1}(x) = \log_e\left(\frac{x-2}{3}\right)$

C. $f^{-1}: [5, \infty) \to R, f^{-1}(x) = 1 + \log_e\left(\frac{x-2}{3}\right)$

D. $f^{-1}: [1, \infty) \to R, f^{-1}(x) = 1 + \log_e\left(\frac{x-2}{3}\right)$

E. $f^{-1}: R \to R, f^{-1}(x) = \frac{x+2}{3} - e^{-1}$

SECTION 1- continued

Question 4

The graph of $y = -\log_2(3-x)+2$ is obtained from the graph of $y = \log_2(x)$ by

A. a reflection in the *x* and *y*-axis, a translation by 3 units right and a translation of 2 units up.
B. a reflection in the *y*-axis, a translation by 3 units right and a translation of 2 units up.
C. a reflection in the *x*-axis, a translation by 3 units right and a translation of 2 units up.
D. a reflection in the *y*-axis, a translation by 3 units left and a translation of 2 down.
E. a dilation factor of 2, a translation by 3 units left and a translation of 2 units up.

Question 5

For the function $f : R \to R$, where $f(x) = a\,sin(bx)+c$, where *a*, *b* and are positive constants, the period, amplitude and range are respectively

A. $\frac{2\pi}{a},\ a,\ [a+c, -a+c]$
B. $\frac{2\pi}{b},\ a,\ [-a+c, a+c]$
C. $\frac{2\pi}{b},\ a,\ [-a, a]$
D. $\frac{2\pi}{a},\ -a,\ [a, b]$
E. $\frac{2\pi}{x},\ c,\ [-a, a]$

Question 6

The sum of the solutions of the equation $2\,cos\left(2x+\frac{\pi}{6}\right) = \sqrt{3}$ in the interval $[0, \pi]$ is equal to

A. $\frac{\pi}{6}$
B. 2π
C. $\frac{11\pi}{6}$
D. π
E. 4π

Question 7

For the graph of $y = |3\,cos\,2x|$ over the domain $[0, 2\pi]$ then the range is

A. $[-3, 3]$
B. R
C. $[0, 3]$
D. $(0, 3]$
E. $(0, 3)$

SECTION 1- continued
TURN OVER

Question 8

If $2\log_e 3x - 1 = \log_e a$ where $a > 0$, then x is equal to

A. $\frac{a}{9}$

B. $\frac{\sqrt{a}}{3}$

C. $\frac{ae}{3}$

D. $\frac{\sqrt{ae}}{3}$

E. $\frac{a}{e}$

Question 9

The derivative of $y = \frac{\cos(2x)}{3e^x - x}$ is

A. $\frac{2\sin 2x\left(3e^x - x\right) - \cos 2x\left(3e^x - 1\right)}{3e^x - x}$

B. $\frac{-2\sin 2x\left(3e^x - x\right) - \cos 2x\left(3e^x - 1\right)}{9e^{2x} - 6xe^x + x}$

C. $\frac{2\sin 2x\left(3e^x - x\right) - \cos 2x\left(3e^x - 1\right)}{9e^{2x} - 6xe^x + x^2}$

D. $\frac{-2\sin 2x\left(3e^x - x\right) - \cos 2x\left(3e^x - 1\right)}{9e^{2x} - 6xe^x + x^2}$

E. $\frac{2\sin 2x\left(3e^x - x\right) - \cos 2x\left(3e^x - 1\right)}{\left(3x^x - x\right)^2}$

Question 10

A spherical balloon of Volume, V cm^3 has a radius, r cm where $V = \frac{4}{3}\pi r^3$.

When $r = 7$ the rate of increase in volume of the balloon with respect to the radius is

A. 49π cm^3/cm

B. 196π cm^3/cm

C. 169π cm^2/cm

D. $\frac{196}{\pi}$ cm^2/cm

E. 196π cm^2/cm

Question 11

If $f(x) = 2x^3 + ax^2 + bx$ has a stationary point at (–2, –4) then the values of a and b respectively are

A. 9 and 12

B. 12 and 9

C. –9 and –12

D. 5 and 4

E. 9 and –12

Question 12

An approximate value for $\frac{1}{\sqrt{9.5}}$ is

A. $-\frac{1}{54} \times 0.5 + \frac{1}{3}$

B. $\frac{1}{54} \times 0.5 + \frac{1}{3}$

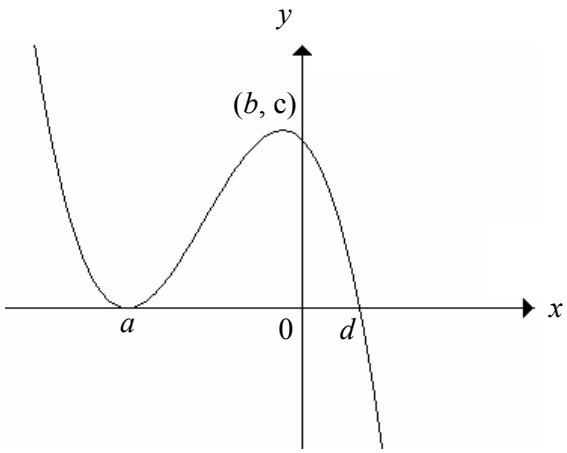
C. $-\frac{1}{54} \times 0.5 - \frac{1}{3}$

D. $\frac{1}{54} \times 0.5 - \frac{1}{3}$

E. $\frac{1}{3} - \frac{1}{54} \times 0.5$

SECTION 1- continued
TURN OVER

Question 13



For the graph of $y = f(x)$ shown above $f'(x)$ is negative when

A. $x > -a$ or $x < a$
B. $-a < x < a$
C. $x < a$ or $x > b$
D. $-a < x < b$ or $x > d$
E. $-\infty < x < d$

Question 14

The hybrid function $f(x) = \begin{cases} 9 - x^2, & x \leq 3 \\ x - 3, & x > 3 \end{cases}$ is

A. differentiable but not continuous at $x = 3$
B. neither continuous nor differentiable at $x = 3$
C. continuous and smooth at $x = 3$
D. continuous and differentiable for all $x \in R$
E. continuous but not differentiable at $x = 3$

SECTION 1- continued

Question 15

A random variable X, has its frequency curve defined as

$$f(x)=\begin{cases}\frac{1}{2}e^{-\frac{1}{2}x} & , \ x>0\\ 0 & , \text{ elsewhere}\end{cases}$$

The probability X, is smaller than 2 is

A. –0.3679
B. 1.367
C. –0.6321
D. 0.5
E. 0.6321

Question 16

A manufacturer who produces sheets of coloured paper believes there is a probability of 0.2 of any one sheet being defective. The coloured paper is packed 10 to a box. The probability that exactly half of the coloured paper per box is defective is

A. 0.0026
B. 0.9736
C. 0.246
D. 0.0264
E. 0.264

Question 17

The discrete random variable X has the probability distribution given by

x	1	2	3	4
$\Pr(X=x)$	0.2	$3k$	$2a$	$3a$

If the mean is 2.6, then the values of a and k respectively are

A. $a=0.1$ and $k=0.1$
B. $a=0.1$ and $k=0.9$
C. $a=0.6$ and $k=0.2$
D. $a=0.6$ and $k=0.3$
E. $a=0.4$ and $k=0.5$

SECTION 1- continued
TURN OVER

Question 18

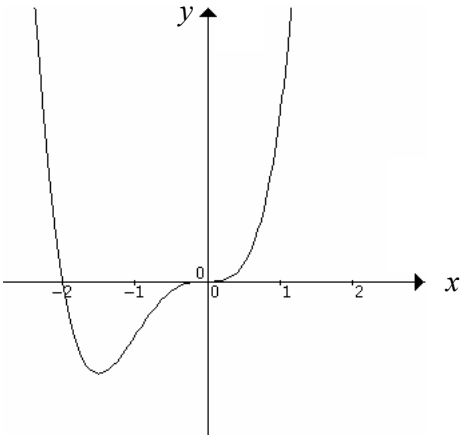
A normal random variable X has a mean of 16 and a standard deviation of 4, then the probability that X is greater than or equal to 20 is

A. $\Pr(X \geq 1)$
B. $1 - \Pr(z \geq 1)$
C. $\Pr(z \geq 20)$
D. $\Pr(z \geq 1)$
E. $\Pr(z \geq -1)$

SECTION 1- continued

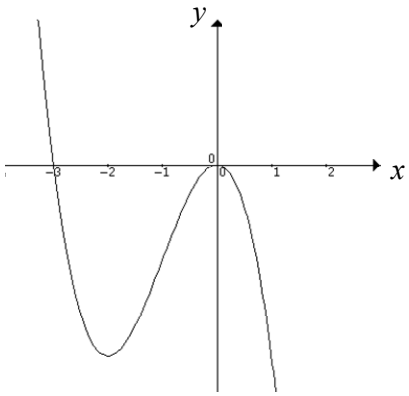
Question 19

The graph of the function f with $y = f(x)$ is shown below.

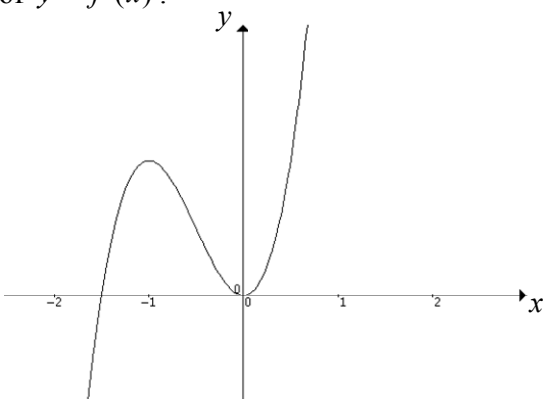


Which one of the following could be the graph of $y = f'(x)$?

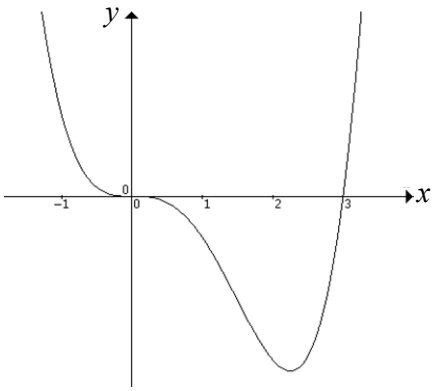
A.



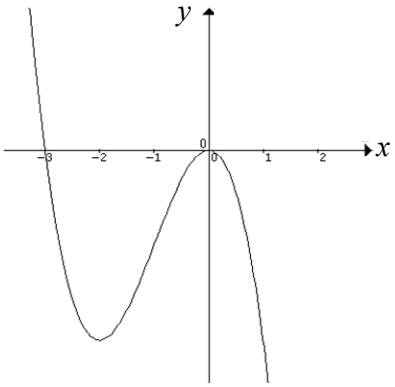
B.



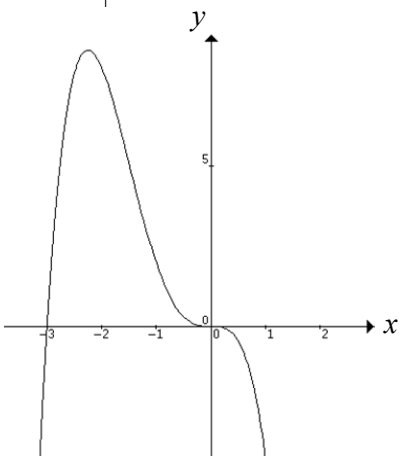
C.



D.



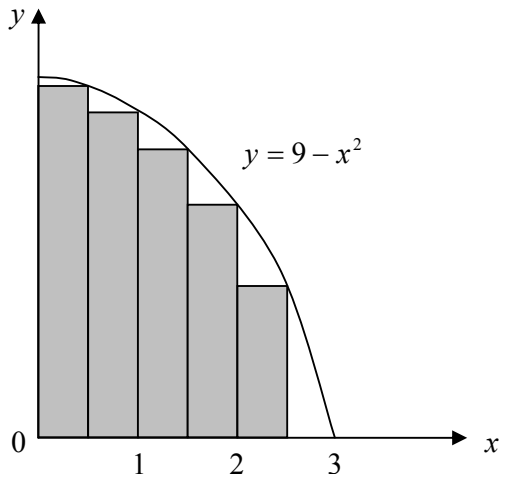
E.



SECTION 1- continued
TURN OVER

Question 20

The total area of the shaded rectangles can be used as an approximation for the area bounded by the curve $y = 9 - x^2$, and the x-axis.



The area of the shaded region shown above is

A. 15.625 units squared

B. 16.625 units squared

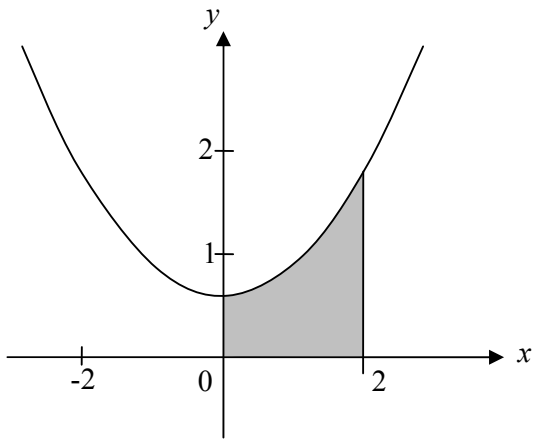
C. 17.295 units squared

D. 31.25 units squared

E. 18 units squared

Question 21

The graph with equation $y = m\left(x^2 + 2\right)$ where $m > 0$ is shown below.



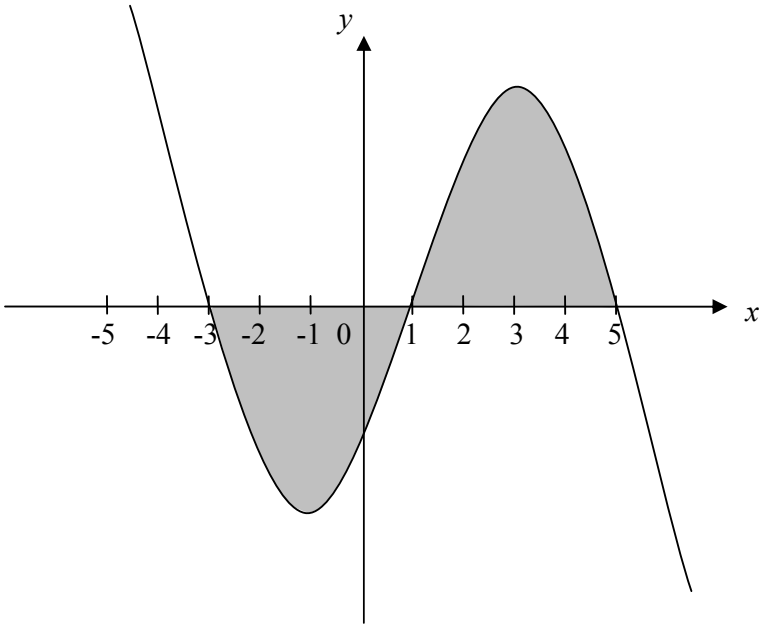
If the area of the shaded region is 2, then the value of m is

A. 10
B. 3
C. $\frac{3}{10}$
D. $\frac{10}{3}$
E. $\frac{1}{10}$

SECTION 1- continued
TURNOVER

Question 22

Which of the following integrals represents the shaded area shown in the graph below?



A. $\int_{-3}^{1} f(x)\,dx + \int_{1}^{5} f(x)\,dx$

B. $-\int_{-3}^{1} f(x)\,dx + \int_{1}^{5} f(x)\,dx$

C. $\left| \int_{-3}^{5} f(x)\,dx \right|$

D. $2\int_{-3}^{1} f(x)\,dx$

E. $\int_{-3}^{5} f(x)\,dx$

END OF SECTION 1

SECTION 2

Instructions for Section 2
Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let f be the function $f : D \to R, f(x) = log_e(5 - 2x)$, where D is the largest possible domain for which f is defined.

a. Find the **exact** co-ordinates of the x and y intercepts of the graph of $y = f(x)$.

_____

_____

_____

_____

2 marks

b. Find D the **largest** possible domain over which f is defined.

_____

_____

_____

1 mark

c. Use calculus to show that the rate of change is always negative.

_____

_____

_____

_____

_____

_____

2 marks

SECTION 2-Question 1- continued
TURN OVER

d.

i. Find $f^{-1}(x)$, the rule for f^{-1}.

_____

_____

_____

_____

_____

_____

ii. State the domain and range of f^{-1}.

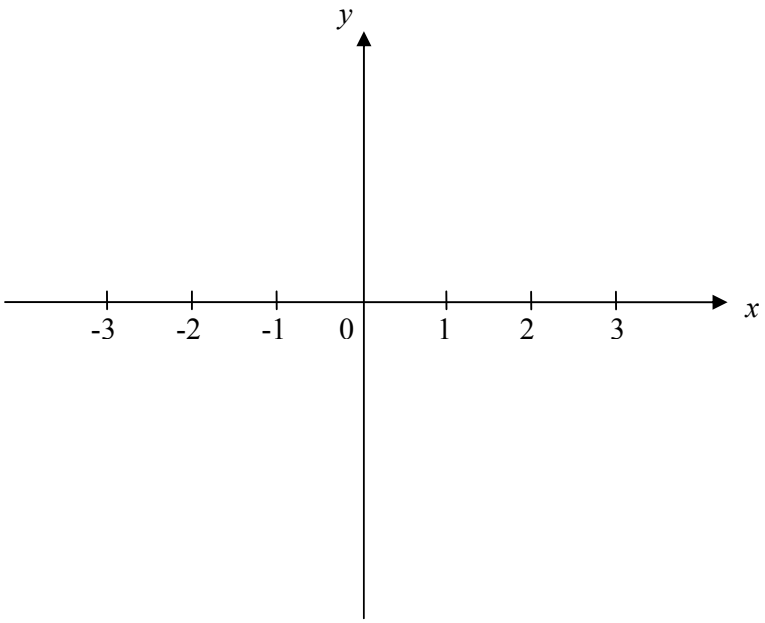
_____

_____

_____

2 + 1 = 3 marks

e. Sketch the graph of $f^{-1}(x)$. Clearly label any intercepts with the axes and vertical or horizontal asymptotes with its equation.



2 marks

SECTION 2-Question 1- continued

f. Using calculus find the exact area bounded by the graph of $y = f^{-1}(x)$ the x-axis and y-axis.

_____

_____

_____

_____

_____

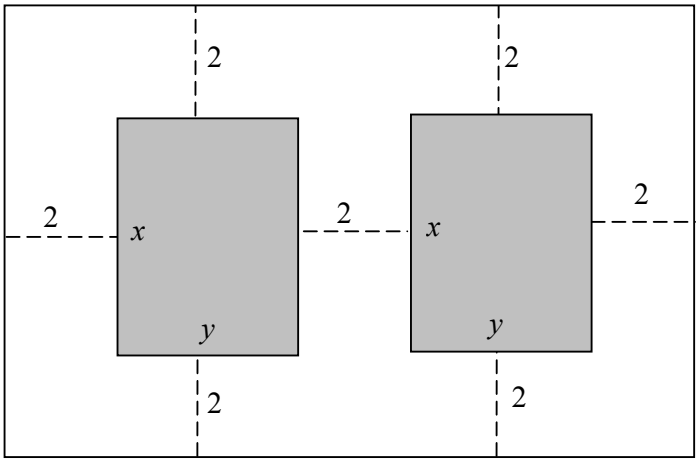
_____

2 marks
Total 12 marks

SECTION 2-continued
TURN OVER

Question 2

The diagram below shows a design for a shop with two identical display windows situated on a wall. The two windows are equally spaced so that all of the dotted lines are 2 metres long. The total area of each window is to be 12 square metres.



a. Show that $y = \frac{12}{x}$ where y is the width of a window.

_____

_____

_____

_____

2 marks

b. Find the length of the wall, L in terms of x.

_____

_____

_____

1 mark

SECTION 2-Question 2- continued

c. Find the height of the wall, H in terms of x.

1 mark

d. Show that the total area, A square metres of the wall excluding the windows is a function of the window height x metres is modeled by

$$A = 24 + 6x + \frac{96}{x}$$

3 marks

SECTION 2-Question 2- continued
TURN OVER

e. Find $\frac{dA}{dx}$.

1 mark

f. Find the minimum value of x for which the area, A is a minimum. Hence find the value of A.

3 marks

g. Using calculus verify that the area is a minimum.

2 marks

SECTION 2-Question 2- continued

h. Sketch the graph of A against x.

2 marks

i. Find the width of the window, y, which will give a minimum area, A.

_____

_____

_____

1 mark
Total 16 marks

SECTION 2-continued
TURN OVER

Question 3
On 'Lee's Celebrity Quiz' show, each team has to answer a total of 20 questions.
There are 5 questions in each category and 4 categories are covered in each show.

The score X, represents the number of correctly answered questions out of a set of 5 questions in a particular category.

The probability of the celebrity team obtaining a particular score of X in the category of **Music** is shown in the table below:

x	0	1	2	3	4	5
Pr ($X = x$)	0.06	0.04	0.3	0.1	0.2	0.3

a. Find the probability that on the next Quiz Show the celebrity team will answer
i. all the Music questions correctly.

_____

_____

_____

ii. less than half of their Music questions correctly.

_____

_____

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_____

_____

_____

1 + 2 = 3 marks

SECTION 2-Question 3- continued

For the celebrity team, the probability of obtaining a particular value of x in the category of **Maths** is shown in the table below:

x	0	1	2	3	4	5
$\Pr(X = x)$	p	$3q$	q	p	p	p

b. If $p = 4q$, find the probability that on the next Quiz Show the celebrity team will answer three of their Maths questions correctly.

3 marks

c. Find the probability that the celebrity team scores a total of ten for their Maths and Music questions on the next Quiz Show.

1 mark

SECTION 2-Question 3- continued
TURN OVER

The Quiz Show has a regular team of celebrities who frequently appear on the show.
The probability of this regular team of celebrities appearing on the show is 0.85.
The teams are chosen at random.

d. Find the probability that the regular celebrity team will compete on the next six shows.

_____

_____

_____

_____

2 marks

e. For how many successive shows coming up will there be a better than even chance of having the regular celebrity team on the show?

_____

_____

_____

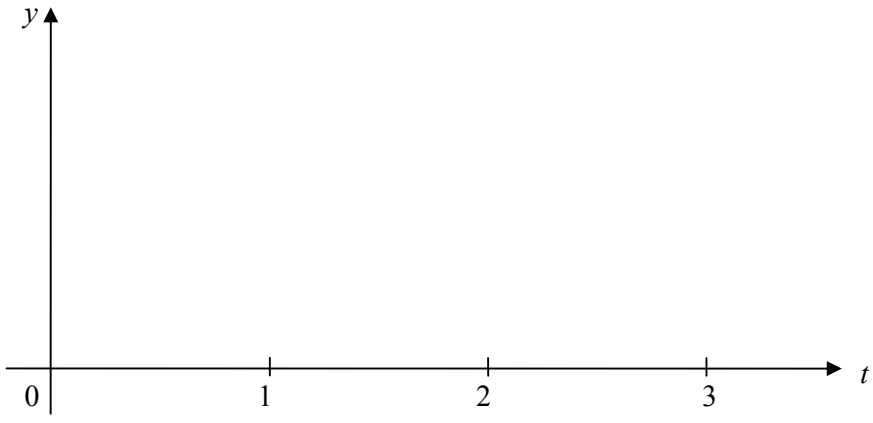
_____

2 marks

When Lee is on the Celebrity Quiz Show, the time, t hours, that he spends on the show is a continuous random variable with probability density function given by

$$f(t) = \begin{cases} t^3 - 9t^2 + 26t - 24 & , \quad 2 \le t \le 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

f. Sketch the graph of $f(t)$ on the axes below. Label any stationary points with their coordinates, correct to two decimal places.



2 marks

SECTION 2-Question 3- continued

g. What is the probability, correct to three decimal places, that Lee spends less than 150 minutes on the quiz show?

_____

_____

_____

_____

_____

_____

2 marks

h. Using calculus, find the expected time, to the nearest minute, that he spends on the quiz show.

_____

_____

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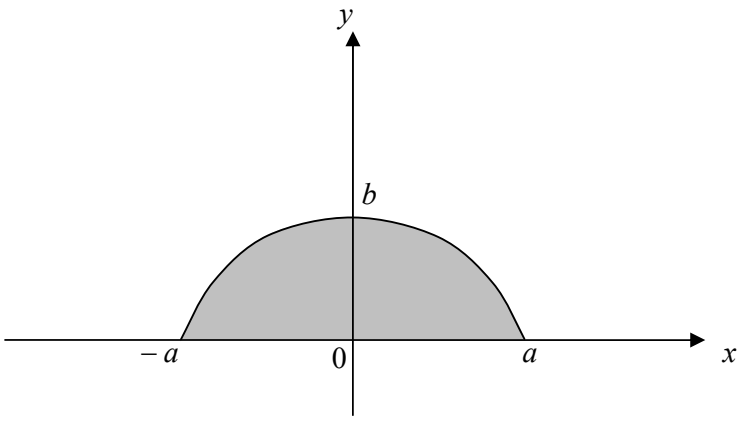
2 marks

Total 17 marks

SECTION 2-continued
TURN OVER

Question 4

The entrance to a shopping centre is given by the curve $y = 4 - \frac{1}{2}e^{\frac{x}{2}} - \frac{1}{2}e^{-\frac{x}{2}}$ as shown in the diagram below:



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a. The y-axis intercept of the graph represents the height of the entrance door is b.
Show that $b = 3$.

_____

_____

_____

1 mark

b. Find the exact value of a. Show your working.

_____

_____

_____

_____

_____

_____

_____

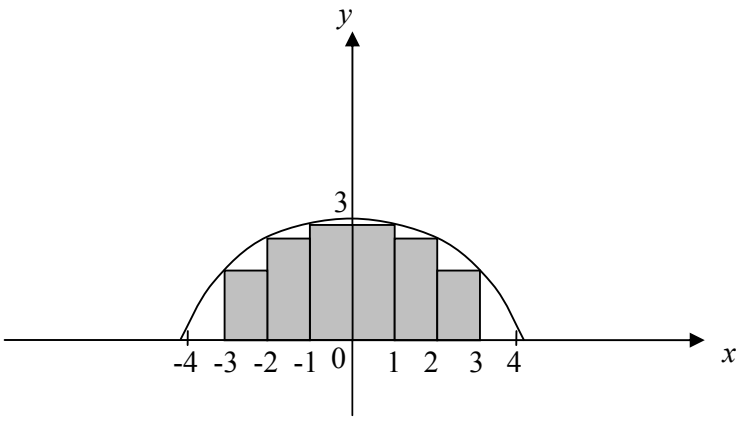
_____

4 marks

SECTION 2-Question 4- continued

c. To approximate the area of the door, assume the value of $a = 4$.

i. Use the curve of $y = 4 - \frac{1}{2}e^{\frac{x}{2}} - \frac{1}{2}e^{-\frac{x}{2}}$ to find the approximate area of the door correct to two decimal places, using rectangles of width 1 metre as shown in the diagram below:



ii. The shopping centre uses this to approximation to estimate the cost of painting the entrance door. If the cost is $35 per square metre, find the cost of painting the door.

2 + 1 = 3 marks

SECTION 2-Question 4- continued
TURN OVER

d. The area of the entrance door to the shopping centre is approximated with a parabola by using the points (0, 3), (4, 0), (–4, 0). Find the equation of this parabola.

3 marks

e. Use calculus to find the area enclosed by the parabola and the x-axis, giving answer correct to two decimal places.

2 marks

Total 13 marks

END OF QUESTION AND ANSWER BOOK

