Year 2007

VCE

Mathematical Methods
and
Mathematical Methods
(CAS)
Solutions
Trial Examination 1

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Question 1

a. $f : R\backslash\{3\} \to R$, $f(x) = \dfrac{2}{x-3} + 4$

$f \qquad y = \dfrac{2}{x-3} + 4$ interchanging y and x

$f^{-1} \qquad x = \dfrac{2}{y-3} + 4$ transposing to make y the subject

$f^{-1} \qquad x - 4 = \dfrac{2}{y-3}$

$f^{-1} \qquad y - 3 = \dfrac{2}{x-4}$

$f^{-1}(x) = \dfrac{2}{x-4} + 3$ A1

b. dom $f^{-1} = R\backslash\{4\}$ A1

Question 2

$y = \dfrac{\tan(2x)}{2x}$ differentiating using the quotient rule

let $u = \tan(2x) \qquad v = 2x$

$\dfrac{du}{dx} = \dfrac{2}{\cos^2(2x)} \qquad \dfrac{dv}{dx} = 2$ M1

$$\frac{dy}{dx} = \frac{\dfrac{4x}{\cos^2(2x)} - 2\tan(2x)}{4x^2}$$

when $x = \dfrac{\pi}{8}$ $\quad \left.\dfrac{dy}{dx}\right|_{x=\frac{\pi}{8}} = \dfrac{\dfrac{\frac{\pi}{2}}{\cos^2\left(\frac{\pi}{4}\right)} - 2\tan\left(\frac{\pi}{4}\right)}{\frac{\pi^2}{16}}$ M1

$\left.\dfrac{dy}{dx}\right|_{x=\frac{\pi}{8}} = \dfrac{16(\pi-2)}{\pi^2}$ A1

Question 3

a. $y = \cos(x)$ into $y = 4\cos\left(2\left(x - \frac{\pi}{3}\right)\right)$

- dilation by a factor of 4 parallel to the *y*-axis (or away from the *x*-axis)
- dilation by a factor of $\frac{1}{2}$ parallel to the *x*-axis (or away from the *y*-axis)
- translation by $\frac{\pi}{3}$ to the right parallel to the *x*-axis (or away from the *y*-axis)

for correct transformations A2

b. $f : [-\pi, \pi] \to R,\ f(x) = 4\cos\left(2\left(x - \frac{\pi}{3}\right)\right)$ the amplitude is 4 and

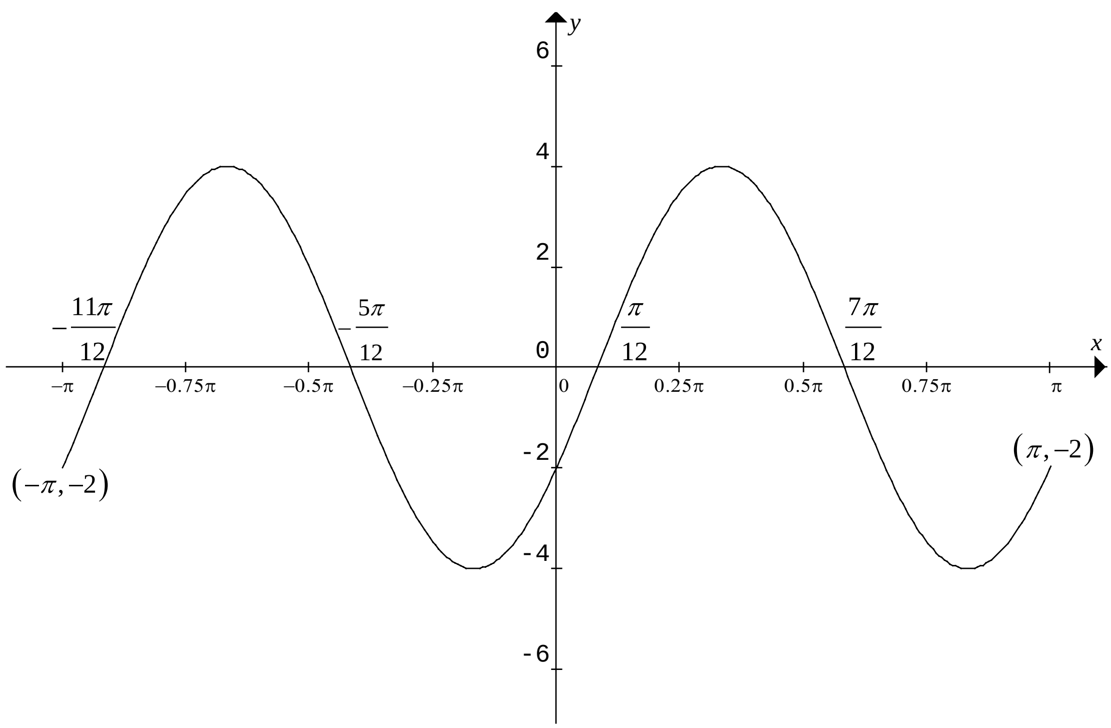
the period is π, so that $f(-\pi) = f(0) = f(\pi) = 4\cos\left(-\frac{2\pi}{3}\right) = -2$ A1

crosses the *x*-axis when $y = 0$ $\cos\left(2\left(x - \frac{\pi}{3}\right)\right) = 0$

$2\left(x - \frac{\pi}{3}\right) = \frac{\pi}{2}$ at $x - \frac{\pi}{3} = \frac{\pi}{4}$ so $x = \frac{7\pi}{12}$ A1

the other *x*-intercepts are $\frac{\pi}{2}$ apart, $\left(-\frac{11\pi}{12}, 0\right), \left(-\frac{5\pi}{12}, 0\right), \left(\frac{\pi}{12}, 0\right), \left(\frac{7\pi}{12}, 0\right)$

correct graph A1



Question 4

$$f(x)=\begin{cases} k(x-4)^2 & 1\le x\le 3 \\ 0 & \text{otherwise} \end{cases}$$

a. $\int_1^3 k(x-4)^2\,dx=1$

$k\left[\frac{(x-4)^3}{3}\right]_1^3=1$ A1

$k\left[\frac{(-1)^3}{3}-\frac{(-3)^3}{3}\right]=1$

$k\left[-\frac{1}{3}+9\right]=1$

$\frac{26k}{3}=1$

$k=\frac{3}{26}$

b. $\Pr(X<2)=\int_1^2 \frac{3}{26}(x-4)^2\,dx$

$\Pr(X<2)=\frac{3}{26}\left[\frac{(x-4)^3}{3}\right]_1^2$ M1

$\Pr(X<2)=\frac{3}{26}\left[\frac{(-2)^3}{3}-\frac{(-3)^3}{3}\right]$

$\Pr(X<2)=\frac{3}{26}\left[-\frac{8}{3}+9\right]$

$\Pr(X<2)=\frac{3}{26}\left(\frac{-8+27}{3}\right)$

$\Pr(X<2)=\frac{19}{26}$ A1

Question 5

X is the weights of chocolate bars

$X \overset{d}{=} N\left(\mu = 51, \sigma^2 = 4^2\right)$

a. $\Pr(X < 50)$

$= \Pr\left(Z < \frac{50-51}{4}\right)$

$= \Pr(Z < -0.25) = \Pr(Z > 0.25)$

$= 1 - \Pr(Z < 0.25)$

$= 1 - 0.6$

$= 0.4$ A1

b. $\Pr(51 < X < 52)$

$= \Pr\left(\frac{51-51}{4} < Z < \frac{52-51}{4}\right)$

$= \Pr(0 < Z < 0.25) = \Pr(Z < 0.25) - \Pr(Z < 0)$

$= 0.6 - 0.5$

$= 0.1$ A1

c. $Y \overset{d}{=} Bi(n = 3, p = 0.6)$

$\Pr(Y \geq 1) = 1 - \left[\Pr(Y = 0)\right]$

$\Pr(Y \geq 1) = 1 - (0.4)^4$ M1

$\Pr(Y \geq 1) = 1 - 0.064$

$\Pr(Y \geq 1) = 0.936$ A1

Question 6

a. $f\left(g(x)\right) = \log_e\left(\cos(2x)\right)$

$g(x) = \cos(2x)$ then $f(x) = \log_e(x)$ A1

b. $\frac{d}{dx}\left[f\left(g(x)\right)\right] = \frac{d}{dx}\left[\log_e\left(\cos(2x)\right)\right]$

$= \frac{-2\sin(2x)}{\cos(2x)}$

$= -2\tan(2x)$ A1

c. hence $\int \tan(2x)\,dx = -\frac{1}{2}\log_e\left(\cos(2x)\right) + C$ A1

Question 7

The line $3y+x+k=0 \quad 3y=-x-k \quad y=-\frac{x}{3}-\frac{k}{3}$

has a gradient of $-\frac{1}{3}$ so $m_N=-\frac{1}{3}$

so the tangent has a gradient of $m_T=3$ A1

$y=x^5+bx$

$\frac{dy}{dx}=5x^4+b=3$ at $x=-1$ M1

$5(-1)^4+b=5+b=3$

$b=-2$

So the curve is $y=x^5-2x$ at the point $x=-1$

$y(-1)=(-1)^5+2=-1+2=1$ M1

the point $P(-1,1)$ is also on the line

$3y+x+k=0 \qquad 3-1+k=0$

so $k=-2$ A1

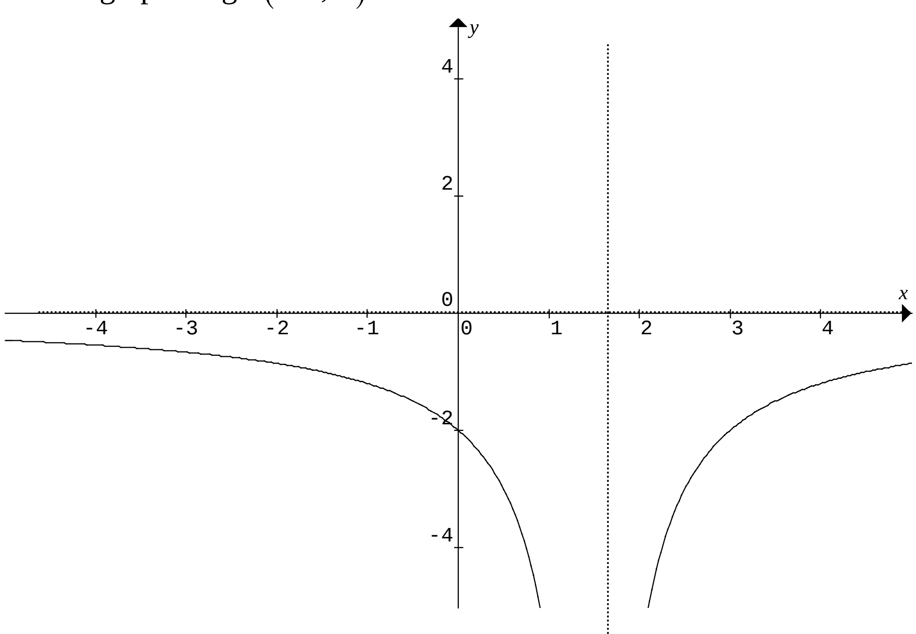
Question 8

a. $f:R\setminus\left\{\frac{3}{2}\right\}\to R$, $f(x)=\frac{-6}{|2x-3|}$

$x=\frac{3}{2}$ is a vertical asymptote and $y=0$ (the *x*-axis) is a horizontal asymptote

crosses the *y*-axis at $x=0 \quad f(0)=\frac{-6}{|-3|}=-2$ at $(0,-2)$ A1

correct graph range $(-\infty,0)$ A1



b. the area $A=\int_0^1 \frac{-6}{|2x-3|}dx=\int_0^1 \frac{6}{3-2x}dx$ since $|2x-3|=3-2x$ for $0<x<1$

$A=\left[-3\log_e(3-2x)\right]_0^1$ and $A>0$ M1

$A=\left[-3\log_e(3)+3\log_e(3)\right]$

$A=3\log_e(3)=\log_e(27)$ A1

Question 9

a. $s=\sqrt{a^2+b^2}$ but the point $P(a,b)$ lies on the line $y+2x-5=0$

so $b+2a-5=0$ $b=5-2a$

$s=\sqrt{a^2+(5-2a)^2}=\sqrt{a^2+25-20a+4a^2}$ M1

$s=\sqrt{5a^2-20a+25}=\left(5a^2-20a+25\right)^{\frac{1}{2}}$

b. for the minimum value of s $\frac{ds}{da}=0$

differentiating using the chain rule M1

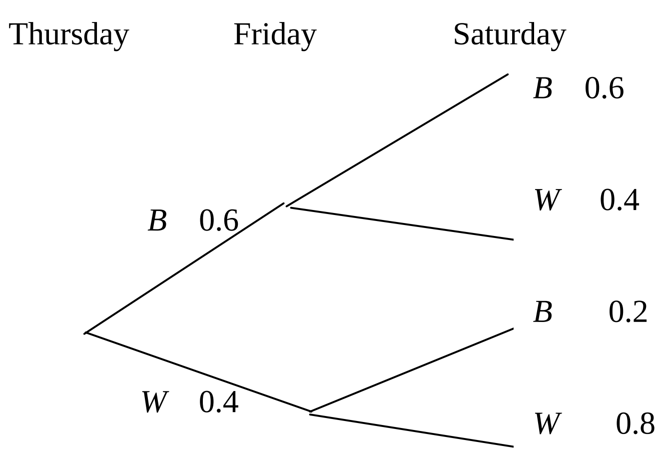
$$\frac{ds}{da}=\frac{\frac{1}{2}(10a-20)}{\sqrt{5a^2-20a+25}}=0$$

$10a=20$ A1

$a=2$ $b=1$

$s=\sqrt{5}$ A1

Question 10



M1

$\Pr(\text{beer on Saturday})=0.6\times0.6+0.4\times0.2=0.36+0.08$ A1

$\Pr(\text{beer on Saturday})=0.44$ A1

Question 11

a. $f:(0,\infty)\to R,\quad f(x)=2x+\dfrac{8}{x^2}=2x+8x^{-2}$

for stationary points $f'(x)=2-16x^{-3}=2-\dfrac{16}{x^3}=0$ M1

$2=\dfrac{16}{x^3}\qquad x^3=8$

$x=2$ and $f(2)=4+\dfrac{8}{4}=6$

the point is $(2\ ,\ 6)$ A1

b. the points $(a\ ,10)$ and $(b, 8.5)$ both lie on the line $x+2y=21$

$a+20=21$ so that $a=1$

$b+17=21$ so that $b=4$ A1

let $y_1=\dfrac{21-x}{2}$ and $y_2=2x+\dfrac{8}{x^2}$

the area between the line and the curve is

$$\int_a^b (y_1-y_2)\,dx$$

$$=\int_1^4\left(\left(\frac{21-x}{2}\right)-\left(2x+\frac{8}{x^2}\right)\right)dx$$ M1

$$=\int_1^4\left(\frac{21}{2}-\frac{5x}{2}-\frac{8}{x^2}\right)dx=\int_a^b\left(p+qx+\frac{r}{x^2}\right)dx$$

so that $p=\dfrac{21}{2}=10.5\quad q=-\dfrac{5}{2}=-2.5$ and $r=-8$ A1

END OF SUGGESTED SOLUTIONS