MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

2007 Trial Examination

SOLUTIONS

Question 1

a.

$$f(g(x)) = f(x-5)$$
$$= 2(x-5)^2 - 1$$

A1

b.

$$\text{dom } f(g(x)) = R$$

A1

Question 2

a.

$$f(x) = \log_e(2x+1)$$
$$\text{inverse} \quad x = \log_e(2y+1)$$
$$e^x = 2y+1$$
$$2y = e^x - 1$$
$$y = \frac{1}{2}(e^x - 1)$$

M1

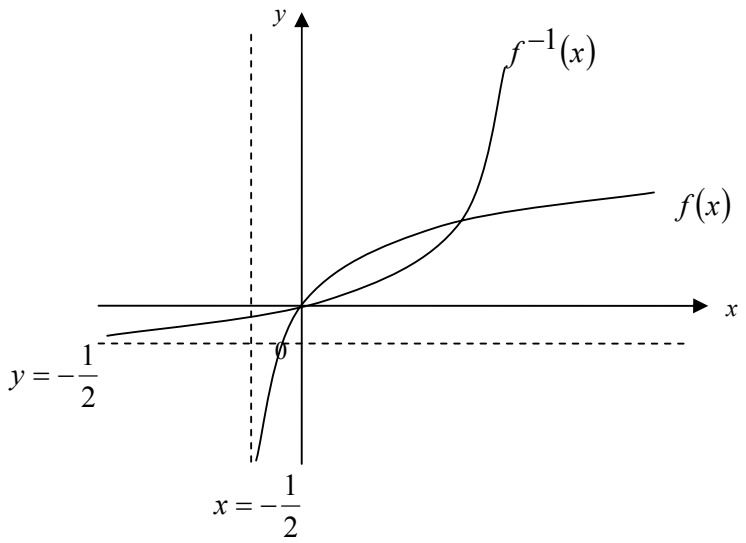
$$f^{-1}(x) = \frac{1}{2}(e^x - 1)$$

A1

b. range $f^{-1} = domf = \left(-\frac{1}{2}, \infty\right)$

A1

c.



M1 correct shape of graph of $f(x)$ with asymptotes labelled.

M1 correct shape of graph of $f^{-1}(x)$ with asymptotes labelled

Question 3

a. $f'(x)=(2x-3)e^{x^2-3}$

A1

b.

$$\begin{aligned} \text{when } x=1 \; f'(1) &= (2-3)e^{1-3} \\ &= -e^{-2} \\ &= -\frac{1}{e^2} \end{aligned}$$

A1

Question 4

$$\frac{dy}{dx}=\cos 2x-2x\sin 2x$$

A1

$$\begin{aligned} \text{when } x=\frac{\pi}{6} \quad \frac{dy}{dx} &= \cos 2\left(\frac{\pi}{6}\right)-2\left(\frac{\pi}{6}\right)\sin 2\left(\frac{\pi}{6}\right) \\ &= \cos\frac{\pi}{3}-\frac{\pi}{3}\sin\frac{\pi}{3} \\ &= \frac{1}{2}-\frac{\pi}{3}\frac{\sqrt{3}}{2} \\ &= \frac{1}{2}-\frac{\pi\sqrt{3}}{6} \end{aligned}$$

A1

Question 5

a. $y = -\dfrac{1}{2(x-3)} + 1$

M1 for 2 correct transformations
M1 for 2 correct transformations

b. Dom = R\{3}
Range = R\{1}

A1 must have both domain and range correct

Question 6

a. $f(x) = g(x)$

$$\frac{3}{2} = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$$

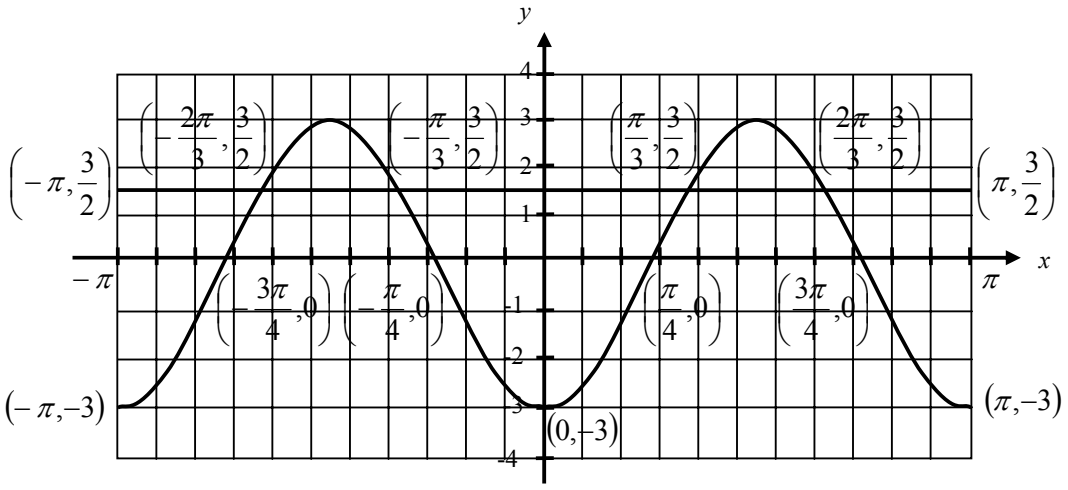
$$\frac{1}{2} = \sin\left(2\left(x - \frac{\pi}{4}\right)\right)$$ M1

$$2\left(x - \frac{\pi}{4}\right) = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$$

$$x - \frac{\pi}{4} = \frac{\pi}{12}, \frac{5\pi}{12}, -\frac{7\pi}{12}, -\frac{11\pi}{12}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{2\pi}{3}$$ A1

b.

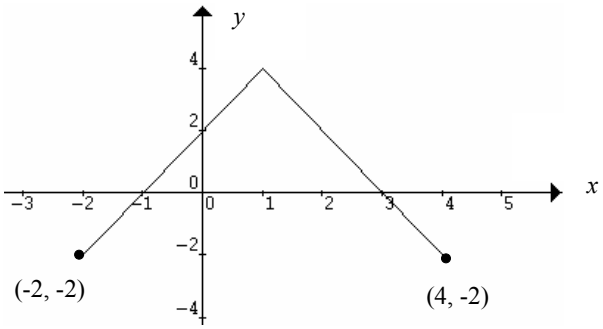


M1 correct shape of graphs

M1 intercepts labelled

M1 end points labelled

Question 7



M1 correct shape of graph

M1 stationary point and all intercepts labelled

M1 both end points labelled

Question 8

a.

$$
\begin{array}{r}
-x^2-2x+8 \\
\overline{)\,-x^3-3x^2+6x+8} \\
\underline{-x^3-x^2} \\
2x^2+6x \\
\underline{-2x^2-2x} \\
8x+8 \\
8x+8
\end{array}
$$

$b = 2,\ c = -8$ M1

$y = -(x+1)(x^2+2x-8)$ M1

b.

$-(x+1)(x-2)(x+4) = 0$

$x = -1,\ x = 2,\ x = -4$ A1

c.

$$A = 2\int_{-1}^{2}\left(-x^3-3x^2+6x+8\right)dx$$ M1

$$= 2\left[-\frac{x^4}{4}-x^3+3x^2+8x\right]_{-1}^{2}$$

$$= 2\left[\left(-4-8+12+16\right)-\left(-\frac{1}{4}+1+3-8\right)\right]$$ M1

$$= 2\left[16-\left(4\frac{1}{4}\right)\right]$$

$$= 40\frac{1}{2}\ \text{units squared}$$ A1

Question 9

a.

$$\int_{-a}^{a} k(a+x)dx = 1$$ M1

$$k\int_{-a}^{a} (a+x)dx = 1$$

$$\left[ax + \frac{x^2}{2}\right]_{-a}^{a} = \frac{1}{k}$$ M1

$$\left(a^2 + \frac{a^2}{2}\right) - \left(-a^2 + \frac{a^2}{2}\right) = \frac{1}{k}$$

$$2a^2 = \frac{1}{k}$$ A1

$$k = \frac{1}{2a^2}$$

b.

$$\mathrm{E(X)} = \int_{-a}^{a} xf(x)dx$$

$$1 = \int_{-a}^{a} x\frac{1}{2a^2}(a+x)dx$$

$$1 = \frac{1}{2a^2}\int_{-a}^{a} x(a+x)dx$$ M1

$$2a^2 = \int_{-a}^{a} x(a+x)dx$$

$$2a^2 = \int_{-a}^{a} (ax + x^2)dx$$

$$2a^2 = \left[\frac{ax^2}{2} + \frac{x^3}{3}\right]_{-a}^{a}$$

$$2a^2 = \left(\frac{a^3}{2} + \frac{a^3}{3}\right) - \left(\frac{a^3}{2} - \frac{a^3}{3}\right)$$ M1

$$2a^2 = \frac{2a^3}{3}$$

$$6a^2 = 2a^3$$

$$3a^2 = a^3$$

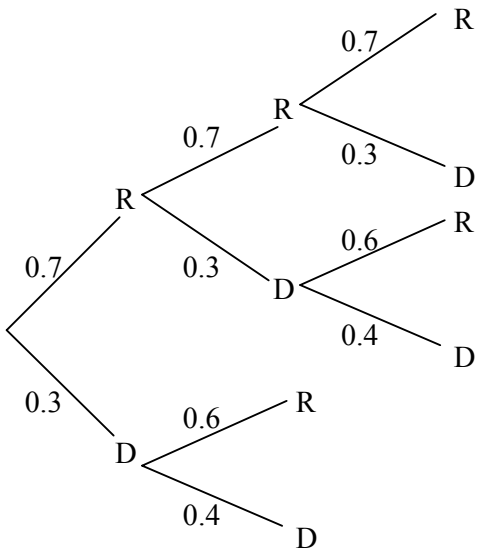
$$a^2(a-3) = 0$$

$$a = 0 \ (reject) \ or \ a = 3$$

$$\therefore a = 3$$

A1

Question 10



$\Pr(\text{if it is raining on Monday, it will be raining on Thursday}) = (0.7 \times 0.7 \times 0.7) + (0.7 \times 0.3 \times 0.6)$ M1

$= 0.343 + 0.126$

$= 0.469$ A1

Question 11

a.

$$\Pr(X > 70) = \Pr\left(z > \frac{70-50}{10}\right)$$
$$= \Pr(z > 2)$$
$$= 1 - \Pr(z > 2)$$
$$= 1 - 0.98$$
$$= 0.02$$

A1

b.

$$Pr(X < 30 \mid X < 70) = \frac{Pr(X < 30 \cap X < 70)}{X < 70}$$ M1

$$= \frac{Pr(X < 30)}{Pr(X < 70)}$$
$$= \frac{Pr(z < -2)}{Pr(z < 2)}$$ M1
$$= \frac{Pr(z > 2)}{0.98}$$
$$= \frac{1 - Pr(z < 2)}{0.98}$$
$$= \frac{0.02}{0.98}$$
$$= \frac{1}{49}$$

A1

