Year 2007

VCE

Mathematical Methods CAS

Trial Examination 2

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Victorian Certificate of Education 2007

STUDENT NUMBER

Figures									Letter
Words									

MATHEMATICAL METHODS (CAS)

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 28 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.

Question 1

The transformation $T: R^2 \to R^2$, which maps the curve with equation $y = x^3$ to the curve with equation $y = (4x-6)^3 + 1$, could have rule

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -6 \\ 1 \end{bmatrix}$

B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -6 \\ -1 \end{bmatrix}$

C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.25 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -6 \\ 1 \end{bmatrix}$

D. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.25 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -6 \\ -1 \end{bmatrix}$

E. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.25 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$

Question 2

The simultaneous linear equations $-3x + (m+1)y = 5$ and $8y - (m+3)x = (2m+4)$

have no solution when

A. $m \in R \setminus \{3, -7\}$

B. $m \in R \setminus \{3\}$

C. $m \in R \setminus \{-7\}$

D. $m = -7$

E. $m = 3$

Question 3

The function $f:(-\infty,2)\rightarrow R$ has the rule $f(x)=\dfrac{9}{(x-2)^2}-4$.

The rule for the inverse function is

A. $f^{-1}(x)=2-\dfrac{3}{\sqrt{x+4}}$

B. $f^{-1}(x)=2+\dfrac{3}{\sqrt{x+4}}$

C. $f^{-1}(x)=2\pm\dfrac{3}{\sqrt{x+4}}$

D. $f^{-1}(x)=\dfrac{3}{\sqrt{x}}$

E. $f^{-1}(x)=2+\dfrac{5}{\sqrt{x}}$

Question 4

For the function with rule $f(x)=\begin{cases} x^2-2x+5 & x\le 2 \\ 15-5x & x>2 \end{cases}$

which one of the following statements is true?

A. The function is continous and differentiable for all $x\in R$.

B. The function is not defined at $x=2$.

C. The function is not continous at $x=2$.

D. The function is continous for $x\in R$ and not differentiable at $x=2$.

E. The function is not continous at $x=2$ and not differentiable at $x=2$.

Question 5

The number of blue whales P, in a colony, varies with time according to the rule $P(t)=\dfrac{500}{0.03+e^{-0.1t}}$, where t is the time in years, and $t\ge 0$. The average rate of change in the number of blue whales over the first two years is closest to

A. 485.5

B. 589.1

C. 47.3

D. 51.8

E. 56.8

Question 6

If $f(x) = \log_e(x)$, then which one of the following is true, where *x* and *y* are any non-zero real numbers?

A. $f(x+y) = f(x) + f(y)$

B. $f(x+y) = f(x)f(y)$

C. $f(xy) = f(x) + f(y)$

D. $f(xy) = f(x)f(y)$

E. $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$

Question 7

The largest set of real values of x, for which $|5-3x| > 2$ is

A. $x < 1$

B. $x > \frac{7}{3}$

C. $x > -1$

D. $1 < x < \frac{7}{3}$

E. $x < 1$ or $x > \frac{7}{3}$

Question 8

The function $f : (-\infty, a] \to R$ with the rule $f(x) = x^3 - 3x^2 - 24x + 5$ will have an inverse function provided

A. $a \le 4$

B. $a \ge 4$

C. $a \le 5$

D. $-2 \le a \le 4$

E. $a \le -2$

Question 9

The graph of $y = \frac{1}{x+a} + c$ has a domain of $R\backslash\{-2\}$ and a range of $R\backslash\{-3\}$.
It follows that

A. $a = 2 \quad c = 3$

B. $a = 2 \quad c = -3$

C. $a = -2 \quad c = 3$

D. $a = -2 \quad c = -3$

E. $a = -3 \quad c = -2$

Question 10

If $f(x) = h(x)\cos(4x)$ and $f'(x) = -4e^{-4x}\left(\sin(4x) + \cos(4x)\right)$, then $h(x)$ is equal to

A. $-4e^{-4x}$

B. $-\frac{1}{4}e^{-4x}$

C. $\frac{1}{4}e^{-4x}$

D. $-e^{-4x}$

E. e^{-4x}

Question 11

If $\int_b^a f(x)\,dx = A$, then $\int_a^b \left(\alpha f(x) + \beta\right)dx$ is equal to

A. $\alpha A + \beta$

B. $\beta x - \alpha A$

C. $\beta(b-a) - \alpha A$

D. $\beta(b-a) + \alpha A$

E. $\beta(a-b) - \alpha A$

Question 12

A right angled isosceles triangle has its hypotenuse increasing at a rate of $\sqrt{2}$ cm/min. When the hypotenuse is equal to $4\sqrt{2}$ cm, the rate of increase of the area of the triangle , in cm^2/min is equal to

A. 2

B. $2\sqrt{2}$

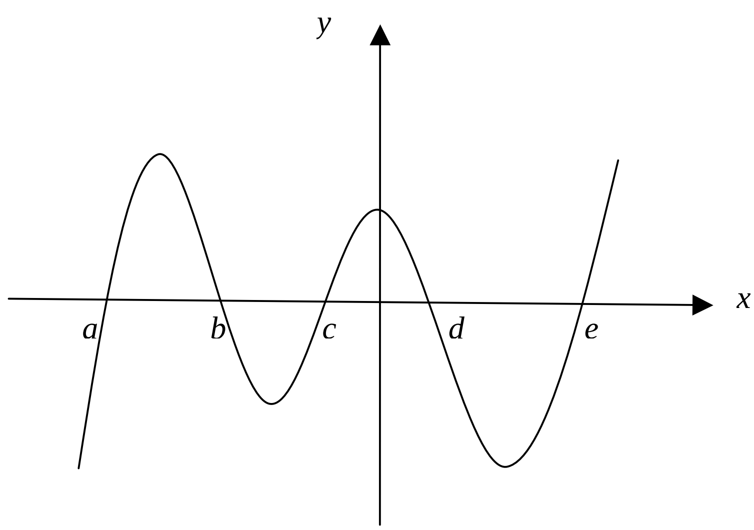
C. 4

D. $4\sqrt{2}$

E. 8

Question 13

Part of the graph of the function f is shown below. The graph crosses the *x*-axis at $x = a$, $x = b$, $x = c$, $x = d$ and $x = e$.



The total area, bounded by the curve of $y = f(x)$ and the *x*-axis on the interval $[a, e]$, is given by

A. $\int_a^e f(x)\,dx$

B. $\int_a^b f(x)\,dx + \int_c^d f(x)\,dx$

C. $\int_a^b f(x)\,dx - \int_b^c f(x)\,dx + \int_c^d f(x)\,dx - \int_d^e f(x)\,dx$

D. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^d f(x)\,dx + \int_d^e f(x)\,dx$

E. $\int_a^b f(x)\,dx - \int_c^b f(x)\,dx + \int_c^d f(x)\,dx - \int_e^d f(x)\,dx$

Question 14

For the graph of $y = \log_e(ax+b)$ where a and b are positive real numbers, which of the following is correct?

A. The graph crosses the x-axis at $x = -\frac{b}{a}$ and has a range of R.

B. The graph crosses the x-axis at $x = \frac{1-b}{a}$ and has a maximal domain of $\left(-\frac{b}{a}, \infty\right)$.

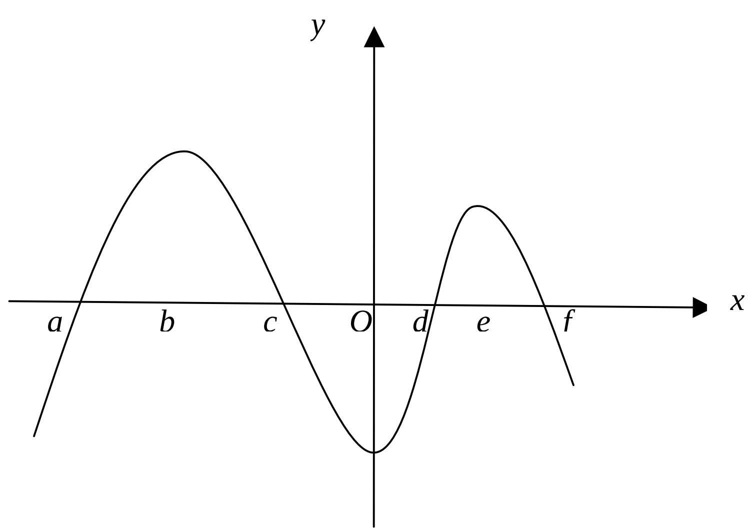
C. The graph crosses the y-axis at $y = \log_e(b)$ and has a maximal domain of R.

D. The graph does not cross the x or y-axis and has a maximal domain of $\left(\log_e(b), \infty\right)$.

E. The graph has a vertical asymptote at $x = -\frac{b}{a}$ and a horizontal asymptote at $y = \log_e(b)$.

Question 15

For the graph of the function $y = f(x)$ shown below, $f'(x)$ is positive when



A. (a, c) and (d, f)

B. $[a, c]$ and $[d, f]$

C. $x < b$ and $0 < x < e$

D. $x \le b$ and $0 \le x \le e$

E. $b < x < 0$ and $x > e$

Question 16

The interval $[0,3]$ is divided into n equal subintervals by the points $x_0, x_1, \ldots x_{n-1}, x_n$ where $0 = x_0 < x_1 < \ldots < x_{n-1} < x_n = \pi$. Let $\delta x = x_i - x_{i-1}$ for $i = 1, 2, \ldots n$.

$\lim\limits_{\delta x \to 0} \sum\limits_{i=1}^{n} 3\sin(3x)\delta x$ is equal to

A. $\int_0^3 3\sin(3x)\,dx$

B. $\int_3^0 3\sin(3x)\,dx$

C. $\int_0^3 \sin(3x)\,dx$

C. $\int_0^3 \cos(3x)\,dx$

E. $-\int_0^3 \cos(3x)\,dx$

Question 17

Let $f(x) = \int 4e^{-4x}\,d(4x)$ and $f(0) = 0$

Then $f(x)$ is given by

A. $f(x) = 4\left(1 - e^{-4x}\right)$

B. $f(x) = 4\left(e^{-4x} - 1\right)$

C. $f(x) = 1 - e^{-4x}$

D. $f(x) = e^{-4x}$

E. $f(x) = -e^{-4x}$

Question 18

If Z has the standard normal distribution and $\Pr(Z < c) = a$, where $0 < c < 3$ and $0 < a < 1$, then $\Pr(|Z| < c)$ is equal to

A. $1 - 2a$

B. $2a - 1$

C. $a - 1$

D. $1 - a$

E. $a - 0.5$

Question 19

The average value of the function $y = 5\sin\left(\frac{\pi x}{5}\right)$ over $0 \le x \le 5$ is

A. 5

B. 25

C. $\frac{50}{\pi}$

D. $\frac{10}{\pi}$

E. $\frac{25}{\pi}$

Question 20

The discrete random variable X has the following probability distribution where $0 < a < 1$ and $0 < b < 1$

X	1	2
$\Pr(X = x)$	a	b

$E(X^2)$ is equal to

A. $1 + 3b$

B. $(a + 2b)^2$

C. $a^2 + 4b^2$

D. $a^2 + 2b^2$

E. $a + 2b$

Question 21

It is known that on average 8 out of 20 people listen to music on an Ipod whilst travelling on a train. On a certain carriage on a train there are 10 people. The probability that exactly 4 are listening to music on an Ipod is equal to

A. 1

B. $\frac{1}{2}$

C. ${}^{10}C_4\, 0.6^4\, 0.4^6$

D. ${}^{10}C_4\, 0.4^4\, 0.6^6$

E. $1 - {}^{20}C_8\, 0.6^8\, 0.4^{12}$

Question 22

An Ipod contains 250 songs. There are 100 songs by artist A, 90 by artist B and 60 by artist C. Three different songs are played in random order. The probability that the three songs are all by different artists is closest to

A. 0.1037

B. 0.2099

C. 0.2602

D. 0.03456

E. 0.03498

END OF SECTION 1

SECTION 2

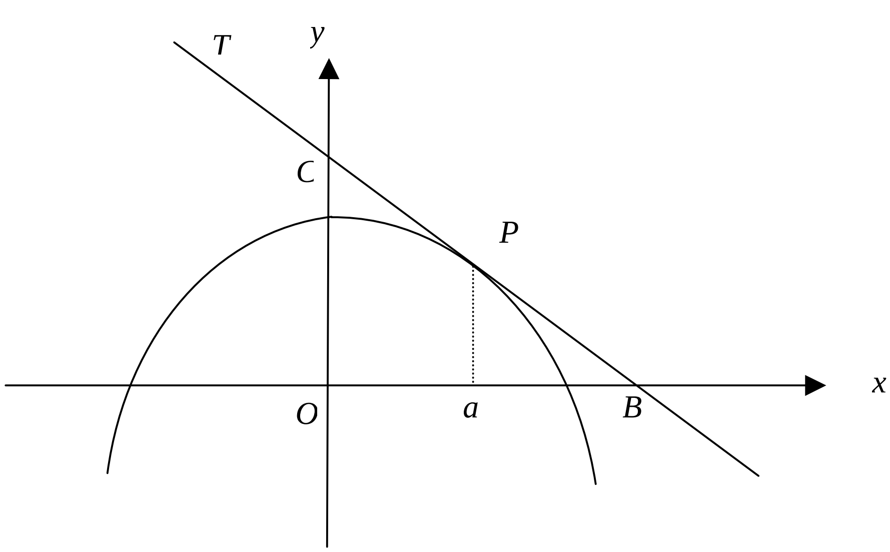
Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1

a. Find the equation of the tangent T to the curve $y = 9 - 4x^2$ at the point P where $x = a$ and $0 < a < \frac{3}{2}$.

2 marks



b. The tangent *T*, to the curve $y = 9 - 4x^2$ crosses the *x*-axis at *B* and crosses the *y*-axis at *C*, as shown in the diagram. *O* is the origin.
The coordinates of *B* and *C* are $(b, 0)$ and $(0, c)$ respectively, where *b* and *c* are positive real numbers.

i. Show that $c = 4a^2 + 9$.

ii. Show that $b = \dfrac{9 + 4a^2}{8a}$.

1 + 1 = 2 marks

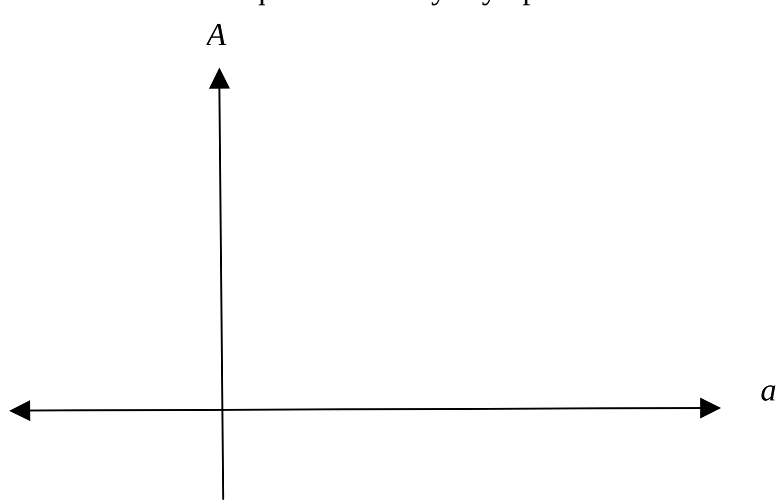
c. Let *A* be the area of the triangle *BOC*.

i. Express the area *A* of the triangle *BOC* in terms of *a*.

ii. Find the exact value of a, for which the area A of the triangle is a minimum.

iii. Verify that it is a minimum and find the exact minimum area of the triangle BOC.

iv. Sketch the graph of A versus a on the axis below, clearly labelling the scale and the equations of any asymptotes.



1 + 2 + 2 + 1 = 6 marks
Total 10 marks

Question 2

Each day Ray either drives to work or catches the train. If he drives to work, the probability that he drives to work the next day is 0.25, and if he catches the train one day, the probability that he catches the train the next day is 0.65.

a. Suppose that on Monday he catches the train.
What is the probability that he drives to work on exactly two of the next three days?
(Give your answer correct to four decimal places.)

2 marks

b. In the long term, what proportion of the days does he take the train to work?
(Give your answer correct to three decimal places.)

1 mark

c. When Ray drives to work, the time spent driving is normally distributed with a mean of 35 minutes. 30% of the time he takes less than 30 minutes. Find the standard deviation in minutes, correct to one decimal place, of the time spent driving to work.

2 marks

When Ray takes the train to work, the time, T hours that is spent on the train ride is a continuous random variable with probability density function given by

$$f(t)=\begin{cases} 2\pi t e^{-\pi t^2} & \text{if} \quad t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

d. Sketch the graph of $y=f(t)$ on the axes below. Label any stationary points with their exact coordinates, correct to three decimal places.

2 marks

e. Find the exact probability that the train ride takes between 20 and 30 minutes.

2 marks

f. What is the probability, correct to three decimal places, that at least one of the next three times he takes the train, the ride takes between 20 and 30 minutes?

3 marks

g. Find a definite integral which gives the mean train travel time and, hence, find the mean travel time, giving your answer in minutes.

2 marks

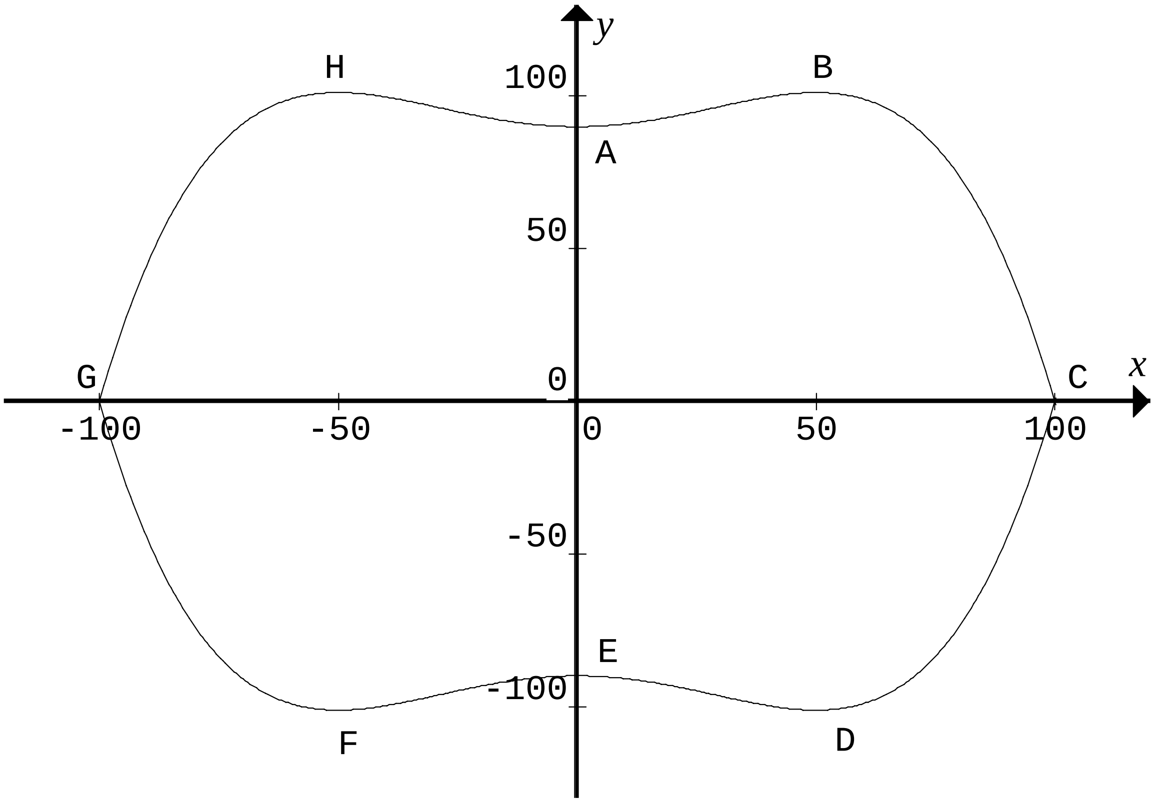
h. Find the median time, to the nearest minute, that he spends travelling on the train.

3 marks

Total 17 marks

Question 3

The cross-section of a fish pond is shown below.



The axes are shown on the drawing and the units are in centimetres.
The points *A* and *C* are $A(0,90)$ and $C(100,0)$.

The curve *ABC* is the graph with equation $f(x) = qx^4 + rx^2 + s$ for $0 \le x \le 100$ where q, r and s are constants.

a. **i**. Show that $s = 90$.

---

ii. Write down an equation involving q and r.

---

1 + 1 = 2 marks

The x coordinate of B is $x = 50$ and is the highest point of the fish pond in the positive y direction.

b. **i.** Use this information to express r in terms of q.

ii. Find the values of q, where q is a real number for which the equation $q\,x^4 - 5000q\,x^2 + 90 = 0$ has more than one solution for x.

iii. Show that $q = -\dfrac{9}{5000000}$ and $r = \dfrac{9}{1000}$.

2 + 4 + 2 = 8 marks

c. The curve *CDE* is the reflection of the curve *ABC* in the x-axis.
Write down the equation of the curve *CDE*.

1 mark

d. The maximum length of the fish pond (in the x direction) is 200 cm.
Find the maximum width of the fish pond (in the y direction).
Give your answer correct to one decimal place.

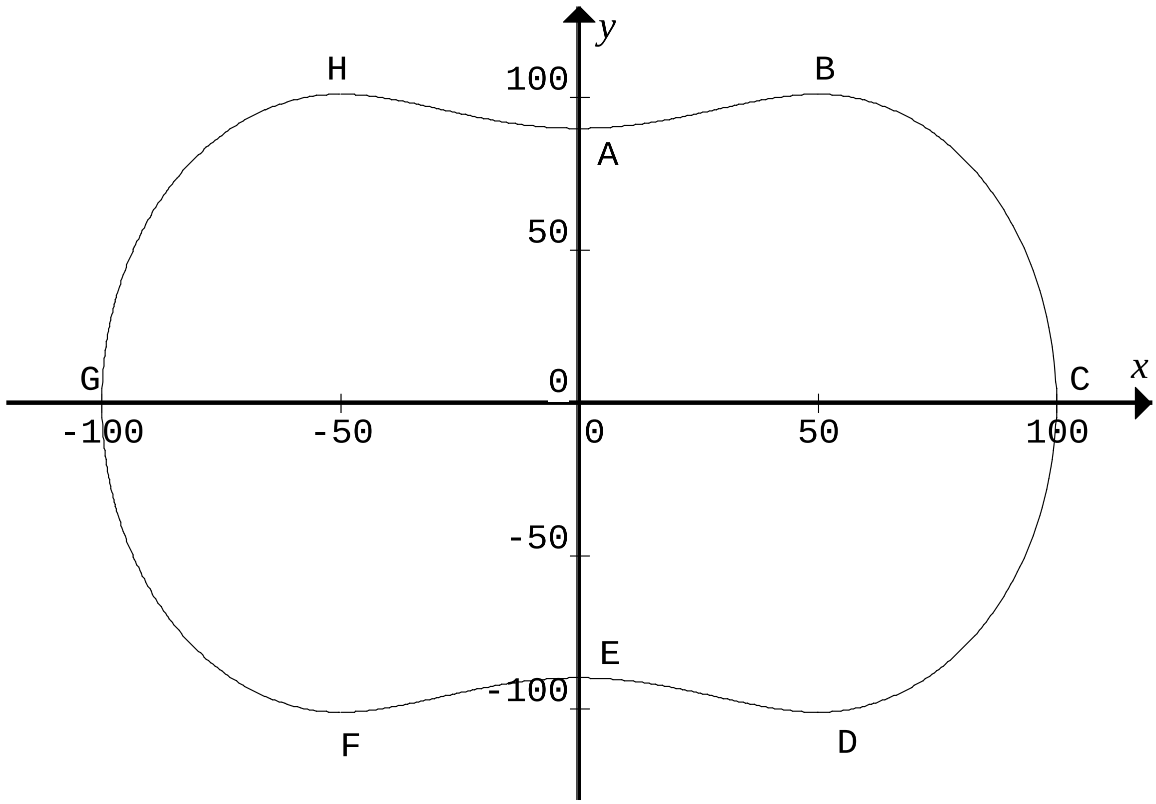
1 mark

The curve *AHGFE* is the reflection of the curve *ABCDE* in the y-axis.

e. Write down a definite integral which gives the total area of the fish pond, and find the total exact area of the fish pond, in square centimetres.

2 marks

The designers of the pond decide that where the two curves *ABC* and *EDC* join at *C*, the join is not smooth. They decide to replace the section of the curve *BC* with a a graph whose equation is $g(x) = a\sqrt{bx - x^2}$ for $50 \le x \le 100$ where *a* and *b* are positive real constants.



f. The designers of the pond like the fact that the pond is still symmetrical.
The section of the curve *GH* is the reflection of the curve $g(x)$ in the *y*-axis, and the section of the curve *GF* is the reflection of the curve *GH* in the *x*-axis.
State the equation of the curve *GF*, in terms of *a* and *b*.

1 mark

g. Find the exact values of *a* and *b*, and show that the join at the point *B* is smooth.

3 marks
Total 18 marks

Question 4

The number of hours *h* of daylight in a certain city in the southern hemisphere can be modelled by a function of the form

$$h(t)=\frac{1}{2}\left(24+5\cos\left(\frac{\pi(t-22)}{183}\right)\right) \quad \text{for} \quad 1\le t\le 366$$

where *t* is the day number, from 1st of January in a leap year.
(So $t=1$ corresponds to 1st January and $t=366$ corresponds to 31st December.)

a. Write down the maximum number of hours of daylight and the date on which this occurs.

1 mark

b. Write down the minimum number of hours of daylight and the day number of the year for which this occurs.

2 marks

c. **i.** Find the general solution to $2\cos\left(\dfrac{\pi(x-22)}{183}\right)+1=0$.

ii. On how many days of the year is the number of hours of daylight at least 10 hours and 45 minutes?

2 + 2 = 4 marks

d. Write down an expression, in terms of *t*, for the rate of change of *h* with respect to the number of days.

1 mark

e. What is the maximum value of the rate of change of *h* with respect to the number of days? Give an exact answer in hours per day.
On what day number does this occur?

2 marks

f. The total number of hours of daylight during the month of January can be expressed as a definite integral. Find the total number of hours of daylight during the entire month of January, giving your answer in hours and minutes correct to nearest minute.

2 marks

g. The number of hours *h* of daylight in a certain city in the nothern hemisphere can be modelled by a function of the form

$$h(t) = a + b\cos\left(\frac{\pi(t-22)}{183}\right) \text{ for } 1 \le t \le 366$$

where *t* is the day number, from 1st of January in a leap year.
This city has a maximum of 16.5 hours of daylight and this occurs on the same day when the city in the southern hemisphere has its minimum number of daylight hours. This city has a minimum of 7.5 hours of daylight and this occurs on the same day when the city in the southern hemisphere has its maximum number of daylight hours.

Write down the values of a and b.

1 mark
Total 13 marks

END OF EXAMINATION

MATHEMATICAL METHODS CAS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
Curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	Area of triangle:	$\frac{1}{2}bc\sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c\ ,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\, f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Mean: $\mu = E(X)$

variance: $\operatorname{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = E(X)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)dx$

ANSWER SHEET

STUDENT NUMBER

									Letter
Figures									
Wo ds									

SIGNATURE ____

SECTION 1

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**