Trial Examination 2007

VCE Mathematical Methods Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	4	4	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 20 pages with a detachable sheet of miscellaneous formulas in the centrefold.
Answer sheet for multiple-choice questions.

Instructions

Detach the formula sheet from the centre of this book during reading time.

Write **your name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The graph with the equation $y = x^2$ is transformed into the graph with equation $y = (2x + 4)^2$.
The resulting effect would be

A. a translation of 2 units to the left.
B. a dilation by a factor of 2 from the *x*-axis.
C. a dilation by a factor of 2 from the *x*-axis and a translation of 4 units to the left.
D. a dilation by a factor of 4 from the *x*-axis and a translation of 2 units to the left.
E. a dilation by a factor of 4 from the *x*-axis and a translation of 4 units to the left.

Question 2

Consider the functions *f*, *g*, *j* and *k* with the following rules.

$$f(x) = \sin\left(\frac{x}{2} + \pi\right)$$

$$g(x) = |\cos(x)|$$

$$j(x) = \sin^2(x)$$

$$k(x) = 1 - \pi\cos\left(2\left(\frac{\pi}{8} - x\right)\right)$$

Which **one** of the following options identifies those of the above functions with a period of π?

A. *f* and *k* only
B. *g* and *j* only
C. *k* only
D. *f*, *j* and *k*
E. *g*, *j* and *k*

Question 3

The range of the function $f: (-1, 0) \cup (0, 1] \to R,\ f(x) = \frac{2}{x}$ is

A. $(-2, 2]$
B. $[-2, 2)$
C. $R\backslash\{0\}$
D. $R\backslash(-2, 2]$
E. $R\backslash[-2, 2)$

 TEVMMU34EX2_QA_07.FM

Question 4

Given that $m > n > 0$, $\log_e\left(\dfrac{1}{m^2 - n^2}\right)$ is equal to

A. $-2\log_e(m) + 2\log_e(n)$

B. $-\log_e(m - n) - \log_e(m + n)$

C. $-\log_e(2m) + \log_e(2n)$

D. $\log_e(m - n) + \log_e(m + n)$

E. $-2\log_e\left(\dfrac{m}{n}\right)$

Question 5

The function $f:(-\infty, a) \to R$ with rule $f(x) = 2e^{x^2}$ will have an inverse function if

A. $a < 0$

B. $a > 0$

C. $a < 2$

D. $a > 2$

E. $-2 < a < 2$

Question 6

The function g with rule $g(x) = \dfrac{1}{\sqrt{x^2 - 4}}$ has a maximal domain of

A. $[2, \infty)$

B. $[-2, 2]$

C. $(-2, 2)$

D. $R\backslash[-2, 2]$

E. $R\backslash(-2, 2)$

Question 7

The graph of the function with rule $y = \log_e|x|$ is transformed as follows.

- A dilation by a factor of $\frac{1}{3}$ from the y-axis;
- a translation of –3 units parallel to the x-axis; and then
- a reflection in the y-axis.

The rule of the function corresponding to the transformed graph is

A. $y = -\log_e|9 - 3x|$

B. $y = \log_e|9 - 3x|$

C. $y = -\log_e|3 - 3x|$

D. $y = -\log_e\left|1 - \frac{1}{3}x\right|$

E. $y = \log_e\left|1 - \frac{1}{3}x\right|$

Question 8

If $f(x) = e^{\frac{1}{2}x}$, then $\log_e(f'(2))$ is equal to

A. $-\log_e(2)$

B. $-\log_e\left(\frac{1}{2}\right)$

C. $1 - \log_e(2)$

D. $1 + \log_e(2)$

E. 1

Question 9

Given that $p(x) = -x^9 - 2x^7 + ax^2 + 1$ is exactly divisible by $2x + 2$, the value of a is equal to

A. 0

B. 2

C. -4

D. $-\frac{769}{4}$

E. $\frac{769}{4}$

 TEVMMU34EX2_QA_07.FM

Question 10

The values of x for which $|3x - 1| \geq |x + 3|$ are

A. $x \leq -\frac{1}{2}$ or $x \geq 2$

B. $x \leq -\frac{1}{2}$

C. $x \geq 2$

D. $x \leq 2$

E. $-\frac{1}{2} \leq x \leq 2$

Question 11

A fair die is rolled 8 times and the number showing uppermost recorded as either even or odd. The probability, correct to four decimal places, of obtaining no more than the expected number of even results is

A. 0.2188
B. 0.2734
C. 0.3663
D. 0.6367
E. 0.8555

Question 12

The function $f: (-\infty, 2) \to R$ has the rule $f(x) = \log_e \sqrt{-x + 2}$.
Its inverse function is given by

A. $f^{-1}: R \to R$ where $f^{-1}(x) = e^{\sqrt{-x+2}}$.

B. $f^{-1}: R \to R$ where $f^{-1}(x) = e^{(x-2)^2}$.

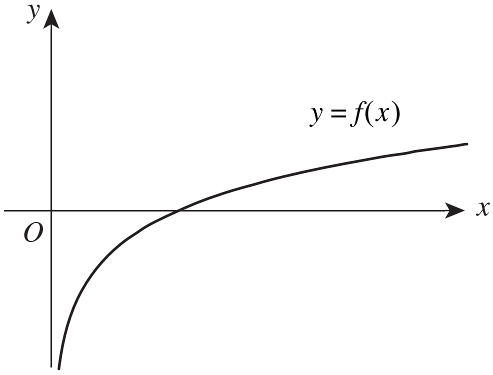
C. $f^{-1}: R \to R$ where $f^{-1}(x) = -e^{2x} + 2$.

D. $f^{-1}: (-\infty, 2) \to R$ where $f^{-1}(x) = -e^{2x} + 2$.

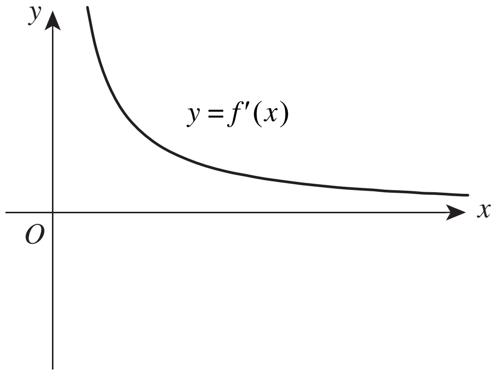
E. $f^{-1}: (-\infty, 2) \to R$ where $f^{-1}(x) = e^{2x} - 2$.

Question 13

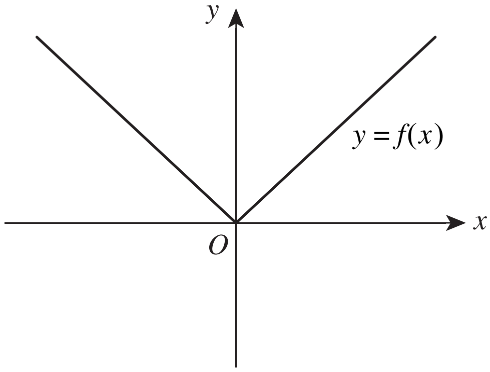
I



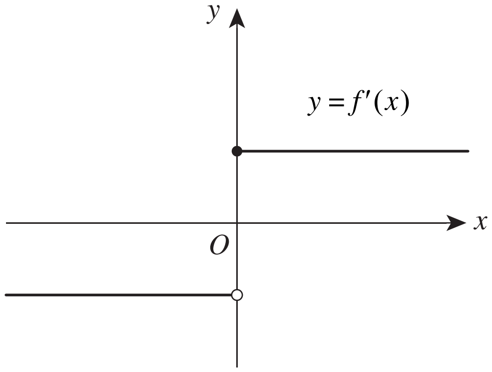
and



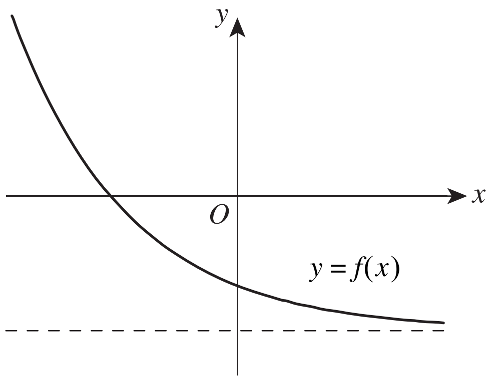
II



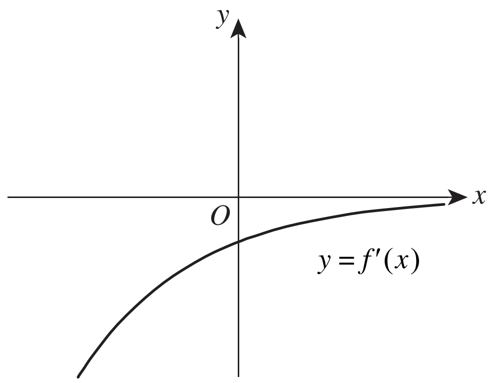
and



III



and



Which of the above pairs of graphs could represent the graph of a function and the graph of its derivative?

A. I only

B. III only

C. II and III

D. I and III

E. I, II and III

Question 14

A function f has the following properties.

- $f'(x)$ exists for all x.
- $f(x) < 0$ for all x.

Which **one** of the following is **not** necessarily true?

A. $\int_{-2}^{2} f(x)dx < 0$

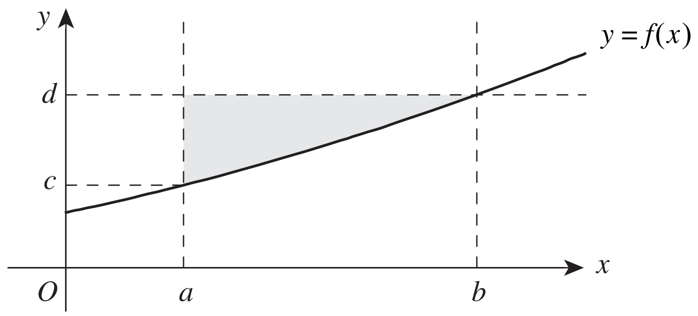
B. $\int_{-2}^{2} 4f(x)dx = 4\int_{-2}^{2} f(x)dx$

C. $\int_{-2}^{2} f(x)dx = 2\int_{0}^{2} f(x)dx$

D. $\int_{-2}^{2} f(x)dx = -\int_{2}^{-2} f(x)dx$

E. $\int_{-2}^{2} f(x)dx = \int_{-2}^{0} f(x)dx + \int_{0}^{2} f(x)dx$

Question 15



Which **one** of the following expressions represents the area of the shaded region in the diagram above?

A. $\int_{a}^{b} (f(x) - d)dx$

B. $d(b - a) - \int_{a}^{b} f(x)dx$

C. $(b - a)(f(b) - f(a))$

D. $\int_{a}^{b} f(x)dx - f(a) \times (b - a)$

E. $\int_{a}^{b} (f(x) - c(b - a))dx$

Question 16

Given that $\frac{1}{(x-1)(x+2)} = \frac{1}{3(x-1)} - \frac{1}{3(x+2)}$, $\int_2^3 \frac{3}{(x-1)(x+2)}dx$ is equal to

A. $\log_e\left(\frac{1}{2}\right)$

B. $\log_e\left(\frac{5}{8}\right)$

C. $\log_e\left(\frac{5}{2}\right)$

D. $\log_e\left(\frac{8}{5}\right)$

E. $\log_e\left(\frac{2}{5}\right)$

Question 17

Let u, v and w be non-zero, differentiable functions of x. Also let $u' = \frac{du}{dx}$, $v' = \frac{dv}{dx}$ and $w' = \frac{dw}{dx}$.

The derivative of $\frac{uv}{w}$ is

A. $\frac{vwu' + uwv' - uvw'}{w^2}$

B. $\frac{vwu' - uwv' - uvw'}{w^2}$

C. $\frac{uv' - vu'}{w^2}$

D. $\frac{uv' + vu'}{w^2}$

E. $\frac{vwu' + uwv' + uvw'}{w^2}$

Question 18

The discrete random variable X has the following distribution.

x	0	1	2	4
$\Pr(X = x)$	p	$2p$	p	0.2

The mean of X is

A. 0.2
B. 1.0
C. 1.4
D. 1.6
E. 1.8

Question 19

The function f is a probability density function with

$$f(x) = \begin{cases} A\sin\left(\frac{\pi}{6}x\right), & 0 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}$$

The value of A is

A. $\frac{\pi}{24}$

B. $\frac{\pi}{12}$

C. $\frac{\pi}{6}$

D. π

E. 6π

Question 20

Using the linear approximation $f(x + h) \approx f(x) + hf'(x)$ to find an approximate value for $\frac{1}{\sqrt{48.5}}$ gives an answer of

A. $\frac{1}{7}\left(1 + \frac{1}{49}\right)$

B. $\frac{1}{7}\left(1 - \frac{1}{49}\right)$

C. $\frac{1}{4 \times 7^3}$

D. $\frac{1}{7}\left(1 - \frac{1}{4 \times 49}\right)$

E. $\frac{1}{7}\left(1 + \frac{1}{4 \times 49}\right)$

Question 21

Better Brew beer is sold in bottles labelled as containing 500 mL. The amount of beer poured into each bottle in the bottle-filling process is normally distributed with a standard deviation of 2 mL. Each bottle can hold no more than 510 mL without spillage occurring. If only 5% of the bottles are over-filled so that spillage occurs, what is the mean amount of beer (correct to the nearest mL) poured into a bottle?

A. 504
B. 505
C. 506
D. 507
E. 508

Question 22

Let the function $f: R \to R$ have the rule $f(x) = a^x$, where $a > 1$. Let g be another continuous function with domain R. For all real values of x, the derivative of $g(f(x))$ will be equal to

A. $g'(a^x)$

B. $a^x g'(x)$

C. $\log_e(a) g'(a^x)$

D. $\log_e(a) g'(x)$

E. $a^x \log_e(a) g'(a^x)$

END OF SECTION 1

Neap:

Trial Examination 2007

VCE Mathematical Methods Units 3 & 4

Written Examination 2

Formula Sheet

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

MATHEMATICAL METHODS FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin(A)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{Var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \Sigma x p(x)$	$\sigma^2 = \Sigma(x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\ dx$	$\mu = \int_{-\infty}^{\infty} x f(x)\ dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\ dx$

END OF FORMULA SHEET

TEVMMU34EX2_FS_07.FM

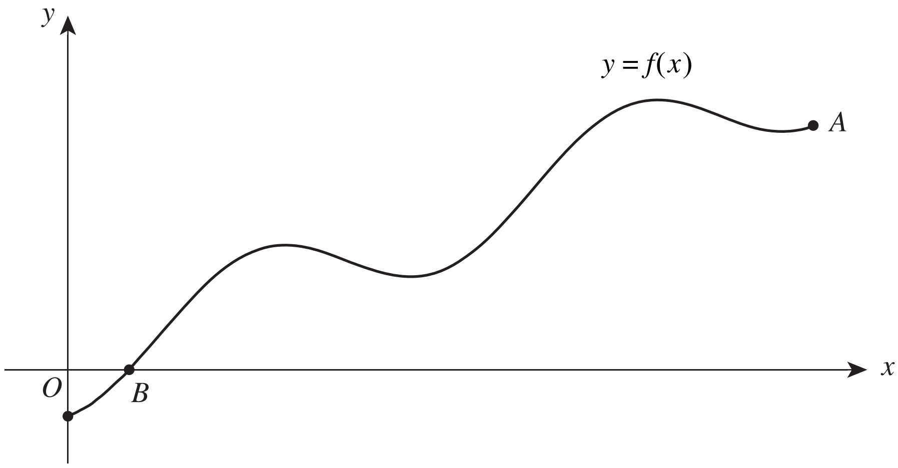
SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

The graph of the function $f:\ [0, 2\pi] \to R,\ f(x) = x - \cos(2x)$ is shown below.



a. **i.** State the exact coordinates of point A.

ii. Find the coordinates of point B, correct to 3 decimal places.

1 + 1 = 2 marks

b. Find the derivative of f and hence find the **exact** x-coordinates of the local maximum turning points.

3 marks

c. Consider the function $g : [0, 2\pi] \rightarrow R,\ g(x) = x + 1$.

i. Solve $\{x : f(x) = g(x)\}$, giving your answer in exact form.

ii. Show that $g(x)$ is tangential to $f(x)$.

iii. Another function which is tangential to *f* has the rule $y = x + c$. Write down the value of *c*.

1 + 3 + 1 = 5 marks
Total 10 marks

Question 2

Jane Blonde is a secret service agent for the Military Intelligence Agency (MIA). Each month she is required to spend a total of 16 hours at the MIA's secret training facility on Swan Island.

a. The amount of time that Jane spends at the training facility on each visit is normally distributed with a mean of 4 hours and a standard deviation of 1 hour.

i. What percentage of Jane's visits to the training facility last for at least 3 hours? Give your answer correct to one decimal place.

ii. Use your answer to **i.** to write an expression for the probability that Jane has spent at least 3 hours at the training facility on each of her last five visits. Hence find this probability correct to three decimal places.

1 + 2 = 3 marks

b. The amount of time it takes Jane to complete security clearance is also normally distributed, with a standard deviation of 6 minutes. There is a 10% chance that it will take Jane more than half an hour to complete security clearance. To the nearest second, find the amount of time than Jane can expect to spend completing a security clearance.

3 marks

c. Jane is an expert markswoman. When she is shooting at a practice target, the probability of any one of her shots hitting the target is 0.85. The magazine of Jane's handgun holds 6 bullets. She must fire off 10 magazines per practice session. Jane considers a magazine to be satisfactory if 5 or more of the 6 bullets hit the target.

i. Find the probability that any one magazine of 6 bullets is satisfactory. Give your answer correct to 3 decimal places.

ii. Find the average number of bullets from each magazine that hit the target.

iii. Find the average number of satisfactory magazines per practice session. Give your answer correct to 2 decimal places.

2 + 1 + 1 = 4 marks

d. There is also an elite driving course at the MIA training facility. The amount of time that Jane spends at the driving course on each visit, *T* hours, is a continuous random variable with a probability density function given by

$$f(t) = \begin{cases} \frac{1}{4}t(4 - t^2), & 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$$

i. Sketch the graph of $y = f(t)$, giving the coordinates of the maximum point correct to two decimal places.

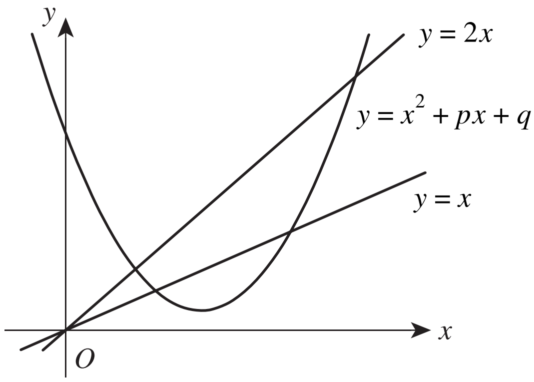
ii. State the mode, correct to the nearest minute, for the amount of time that Jane spends at the driving course on each visit.

iii. Write an expression for the probability that Jane spends between 45 minutes and 90 minutes on a visit to the driving course. Hence find this probability correct to three decimal places.

3 + 1 + 2 = 6 marks
Total 16 marks

Question 3

The diagram below shows the graphs of the lines given by $y = x$ and $y = 2x$ and the parabola with equation $y = x^2 + px + q$.



a. State the values of p and q for which the x-axis is tangential to the parabola at $x = 2$.

2 marks

b. The parabola found in **a.** is translated k units vertically so that it is tangential to the line $y = 2x$. Find the value of k.

3 marks

c. If the line with equation $y = x$ intersects the parabola $y = x^2 + px + q$ at two distinct points, show that $(p - 1)^2 > 4q$.

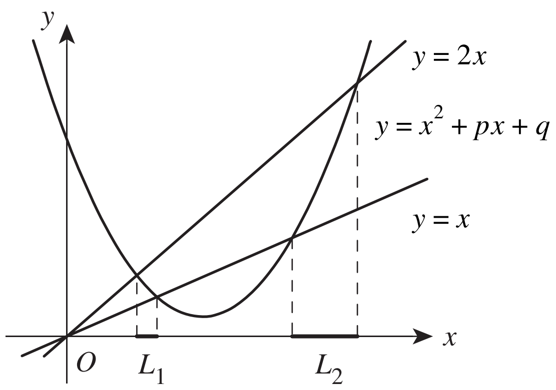
2 marks

In the remainder of this question, assume that the lines given by $y = x$ and $y = 2x$ each intersect the parabola $y = x^2 + px + q$ at two distinct points.

d. The x-coordinates of the points where the line with equation $y = x$ meets the parabola with equation $y = x^2 + px + q$ are given by $x = \frac{1 - p \pm \sqrt{p^2 - 2p + 1 - 4q}}{2}$. Find the x-coordinates of the points where the line with equation $y = 2x$ meets the parabola.

2 marks

e. Consider the two arcs of the parabola $y = x^2 + px + q$ that lie between $y = x$ and $y = 2x$ for $x > 0$. These arcs are projected onto the x-axis to give segments of lengths L_1 and L_2 as shown in the diagram below.



i. Express L_1 and L_2 in terms of p and q.

ii. Show that $|L_1 - L_2|$ is constant.

3 + 1 = 4 marks

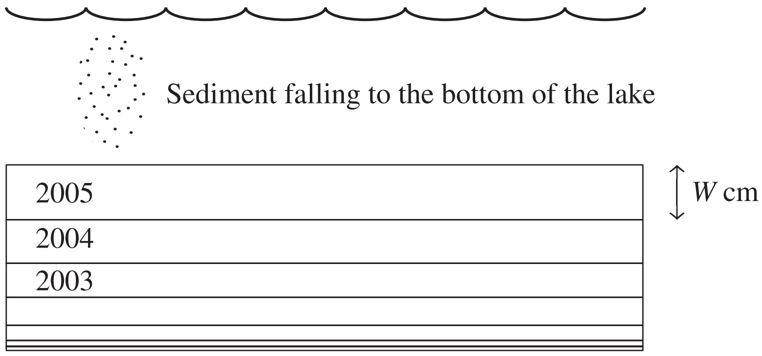
 TEVMMU34EX2_QA_07.FM

f. Now consider the two arcs of the parabola $y = ax^2 + px + q$ with $a > 0$ which lie between the lines with equations $y = x$ and $y = 2x$ for $x > 0$. These arcs are projected onto the *x*-axis to give similar segments L_3 and L_4. Find $|L_3 - L_4|$.

3 marks
Total 16 marks

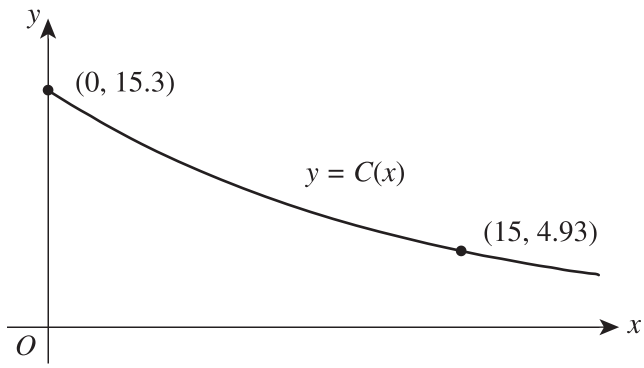
Question 4

When environmental scientists design strategies to deal with pollution in lakes and rivers, they need information about the depth of sediment and the rate of sedimentation. To obtain this information they take core samples of the beds of the lakes and rivers. These core samples show layering, a characteristic of the various sediments that have found their way into the lake or river (often as a result of mining activity).



The diagram above shows the profile of Lake Wilba. Deeper layers are more compacted than layers nearer the surface, as the weight of the upper layers squeezes out water from the space between the grains of sediment in the deeper layers.

Assume that compaction is continuous and that the top layer of sediment is not compacted at all. Also assume that the rate of sediment deposit is constant at W cm per year. Let $C(x)$ be the compacted thickness (in cm) of a layer at a depth of x cm. It can be shown that $C(x) = We^{-bx}$, where is b is a compaction coefficient. The graph below shows $C(x)$ for Lake Wilba.



a. **i.** State the amount of sediment deposited each year (in cm).

ii. Find the compaction coefficient, b. Give your answer correct to four decimal places.

iii. At what range of depths in the sediment will compacted layers be between 0.5 cm and 1 cm in thickness? Give your answer correct to two decimal places.

1 + 1 + 2 = 4 marks

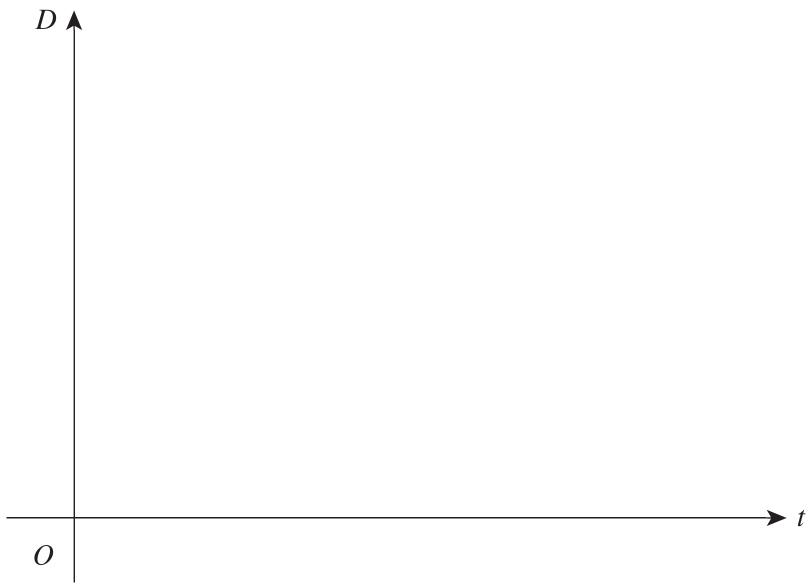
 TEVMMU34EX2_QA_07.FM

b. Use the general compactness equation $C(x) = We^{-bx}$ to show that the compacted thickness of a layer is half of its original thickness when the depth of the layer is $\frac{\log_e(2)}{b}$.

2 marks

c. It can be shown that a layer deposited t years ago will lie at a depth D cm, where $D = \frac{W}{b}(1 - e^{-bt})$.

On the axes below, sketch a graph of D against t. Carefully label the coordinates of any axis intercepts and give the general equations of any asymptotes in terms of W and b.



3 marks

d. Core samples taken from a second lake, Lake Alein, at the end of 2005 $(t = 0)$ indicate a sedimentation rate of 2.06 centimetres per year. The samples show a layer of caesium at the end of 1985 at a depth of 36.2 cm.

i. Use the information given to find the value of the compaction coefficient. Give your answer correct to five decimal places.

ii. Hence find the maximum depth that the model predicts for the sediment in Lake Alein. Give your answer correct to the nearest centimetre.

iii. In the layer from which year is the rate of change of the layer's depth (i.e. $\frac{dD}{dt}$) equal to 1.3 centimetres per year?

2 + 2 + 3 = 7 marks
Total 16 marks

END OF QUESTION AND ANSWER BOOKLET

