THE HEFFERNAN GROUP

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Student Name……………………………………

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

2007

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 11 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where an exact answer is required a decimal approximation will not be accepted.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on page 12 of this exam.

Question 1

Write down the maximal domain of the function $f(x)=\log_e(x-2)$.

1 mark

Question 2

Let $f(x)=\frac{1}{2x}$ and let $g(x)=x+1$.

a. Explain whether or not $f(g(x))$ exists.

1 mark

b. Find

i. the rule for $g(f(x))$

ii. the domain of $g(f(x))$

$1+1=2$ marks

Question 3

For the function $f:(1,\infty)\to R, f(x)=\dfrac{3}{x-1}+2$,

a. find the rule of the inverse function f^{-1}.

2 marks

b. find the domain of the inverse function f^{-1}.

1 mark

Question 4

a. If $f(x)=\sin\left(e^{2x}\right)$, find $f'(x)$.

2 marks

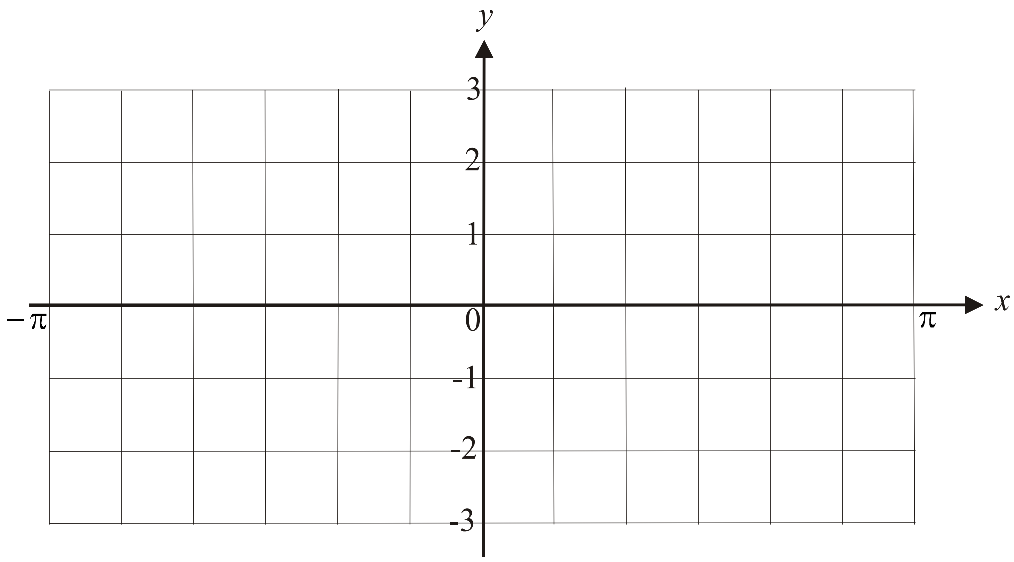
b. Find $\frac{dy}{dx}$ if $y=\frac{\log_e(x)}{x^3+2x}$, $x>0$

2 marks

Question 5

For the function $f:[-\pi,\pi]\to R,\ f(x)=2\sin\left(2\left(x-\dfrac{\pi}{3}\right)\right)$

Sketch the graph of the function *f* on the set of axes below.
Label axes intercepts as well as endpoints with their coordinates.



5 marks

Question 6

Let X be a random variable with a normal distribution with a mean of 7 and a standard deviation of 2. Let Z be a random variable with the standard normal distribution and $\Pr(Z < -1) = 0 \cdot 16$.

a. Find $\Pr(X > 9)$.

1 mark

b. Find the probability that $X < 7$ given that $X < 9$. Express your answer as a fraction with whole numbers in the numerator and denominator.

3 marks

Question 7

The probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases} ax+1, & 1 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

where a is a constant.

a. Show that $a=-\frac{1}{4}$.

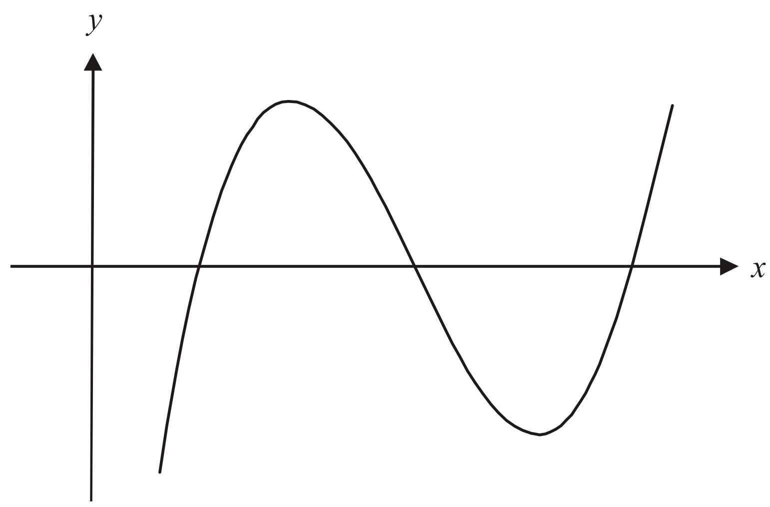
2 marks

b. Find $\Pr(X<2)$.

2 marks

Question 8

The graph of $f : R \to R,\ f(x) = (x-1)(x-3)(x-5)$ is shown below.



a. Sketch the graph of $y = |f(x)|$ on the same set of axes.

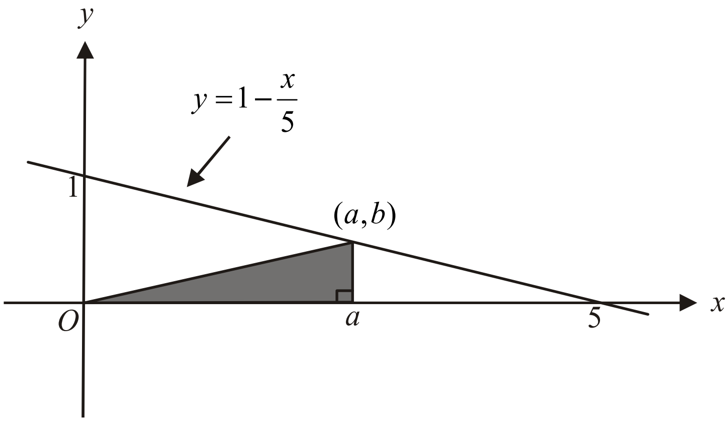
1 mark

b. Let the area of the region enclosed by the graph of $y = |f(x)|$ and the x-axis be A.
Write down an expression for A involving definite integrals in terms of $f(x)$.
Do not evaluate A.

1 mark

Question 9

The shaded right-angled triangle in the diagram below has one vertex at $(0,0)$, a second at the point $(a,0)$ and a third at the point (a,b) which lies on the line with equation $y = 1 - \frac{x}{5}$.
The coordinates a and b are positive, real numbers.



a. Show that the area of the shaded triangle can be expressed as $\frac{a}{2} - \frac{a^2}{10}$.

1 mark

b. Find the maximum area of the shaded triangle as well as the value of a for which this maximum occurs.

3 marks

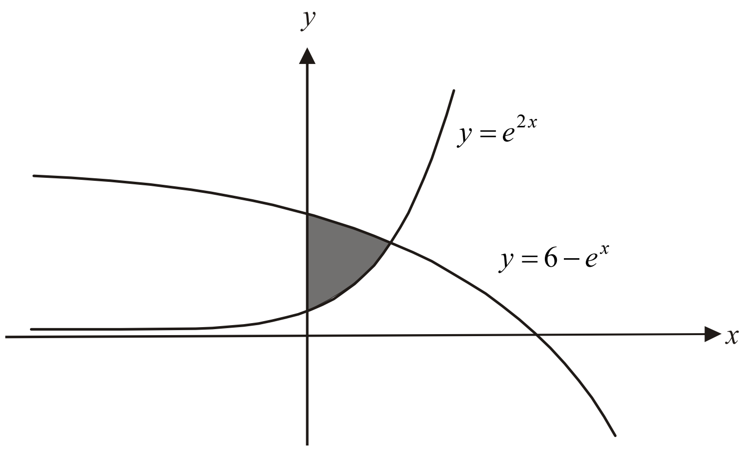
Question 10

The graph of $y = 3x^2 + a$; where a is a real constant, has a normal with equation $y = \frac{x}{3} + 1$. Find the value of a.

4 marks

Question 11

The graphs with equations $y = e^{2x}$ and $y = 6 - e^{x}$ are shown below.



a. Find the x-coordinate of the point of intersection of the two graphs.

2 marks

b. Find the area of the shaded region in the diagram.

4 marks

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$$

$$\int \frac{1}{x}\,dx=\log_e|x|+c$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$$

$$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$$

$$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h)\approx f(x)+hf'(x)$

Probability

$\Pr(A)=1-\Pr(A')$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A/B)=\frac{\Pr(A\cap B)}{\Pr(B)}$

mean: $\mu=E(X)$

variance: $\text{var}(X)=\sigma^2=E((X-\mu)^2)=E(X^2)-\mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\Sigma x\,p(x)$	$\sigma^2=\Sigma(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)dx$	$\mu=\int_{-\infty}^{\infty} x\,f(x)dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$