Year 2007

VCE

Mathematical Methods and Mathematical Methods (CAS)

Trial Examination 1

KILBAHA MULTIMEDIA PUBLISHING **PO BOX 2227** **KEW VIC 3101** **AUSTRALIA**	**TEL: (03) 9817 5374** **FAX: (03) 9817 4334** **chemas@chemas.com** www.chemas.com

Victorian Certificate of Education
2007

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHODS
AND
MATHEMATICAL METHOD (CAS)

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For the function $f : R \setminus \{3\} \to R$, $f(x) = \dfrac{2}{x-3} + 4$

a. Find the rule for the inverse function f^{-1}.

1 mark

b. Find the domain of the inverse function f^{-1}.

1 mark

Question 2

Let $y = \dfrac{\tan(2x)}{2x}$. Evaluate $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{8}$.

3 marks

Question 3

a. State in words, giving scale factors, the transformations required to sketch the graph of $y = 4\cos\left(2\left(x - \frac{\pi}{3}\right)\right)$ from the graph of $y = \cos(x)$.

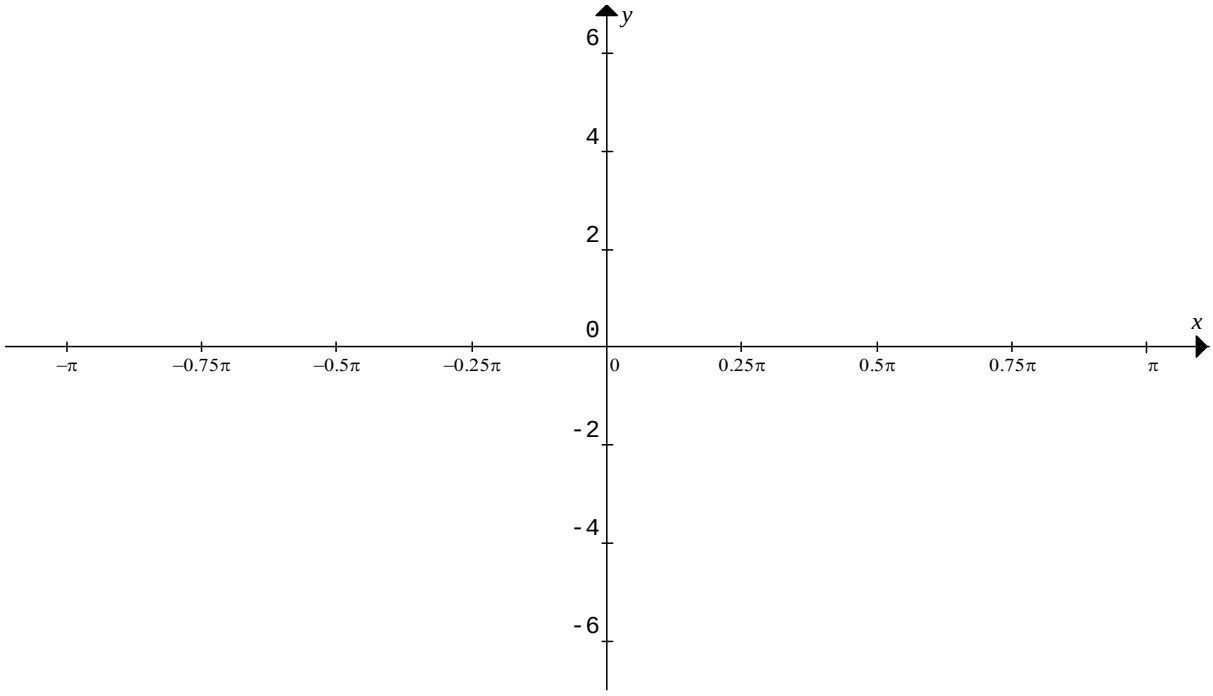
_____

_____

_____

2 marks

b. Sketch the graph of the function $f : [-\pi, \pi] \to R,\ f(x) = 4\cos\left(2\left(x - \frac{\pi}{3}\right)\right)$ on the set of axes below. Label axes intercepts with their coordinates. Label endpoints with their coordinates.



3 marks

Question 4

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} k(x-4)^2 & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

a. Show that $k = \frac{3}{26}$.

1 mark

b. Find $\Pr(X < 2)$.

2 marks

Question 5

The weights of chocolate bars are normally distributed with a mean of 51 grams and a standard deviation of 4 grams. Let Z be the standard normal random variable.
Given that $\Pr(Z < 0.25) = 0.6$,

a. Find the probability that a randomly selected chocolate bar has a weight of less than 50 grams. Give your answer correct to one decimal place.

1 mark

b. Find the probability that a randomly selected chocolate bar has a weight of between 51 and 52 grams. Give your answer correct to one decimal place.

1 mark

c. One night Lilly eats three such chocolate bars. Find the probability that at least one of the chocolate bars has a weight of more than 50 grams.
Give your answer correct to three decimal places.

2 marks

Question 6

Let $g(x) = \cos(2x)$ and $f(g(x)) = \log_e(\cos(2x))$

a. Write down the rule for the function $f(x)$.

1 mark

b. Find the derivative of $f(g(x))$.

1 mark

c. **Hence,** find an anti-derivative of $\tan(2x)$.

1 mark

Question 7

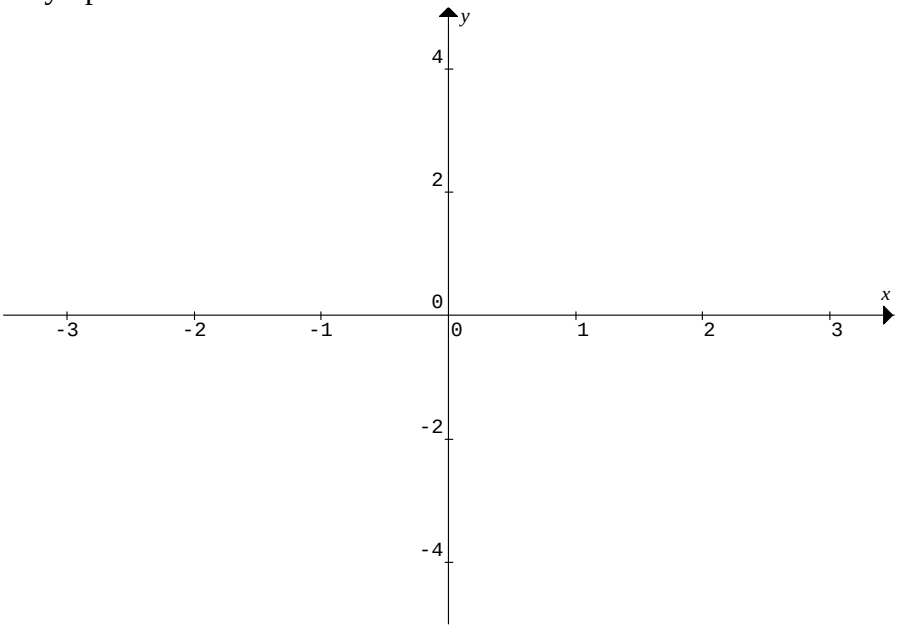
The line $3y + x + k = 0$ is a normal to the curve $y = x^5 + bx$ at the point $x = -1$.
Find the values of b and k.

4 marks

Question 8

a. Sketch the graph of the function $f : R \setminus \left\{\frac{3}{2}\right\} \to R$, $f(x) = \frac{-6}{|2x-3|}$

on the axes below, stating the coordinates of any axial intercepts and the equations of all asymptotes.



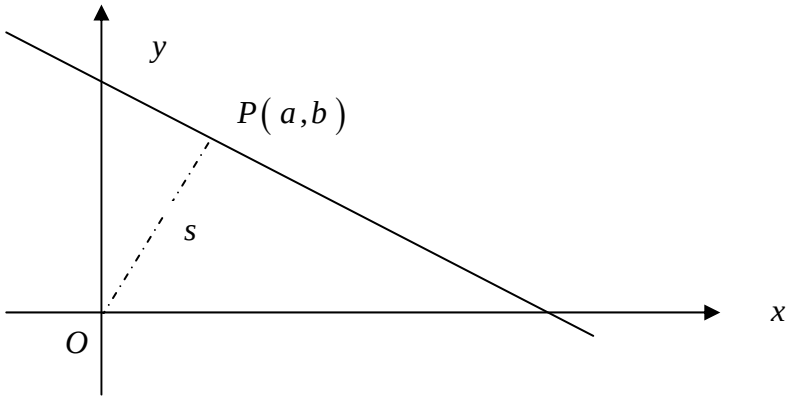
2 marks

b. Find the area bounded by the graph of $y = \frac{-6}{|2x-3|}$, the coordinates axes and the line $x = 1$.

2 marks

Question 9

The point $P(a,b)$ where *a* and *b* are positive real numbers lies on the line $y + 2x - 5 = 0$.



a. Let *s* be the distance from the origin *O* to the point *P*.
Find the distance *s* in terms of *a*.

1 mark

b. Find, using **calculus,** the minimum value of *s* and the value of *a* for which this occurs.

3 marks

Question 10

Each night at dinner Bill has either a glass of wine or a glass of beer. If he has beer one night, the probability he has a beer the next night is 0.6. If he has wine one night, the probability he has a beer the next night is 0.2. Suppose he has a beer on Thursday night. What is the probability that he has a beer on the following Saturday night?

3 marks

Question 11

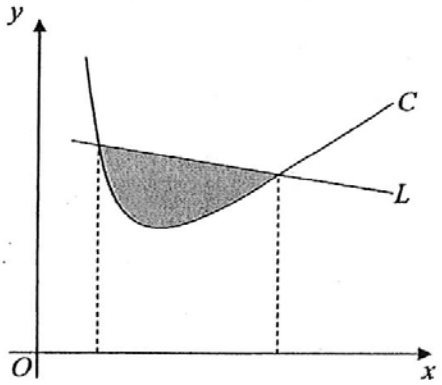
a. Show that the graph of the function $f:(0,\infty)\to R,\quad f(x)=2x+\frac{8}{x^2}$ has a single stationary point and find its coordinates.

2 marks

b. The curve *C* with the equation $y = 2x + \frac{8}{x^2}$ for $x > 0$ is sketched below.

The line *L* with the equation $x + 2y = 21$ intersects the curve *C* at the points $(a, 10)$ and $(b, 8.5)$. If the area of the shaded region bounded by the curve *C* and the line *L* can be expressed as the definite integral $\int_a^b \left(p + qx + \frac{r}{x^2} \right) dx$, find the values of a, b, p, q and r.

(There is no need to evaluate the definite integral or find the area.)



3 marks

END OF QUESTION AND ANSWER BOOKLET

EXTRA WORKING SPACE

END OF EXAMINATION

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods and CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of triangle: $\frac{1}{2}bc\sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c \ , \ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h\,f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E\left(X^2\right) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = E(X)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$