THE SCHOOL FOR EXCELLENCE (TSFX)
UNIT 4 MATHEMATICAL METHODS 2007

WRITTEN EXAMINATION 2

Reading Time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOKLET

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

This examination has two sections: Section 1 (multiple-choice questions) and Section 2 (extended-answer questions).

You must complete both parts in the time allocated. When you have completed one part continue immediately to the other part.

Students are permitted to bring into the examination rooms: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory **DOES NOT** need to be cleared) and, if desired, one scientific calculator.

Students are **NOT** permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Students are **NOT** permitted to bring mobile phones and/or any electronic communication devices into the examination room.

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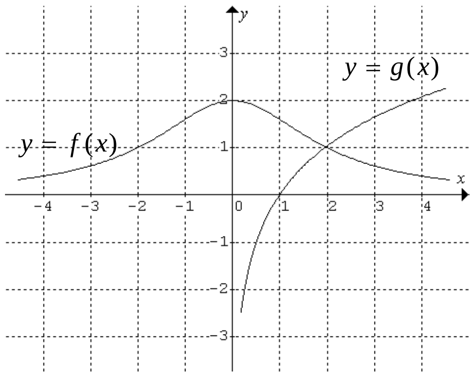
SECTION 1 – MULTIPLE CHOICE QUESTIONS

Instructions for Section 1
Answer all questions in this part on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers. You should attempt every question.
No marks will be given if more than one answer is completed for any question.

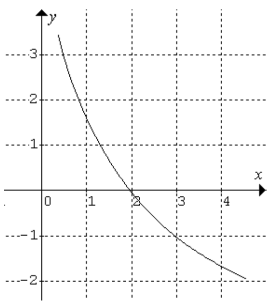
QUESTION 1

The graphs of $y = f(x)$ and $y = g(x)$ are shown below.



The graph of $y = f(x) - g(x)$ is

A.



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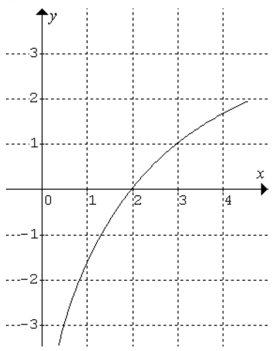

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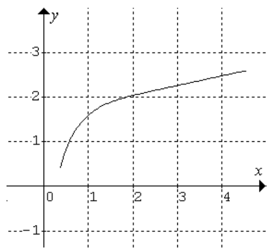
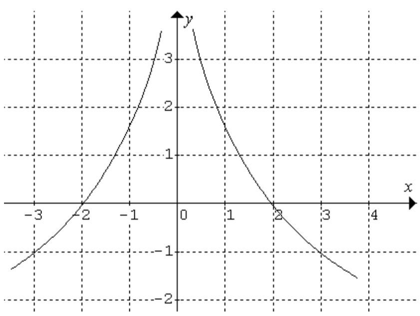
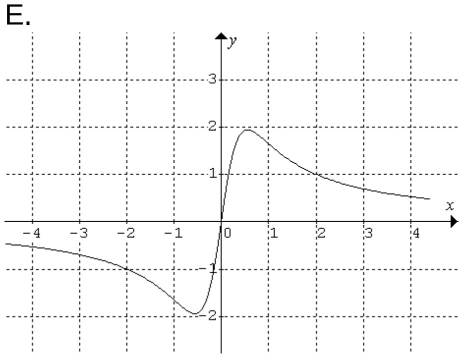


D.



E.



QUESTION 2

Let $f(x) = a\sin(2x)$ and $g(x) = b$, where $0 < x < \frac{3\pi}{2}$ and a and b are positive integers.

Which of the following statements is **not** true?

A. If $b > a$ there are no real solutions to the equation $f(x) = g(x)$.

B. If $0 < b < a$ there are 4 solutions to the equation $f(x) = g(x)$.

C. If $0 < b < a$ there are 4 solutions to the equation $f(x) = -g(x)$.

D. If $a = b$ there is 1 solution to the equation $f(x) = -g(x)$.

E. If $a = b$ there are 2 solutions to the equation $f(x) = g(x)$.

QUESTION 3

The solution to $|2x-3| \geq x^2 - 2$ is

A. $x = 1$

B. $\{x : -1-\sqrt{6} \leq x \leq -1+\sqrt{6}\}$

C. $\{1\} \cup \{x : -1-\sqrt{6} \leq x \leq -1+\sqrt{6}\}$

D. 1, $-1+\sqrt{6}$ or $-1-\sqrt{6}$

E. $x \geq -1$

QUESTION 4

The range of the function $f : [-3, 2] \rightarrow R,\ f(x) = 3-(x+1)^2$ is

A. $[-6, -1]$

B. $[-\infty, 3]$

C. $[-6, 7]$

D. $[-3, -1]$

E. $[-6, 3]$

QUESTION 5

The function $f:[0,a]\to R$, $f(x)=3\cos(2x)$ will have an inverse if

A. $a \le \frac{\pi}{2}$

B. $a \le \pi$

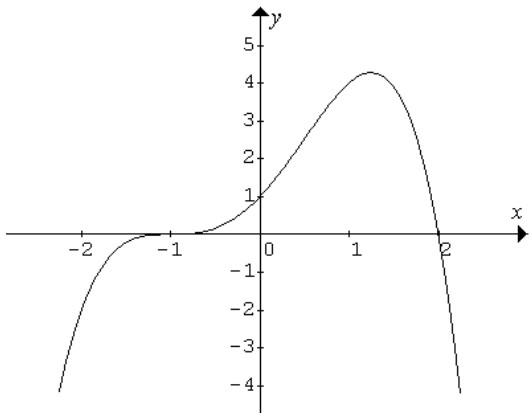
C. $a \ge \frac{\pi}{2}$

D. $a \le 3$

E. $a \le 2.5$

QUESTION 6

Part of the graph of $y=a(x+b)^3(x+c)$ is shown below.



The values of a, b and c could be

	a	b	c
A	$-\frac{1}{2}$	1	-2
B.	$\frac{1}{2}$	-1	2
C.	-2	1	-2
D.	$-\frac{1}{2}$	1	2
E.	1	1	-2

QUESTION 7

The function, f is described by the rule: $f(x)=\frac{x-1}{\sqrt{x}}$.

Given $f(1)=0$, the approximate change in f as x decreases from 1 to 0.9 is

A. 0.2

B. 0.1

C -0.1

D. 0.9

E. 1.1

QUESTION 8

A sine function f has an amplitude of 4 and a period of $\frac{1}{8}$. The rule for f could be

A. $f(t)=2\sin\left(\frac{\pi t}{4}\right)$

B. $f(t)=4\sin(16\pi t)$

C. $f(t)=4\sin(8\pi t)$

D. $f(t)=4\sin\left(\frac{\pi t}{4}\right)$

E. $f(t)=2\sin\left(\frac{\pi t}{8}\right)$

QUESTION 9

The function $f(x)=x+\sqrt{x+a}$ has

A. a stationary point at $x=\frac{1}{4}-a$

B. a stationary point at $x=a-\frac{1}{4}$

C. stationary points at $x=a-\frac{1}{4}$ and $x=\frac{1}{4}-a$

D. no stationary points

E. a stationary point at $x=-a$

QUESTION 10

If $\log_e x = \log_e(x-1) + b$ then x is equal to

A. $\dfrac{e^b}{1-e^b}$

B. $\dfrac{1}{e^b - 1}$

C. $\log_e \dfrac{x}{x-1}$

D. $\dfrac{1}{1-e^b}$

E. $\dfrac{e^b}{e^b - 1}$

QUESTION 11

The function f has the rule $f(x) = a\log_e(bx-c)$ where a, b and c are positive real constants. A possible domain for f is

A. $\left[\dfrac{a}{b}, \infty\right)$

B. $\left(\dfrac{b}{c}, \infty\right)$

C. $\left(-\infty, \dfrac{b}{c}\right)$

D. $\left(\dfrac{c}{b}, \infty\right)$

E. $(-\infty, a]$

QUESTION 12

The graph of $y = 3 + e^{2-x}$ is obtained from the graph of $y = e^x$ by

A. a reflection in the y-axis, a translation by 2 units left and a translation 3 units up.

B. a reflection in the y-axis, a translation by 2 units right and a translation 3 units up.

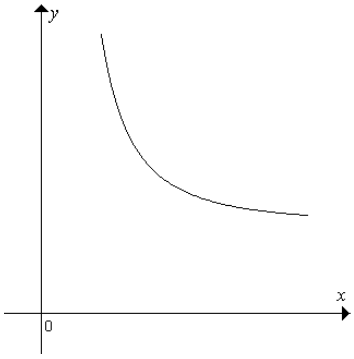
C. a translation by 2 units right and a reflection in the x-axis.

D. reflection in the x-axis and dilation of 3 from the x-axis.

E. a reflection in the x-axis, a translation by 3 units up and a translation of 2 units right.

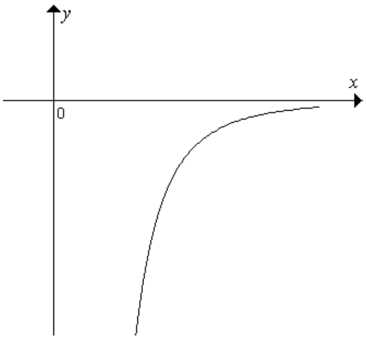
QUESTION 13

The graph of the function f , with rule $y = f(x)$ is shown below.

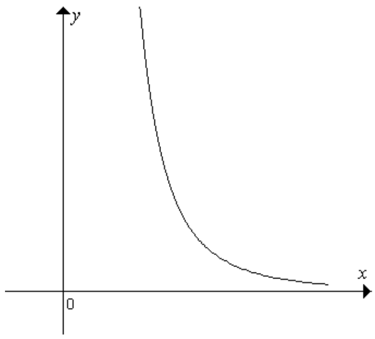


Which one of the following could be the graph of $y = f'(x)$?

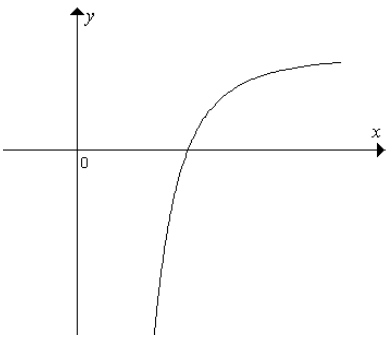
A.



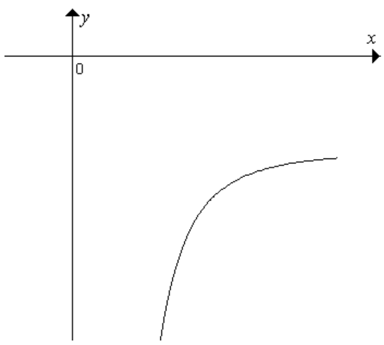
B.



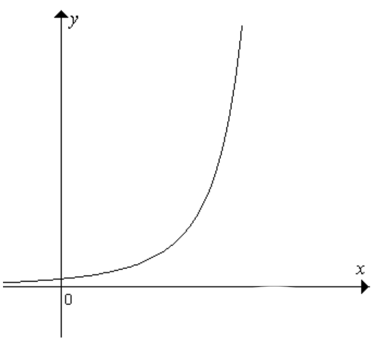
C.



D.



E.



QUESTION 14

For the function, $y = x^n e^{3x-n}$, $n \in R^+$, the gradient of the tangent at the point $(1, e^{3-n})$ is equal to

A. e^{3-n}

B. $4e^2$

C. $(n-1)(n+3)e^{3-n}$

D. $(n+3)e^{3-n}$

E. $(n+1)e^{3-n}$

QUESTION 15

The derivative of $\log_e \sqrt{2x^2+1}$ with respect to x is

A. $\dfrac{x}{\sqrt{2x^2+1}}$

B. $\dfrac{4x}{2x^2+1}$

C. $\dfrac{4x}{\sqrt{2x^2+1}}$

D. $\dfrac{2x}{2x^2+1}$

E. $\dfrac{\sqrt{2x^2+1}}{4x}$

QUESTION 16

If $\int_a^b f(x)\,dx = 1$ then $\int_a^b 1-3f(x)\,dx$ is equal to

A. $b-a-3$

B. $b-a+3$

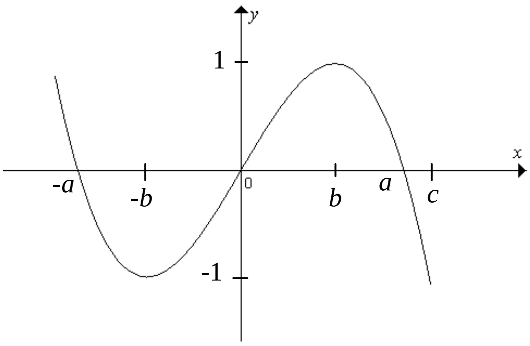
C. $a-b+3$

D. -2

E. -3

QUESTION 17

The graph of the function f with rule $y = f(x)$ is shown below:



The total area bounded by the curve $y = f(x)$ and the x-axis on the interval $[-a, c]$ could be given by

A. $2\int_0^a f(x)\,dx + \int_a^c f(x)\,dx$

B. $4\int_0^a f(x)\,dx - \int_a^c f(x)\,dx$

C. $2\int_0^a f(x)\,dx + \int_c^a f(x)\,dx$

D. $4\int_0^b f(x)\,dx + \int_a^c f(x)\,dx$

E. $2\int_{-a}^a f(x)\,dx - \int_a^c f(x)\,dx$

QUESTION 18

Given that the derivative of $x\log_e(3x)$ is equal to $1+\log_e(3x)$, then $\int 2\log_e(3x)\,dx$ is equal to

A. $x\log_e(3x) - c$

B. $2x\log_e(3x-1) - c$

C. $2x\log_e(3x) - c$

D. $2x(\log_e(3x)-1) - c$

E. $x\log_e(3x) - 1 - c$

QUESTION 19

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \dfrac{3\sqrt{x}}{2}, & \text{for } 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

The value of k such that $\Pr(X > k) = \dfrac{7}{8}$ is

A. $\dfrac{7}{8}$

B. $\dfrac{1}{8}$

C. $\dfrac{1}{4}$

D. $\dfrac{3}{2}$

E. $\dfrac{2}{3}$

QUESTION 20

A company which produces MP3 players knows from experience that 98% of their items are not defective. The probability that in a random sample of n there is more than one defective MP3 player is

A. $1 - 0.02^n$

B. $1 - 0.98^n - n(0.02)(0.98)^{n-1}$

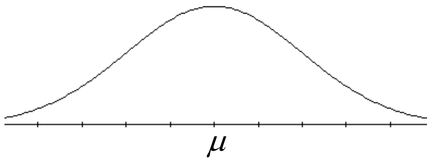
C. $1 - 0.02n - n(0.98)(0.02)^{n-1}$

D. 0.98^n

E. $1 - 0.98^n$

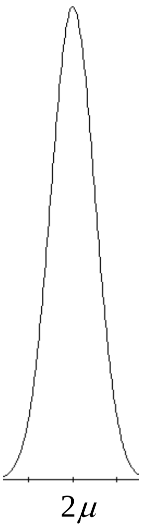
QUESTION 21

A normal distribution where the mean is μ and the standard deviation is σ is shown below

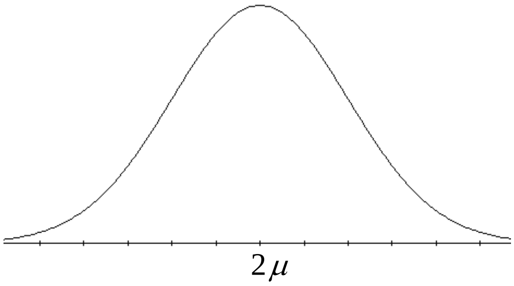


Using the same scale a normal distribution with mean 2μ and standard $\frac{\sigma}{2}$ would look most like

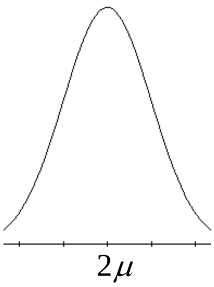
A.



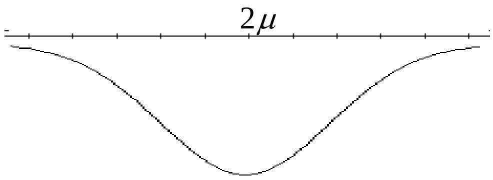
B.



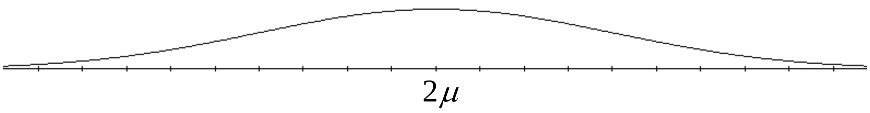
C.



D.



E.



QUESTION 22
A basketballer finds on average that he is successful in shooting a goal in 4 out of every ten shots. The minimum number of shots the basketballer must make so that the probability he shoots a goal at least once is at least 0.9 is:

A. 4

B. 9

C. 8

D. 4.5

E. 5

SECTION 2 – EXTENDED ANSWER QUESTIONS

Instructions For Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1

Let $f : R \to R$, where $f(x) = e^{2x}$ and $g : D_g \to R$, where $g(x) = \log_e |x-1|$ and D_g is the maximal domain of g.

a. (i) State the rule for $f(g(x))$.

1 mark

(ii) Find $(fog)'(-1)$.

1 mark

b. Show that $g(f(x))$ does not exist.

1 mark

By suitably restricting the domain of f , a new function f_1 can be defined such that $g\left(f_1(x)\right)$ exists.

c. (i) Find the maximal domain of f_1 that includes $x = 2$.

1 mark

(ii) Show why or why not the domain of f can be restricted to define a new function f_2 such that $g\left(f_2(x)\right)$ exists and the maximal domain of f_2 includes $x = -2$.

1 mark

d. (i) Find, correct to one decimal place, the coordinates of the point of intersection of $f\left(g(x)\right)$ and $g\left(f_1(x)\right)$ that lies to the right of the line $x = 1$.

1 mark

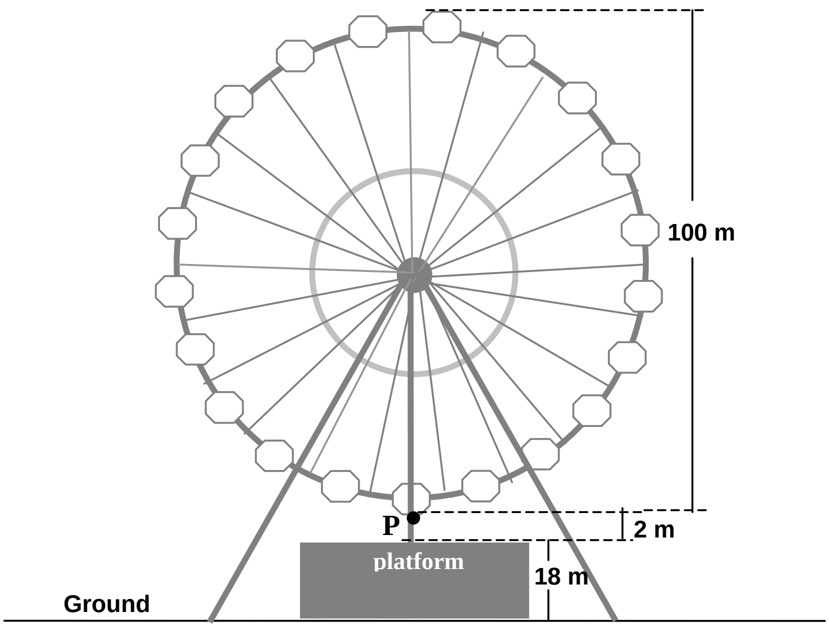
(ii) Find, correct to one decimal place, the area bounded by the curves $y = f\left(g(x)\right)$ and $y = g\left(f_1(x)\right)$ and the lines $x = 1$ and $x = 5$.

3 marks

Total Marks = 9

QUESTION 2

The Southern Star Observation Wheel is currently been constructed at the docklands.
The circular ferris wheel will have a diameter of 100 metres and rise 120 metres above the ground at its highest point. The observation wheel will have 21 air-conditioned glass cabins evenly spaced out. The wheel will be accessed via a platform 18 metres above ground level. It will rotate at a constant speed in an anticlockwise direction. Each ride lasts **36 minutes**. During the time it is in operation the wheel will not stop at any time.



The height above ground level of a point, P on the floor of one of the cabins will be at its lowest point at the start of the ride. The height, h (measured in metres) of the point P after t minutes is given by the equation

$$h = b + a\sin c(t + d)\,.$$

a. (i) Show that $a = 50$.

____

____

____

1 mark

(ii) Show that $b = 70$.

1 mark

b. If the point P takes 80 seconds to complete one full rotation, show that $c = \frac{3\pi}{2}$.

1 mark

c. The point P is initially at its lowest point above the ground. Show that a possible value for d is 1 .

2 marks

d. How many times during the 36 minute ride will the point P reach its maximum height?

1 mark

e. Find the exact value of t for which the point P first reaches a height of at least 95 metres above ground level.

2 marks

f. Find, correct to the nearest second, the time during one rotation that the point P is at least 95 metres above ground level.

2 marks

The average person feels sick when the magnitude of their rate of change of height with respect to time is greater than 200 m/s for more than 20 seconds at a time.

g. Using calculus, determine whether or not the average person will feel sick when riding the Southern Star Observation Wheel.

3 marks

The designers of the Southern Star Observation Wheel decide to change the value of c so that the magnitude of the rate of change of height with respect to time will never be greater than 200 m/s for more than 10 seconds at a time. However, they also want the Wheel to turn as quickly as possible.

h. Find, correct to three decimal places, the smallest positive value of c that can be used.

3 marks

Total Marks = 16

Question 3

Equine influenza is a major viral disease that causes flu like symptoms in horses.
The virus is highly contagious and can be spread rapidly from horse to horse, particularly if (as in Australia) they have not previously been exposed to the virus. In Sydney, equine influenza has led to the closing of the spring race carnival.

The number A (measured in hundreds) of new influenza cases after t days in a population of 1200 horses **not** previously exposed to the virus can be modelled by the equation

$$A = \frac{at}{3t+4},\ 0 \le t \le 14$$

where a is a positive constant.

The number B (measured in hundreds) of new influenza cases after t days in a population of 1200 horses previously **exposed** to the virus can be modelled by the equation

$$B = \frac{12t}{t+3},\ 0 \le t \le 14$$

a. After 4 days, 900 horses **not** previously exposed to the virus will have influenza.

(i) Show that $a = 36$.

1 mark

(ii) Show that $A = 12\left(1 - \frac{4}{3t+4}\right)$.

1 mark

b. Show that the difference in new influenza cases between those previously not exposed and those previously exposed to the virus is given by the equation

$$D = 12\left(\frac{3}{t+3} - \frac{4}{3t+4}\right)$$

1 mark

c. Write a formula for the rate at which the difference in new influenza cases between those previously not exposed and those exposed to the virus is changing with time.

2 marks

d. **Hence** find the maximum difference in new influenza cases between those previously unexposed and those exposed to the virus.

3 marks

A drug can be administered to vaccinate horses against equine flu. The concentration, $C(t)$, in milligrams per litre of the drug in the bloodstream of a horse, t hours after it is administered can be modelled by

$$C(t) = 4t(t+1)e^{-\frac{t}{k}}$$

where k is a positive constant.

e. If the drug reaches its maximum concentration in the bloodstream after 4 hours, use calculus to show that $k = \dfrac{20}{9}$.

3 marks

f. According to this model, the derivative gives a measure of the rate of absorption of the drug into the bloodstream. Using $k = \frac{20}{9}$, find using calculus, the value of t when the rate of absorption is greatest. State your answer correct to 2 decimal places.

3 marks

Total Marks = 14

QUESTION 4

On any one day, thousands of passengers wait for trains at Southern Cross Station. Trains can arrive on time or be late. Assume that the number of minutes, T, that a train is late by is a normally distributed random variable with a mean of 10 and a standard deviation of 2. The table below shows the number of minutes a train may be late by and whether a particular worker will consequently arrive on time or be late to work.

Time Train is Late by	Arrival at Work
More than 15 minutes	Late
Between 5 and 15 minutes	On time
Less than 5 minutes	Early

a. Find correct to four decimal places:

(i) the probability that a train is between 5 and 15 minutes late.

1 mark

(ii) the probability that a train is more than 15 minutes late.

1 mark

(iii) the probability that if a train was less than 15 minutes late, it was more than 5 minutes late.

2 marks

b. If the probability of a train arriving more than k minutes late is 0.0095, find the value of k correct to the nearest minute.

2 marks

c. At 8 a.m. on a Monday morning there are 10 trains due to arrive at Southern Cross Station. The number of minutes each train is late is independent of each other. Find correct to four decimal places:

i. the probability that exactly 2 trains are more than 15 minutes late.

1 mark

ii. the probability that the first train is more than 15 minutes late and the rest are not.

1 mark

d. At 8 a.m. on a Monday morning 200 trains pass through Southern Cross Station. Find the number of trains expected to arrive more than 15 minutes late.

1 mark

The performance of the company which manages the trains was found to be unsatisfactory. In the following month the company therefore tried to improve its performance. The number of minutes that trains are late by is still normally distributed random variable but the mean and standard deviation have changed.

e. Find, correct to four decimal places, the mean and standard deviation of the new distribution if the probability that a train is late by more than 5 minutes is 0.0095 and the probability that a train is late by less than 4 minutes is 0.205.

4 marks

f. On any given day, the number of minutes that Elaine will be late to work to due a train being late is continuous random variable with probability density function given by

$$f(x) = \begin{cases} 0.04x, & 0 \le x \le 5 \\ 0.04(10 - x), & 5 < x \le 10 \\ 0, & x < 0 \text{ and } x > 10 \end{cases}$$

i. Find the exact probability that Elaine is late for work by more than 3 minutes.

2 marks

ii. Use calculus to find the exact mean number of minutes that Elaine is late to work.

2 marks

iii. Find the exact value of k such that $\Pr(X > k) = 0.8$.

2 marks

Total Marks = 19