Mathematical Methods
Written examinations 1 and 2 – October/November

Introduction

Mathematical Methods Examination 1 is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways without the use of technology. Mathematical Methods Examination 1 and Mathematical Methods Examination 1 CAS will be a common examination.

Mathematical Methods Examination 2 is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems. Students are required to respond to multiple-choice questions in Part I of the paper and to extended answer questions, involving multi-stage solutions of increasing complexity, in Part II of the paper.

A formula sheet will be provided with each examination. Details of the formulas to be provided are published with the examination. The formula sheets for Mathematical Methods and Mathematical Methods (CAS) are exactly the same and common to both examinations 1 and 2.

Structure and format

Examination 1

The examination will consist of short answer questions which are to be answered without the use of technology.

The examination will be out of a total of 40 marks.

Examination 2

The examination will consist of two parts. Part 1 will be a multiple-choice section containing 22 questions worth one mark each and Part II will consist of extended answer questions, involving multi-stage solutions of increasing complexity worth 58 marks. Examination 2 will be out of a total of 80 marks.

Approved materials

Examination 1

The following materials are permitted in this examination.

- Normal stationery: this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- A calculator is not allowed in this examination.
- Notes are not permitted in this examination.

Note: protractors, set squares, aids for curve sketching are no longer required for this examination and have been **removed** from the list of approved materials.

Examination 2

The following materials are permitted in this examination.

- Normal stationery: this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- One bound reference that may be annotated. The reference may be a textbook.
- Protractors, set squares, aids for curve sketching.
- Graphics calculator and, if desired, one scientific calculator.

The memories of calculators do not need to be cleared for this examination.

The VCAA publishes details of approved technology for use in mathematics examinations annually. Details of approved calculators for 2006 were published in the October 2005 *VCAA Bulletin*, No. 31. The current list may be found at the VCE Mathematical Methods (CAS)Study page on the VCAA website. Details concerning VCAA approved reference material and technology for use in the 2006 Mathematical Methods examinations were published in the October 2005 *VCAA Bulletin*, No. 31 and November 2005 *VCAA Bulletin*, No. 32.

Other resources

Teachers should refer to the Examination section of the *VCE and VCAL Administrative Handbook 2006, VCE Mathematics Assessment Handbook*, the VCE Mathematical Methods (CAS) Study page on the VCAA website and to the *VCAA Bulletin* for further advice during the year.

Sample examinations

The sample examination papers for Mathematical Methods examinations 1 and 2 address content that remains unchanged and new content areas including continuous probability distributions, composition of functions, the modulus function and related rates of change.

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2006

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures										
Words										

MATHEMATICAL METHODS

Written examination 2

Day Date 2006

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (2 hours)**

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one approved **graphics** calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator, one bound reference.

Materials supplied
- Question and answer book of 22 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1
The sum of the first four positive solutions of the equation $2\sin(3x) - 1 = 0$ is

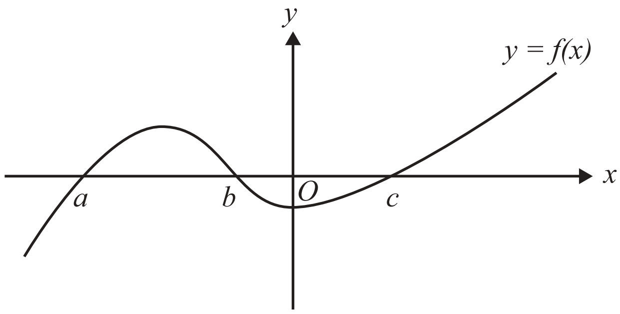
A. $\frac{\pi}{3}$
B. π
C. 2π
D. 4π
E. 6π

Question 2
The solutions to the equation $e^{2x} - 3e^{x} + 2 = 0$ are
A. 1, 2
B. 0, $\log_e 2$
C. e, e^2
D. 1, $\log_e 2$
E. 1, $\log_e 3$

Question 3
The largest set of real values of p for which $|p + 3| > 3$ is
A. $p > 0$ or $p < -6$
B. $p > 0$
C. $p < -6$
D. $p > -6$
E. $p > 0$ or $p < -3$

Question 4

Part of the graph of the function f is shown below.



The total area, bounded by the curve of $y = f(x)$ and the x-axis on the interval (a, c), is given by

A. $\int_a^c f(x)dx$

B. $\int_a^b f(x)dx + \int_b^c f(x)dx$

C. $-\int_a^0 f(x)dx + \int_0^c f(x)dx$

D. $\int_a^b f(x)dx + \int_c^b f(x)dx$

E. $\int_a^b f(x)dx - \int_b^0 f(x)dx + \int_0^c f(x)dx$

Question 5

A small object is oscillating up and down at the end of a vertical spring. The object is h metres above its starting point at time t seconds, where

$$h = 0.5\left(1 - e^{-0.05t}\cos\left(\frac{3\pi t}{2}\right)\right) \qquad \text{and} \quad t > 0$$

The rate in m/s, correct to two decimal places, at which the object is rising 2.5 seconds after motion starts is

A. 0.03

B. –0.43

C –0.40

D. –1.45

E. 1.67

Question 6

The number of ants, N, in a colony varies with time according to the rule $N(t) = 1000e^{0.1t}$, where t is the time measured in days, and $t \geq 0$.

The average rate of change in the number of ants over the first 10 days is closest to

A. 172

B. 183

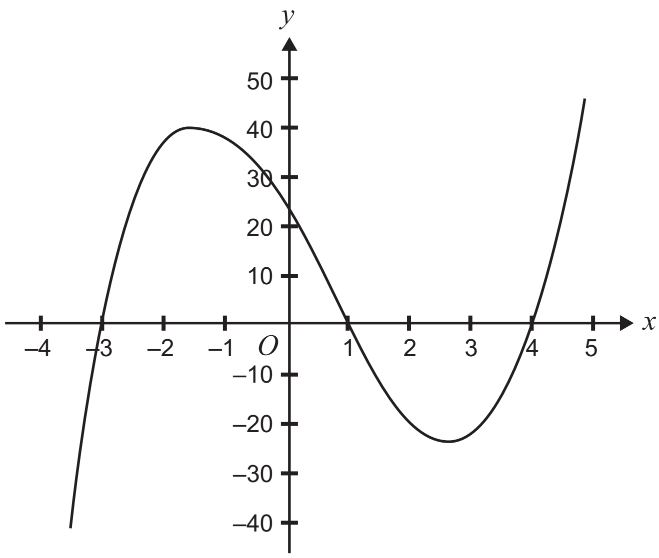
C. 272

D. 1718

E. 2718

Question 7

Part of the graph of $y = ax^3 + bx^2 + cx + d$ is shown below.



The values of *a, b, c* and *d* could be

	a	*b*	*c*	*d*
A.	1	–2	–11	12
B.	2	–4	–22	24
C.	1	2	–22	12
D.	–2	–6	–2	24
E.	3	–1	1	24

Question 8

The best approximate value for $\log_5(6)$ is

A. −0.079

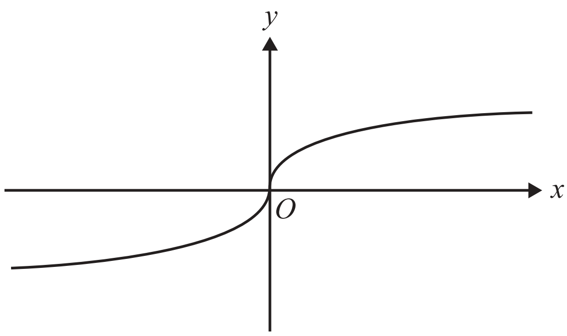
B. 0.778

C. 0.898

D. 1.113

E. 3.890

Question 9



The rule of the graph shown could be

A. $y = \frac{1}{x^3}$

B. $y = x^{\frac{3}{2}}$

C. $y = x^{\frac{2}{3}}$

D. $y = x^{\frac{1}{3}}$

E. $y = |x|^{\frac{1}{3}}$

Question 10

The maximal domain, D, of the function $f : D \to R$ with rule $f(x) = \log_e(x^2) + 1$ is

A. $R \setminus \{0\}$

B. $(-1, \infty)$

C. R

D. $(0, \infty)$

E. $(-\infty, 0)$

Question 11

The function $f : [a, \infty) \to R$ with rule $f(x) = 2x^3 - 3x^2 + 6$ will have an inverse function provided

A. $a \geq 1$

B. $a \geq 0$

C. $a \leq 0$

D. $a \leq \frac{3}{2}$

E. $a \leq 1$

Question 12

If $\int_a^b f(x)dx = 3$, then $\int_a^b (4-2f(x))\,dx$ is equal to

A. $4(b-a)-6$

B. $4(b-a)+6$

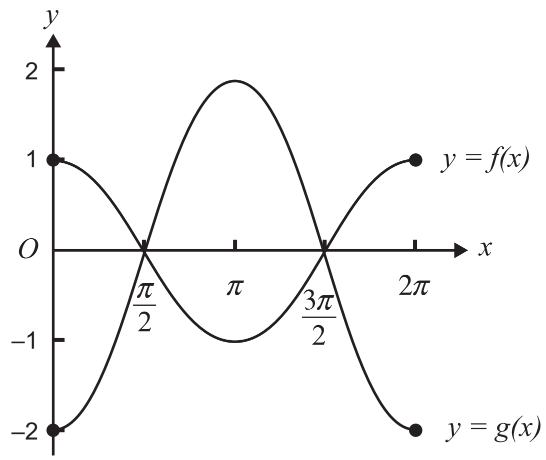
C. $4(a-b)+6$

D. -2

E. 10

Question 13

The diagram below shows the graph of two circular functions, f and g.



The graph of the function with equation $y = f(x)$ is transformed into the graph of the function $y = g(x)$ by

A. a dilation by a scale factor of $\frac{1}{2}$ from the y-axis and a reflection in the x-axis.

B. a dilation by a scale factor of $\frac{1}{2}$ from the x-axis and a reflection in the x-axis.

C. a dilation by a scale factor of 2 from the x-axis and a reflection in the x-axis.

D. a dilation by a scale factor of 2 from the y-axis and a reflection in the y-axis.

E. a dilation by a scale factor of 2 from the x-axis and a reflection in the y-axis.

Question 14

If $y = |\sin(x)|$, then the rate of change of y with respect to x at $x = k$, $\pi < k < 2\pi$, is

A. $-\cos(k)$

B. $\cos(k)$

C. $-\sin(k)$

D. $\sin(k)$

E. $k\cos(1)$

Question 15

A function $f : R \to R$ is such that

$f'(x) = 0$ at $x = 0$ and $x = 2$

$f'(x) < 0$ for $0 < x < 2$ and $x > 2$

$f'(x) > 0$ for $x < 0$

Which one of the following is true?

A. The graph of f has a stationary point of inflection at $x = 0$.

B. The graph of f has a local maximum point at $x = 2$.

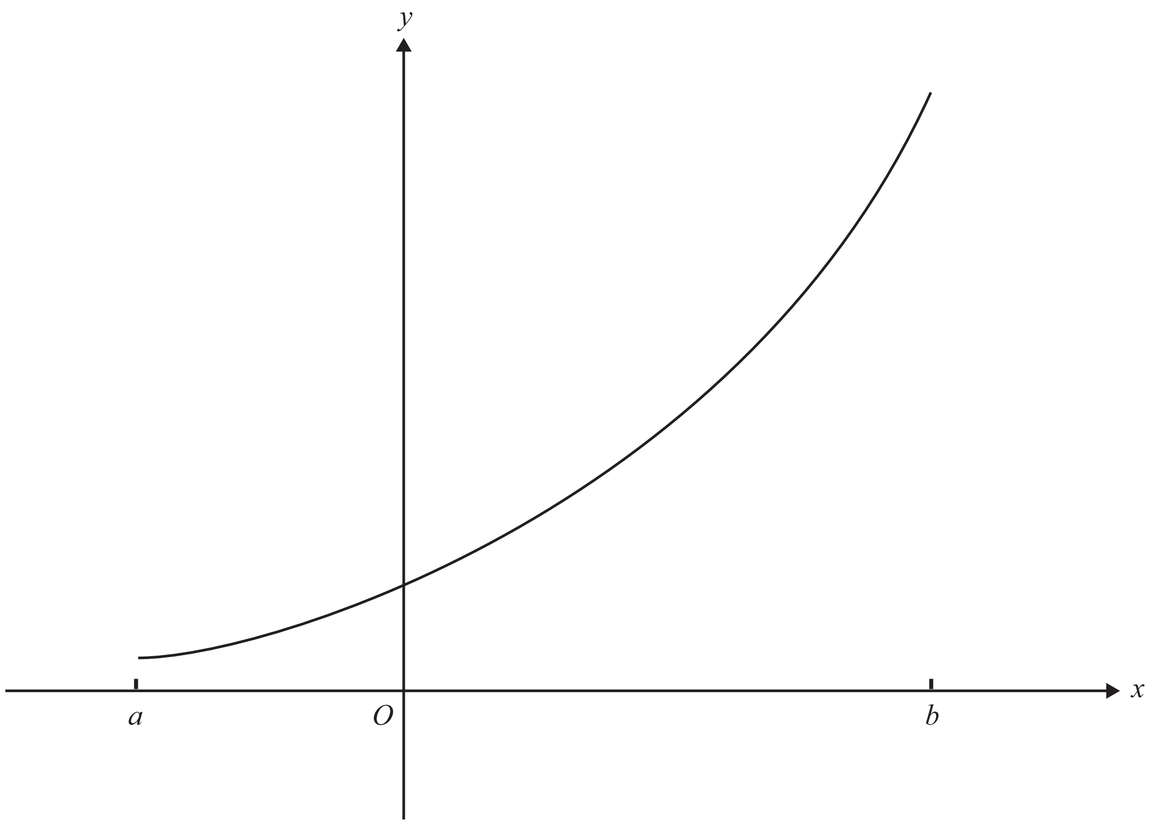
C. The graph of f has a stationary point of inflection at $x = 2$.

D. The graph of f has a local minimum point at $x = 0$.

E. The graph of f has a local minimum point at $x = 2$.

Question 16

The graph of the function $y = f(x)$, where $f : [a, b] \to R$ and a is a negative constant and b a positive constant, is shown below.



The function $F : [a, b] \to R$ is defined by $F(t) = \int_0^t f(x)dx$.

Then $F(t) > 0$ for

A. $t \in [a, b]$

B. $t \in [a, b]\backslash\{0\}$

C. $t \in [0, b]$ only

D. $t \in (0, b]$ only

E. $t \in [a, 0)$ only

Question 17

The equation of the normal to the curve with equation $y = 2x^{\frac{3}{2}}$ at the point where $x = 4$ is

A. $y = -\frac{1}{6}x + \frac{50}{3}$

B. $y = 6x - 18$

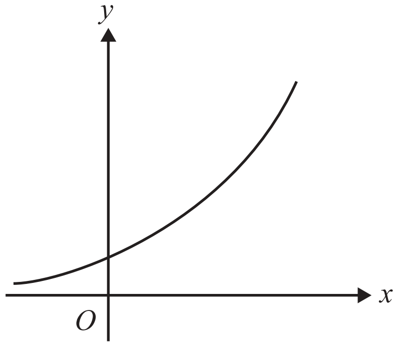
C. $y = 6x - 8$

D. $y = 6x + 40$

E. $y = 160x - 624$

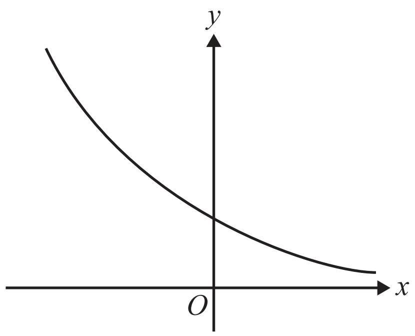
Question 18

The graph of the function f, with rule $y = f(x)$, is shown below.

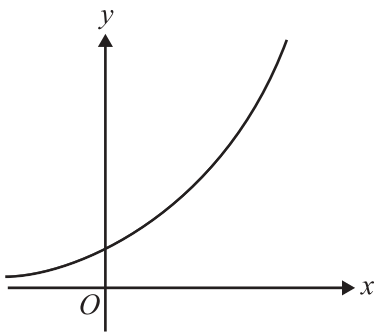


Which one of the follwing could be the graph of the curve with equation $y = f'(x)$?

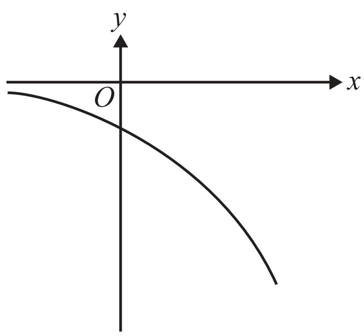
A.



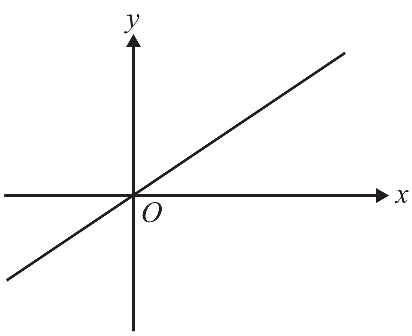
B.



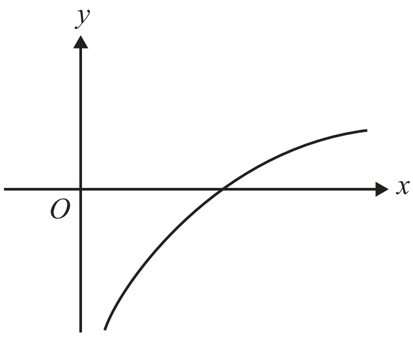
C.



D.



E.



Question 19

The random variable X has a normal distribution with mean 12.2 and standard deviation 1.4.

If Z has the standard normal distribution, then the probability that X is greater than 15 is equal to

A. $\Pr(Z < 2)$

B. $\Pr(Z > 2)$

C. $\Pr(Z > -2)$

D. $1 - \Pr(Z > 2)$

E. $1 - \Pr(Z < -2)$

Question 20

The probability of winning a single game of chance is 0.15, and whether or not the game is won is independent of any other game. Suppose Jodie plays a sequence of n games.

If the probability of Jodie winning at least one game is more than 0.95, then the smallest value n can take is closest to

A. 19

B. 15

C. 8

D. 7

E. 4

Question 21

The number, X, of children in a family is a random variable with the following probability distribution.

x	0	1	2	3
$\Pr(X = x)$	0.4	0.3	0.2	0.1

If two families are selected at random, the probability that they have the same number of children is

A. 0.10

B. 0.20

C. 0.30

D. 0.40

E. 0.50

Question 22

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}\sin(x) & \text{if} \quad 0 < x < \pi \\ 0 & \text{elsewhere} \end{cases}$$

The value of a such that $\Pr(X > a) = 0.25$ is closest to

A. 0.25

B. 0.75

C. 1.04

D. 1.05

E. 2.09

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Consider the function $f: R \to R, f(x) = (x-1)^2(x-2) + 1$.

a. If $f'(x) = (x-1)(ux+v)$, where u and v are constants, use calculus to find the values of u and v.

3 marks

b. The coordinates of the turning points of the graph of $y = f(x)$ are $(a, 1)$ and $\left(b, \frac{23}{27}\right)$.
Find the values of a and b.

2 marks

c. Find the real values of p for which the equation $f(x) = p$ has exactly one solution.

2 marks

d. **i.** Describe a sequence of transformations which maps the graph of $y = f(x)$ on to the graph of $y = f\left(\frac{x}{2}\right) - 1$.

ii. Find the x-axis intercepts of the graph of $y = f\left(\frac{x}{2}\right) - 1$.

iii. Use the answers to **i.** and **ii.** above to write down the area of the region bounded by the graph of $y = f\left(\frac{x}{2}\right) - 1$ and the x-axis, correct to two decimal places.

2 + 1 + 1 = 4 marks

e. Find the real values of h for which **only one** of the solutions of the equation $f(x + h) = 1$ is positive.

2 marks

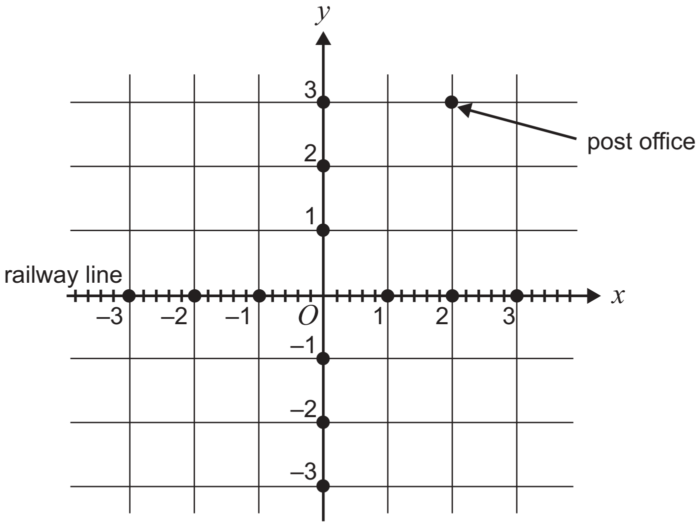
Total 13 marks

Question 2

In a country town, it is decided that a new road should be built.

The grid below shows the positions of the railway line and the post office.

In each direction, 1 unit represents 1 kilometre.



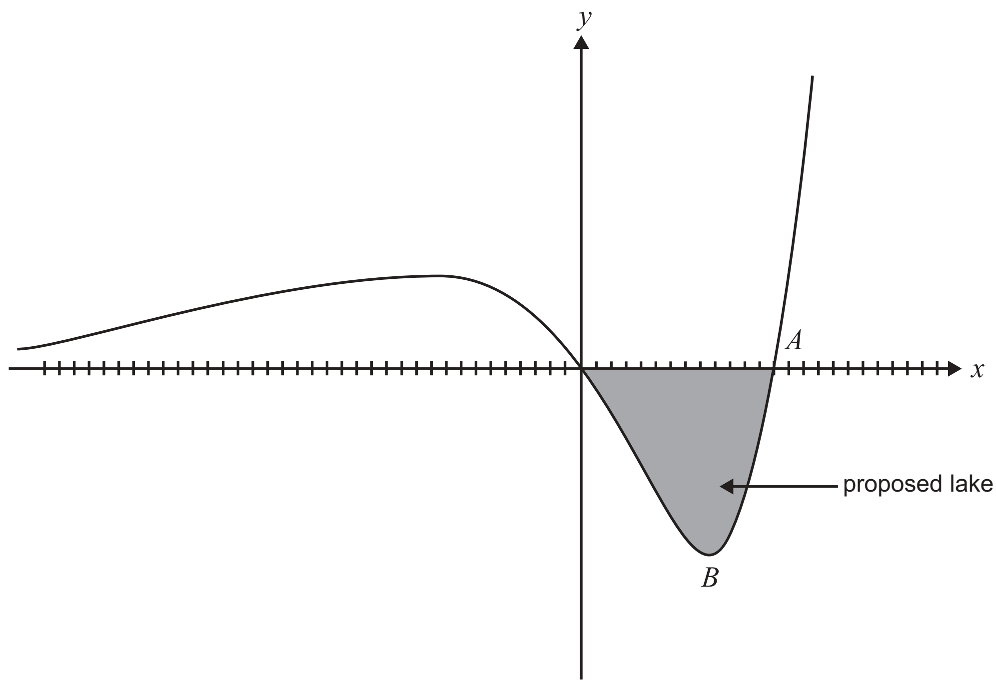
It is decided that the road should follow the path whose equation is

$$y = (2x^2 - 3x)e^{ax} \qquad \text{where } a > 0.$$

a. Find the value of a for which the road will pass through the post office. Give your answer correct to three decimal places.

3 marks

In fact they decided to build the road for which $a = 1$ as shown in the diagram below.



b. Find the x-coordinate of the point A where the road crosses the railway line.

2 marks

c. **i.** Given that $\frac{dy}{dx} = e^x(px^2 + qx + r)$, find the exact values of p, q and r.

ii. Hence find the coordinates of the turning point B. Give your answer correct to three decimal places.

4 + 2 = 6 marks

The town council wishes to develop the shaded area bounded by the road and the railway line as a lake for native water birds.

d. Find the values of m and n for which

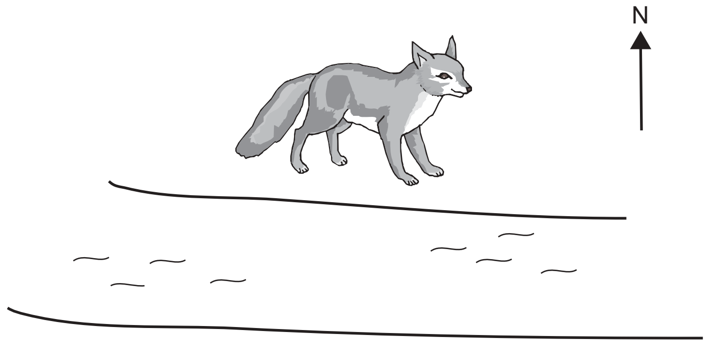
$$\frac{d}{dx}\left\{\left(2x^2+mx+n\right)e^x\right\}=\left(2x^2-3x\right)e^x$$

and hence find the exact area of the lake.

7 marks

Total 18 marks

Question 3



A fox hunts each night in one of two areas, either on the north side of a creek or the south side. The side it hunts on each night depends only on the side that it hunted the night before. If the fox hunts on the north side of the creek one night, then the probability of the fox hunting on the north side of the creek the next night is $\frac{2}{5}$. If the fox hunts on the south side of the creek one night, then the probability of the fox hunting on the south side of the creek the next night is $\frac{1}{5}$.

a. Suppose the fox hunts on the north side of the creek one Monday night.

i. What is the probability that it hunts on the north side on each of the next three nights?

ii. What is the probability that it hunts on the north side on exactly two of the next three nights?

iii. What is the probability that the first time it hunts on the south side that week is on the Thursday or the Friday?

1 + 3 + 2 = 6 marks

The time, t, in hours that the fox spends hunting each night is independent of the area that it hunts in and is a random variable with probability density function

$$f(x) = \begin{cases} \dfrac{3}{32}t(4-t) & \text{if} \quad 0 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$$

b. What is the probability that the fox spends longer than 3 hours hunting on a night?

2 marks

c. What is the probability, correct to three decimal places, that the fox spends longer than 3 hours hunting on at least two out of three nights?

2 marks

d. On 10.4% of nights the fox hunts for less than n minutes. Find the value of n.

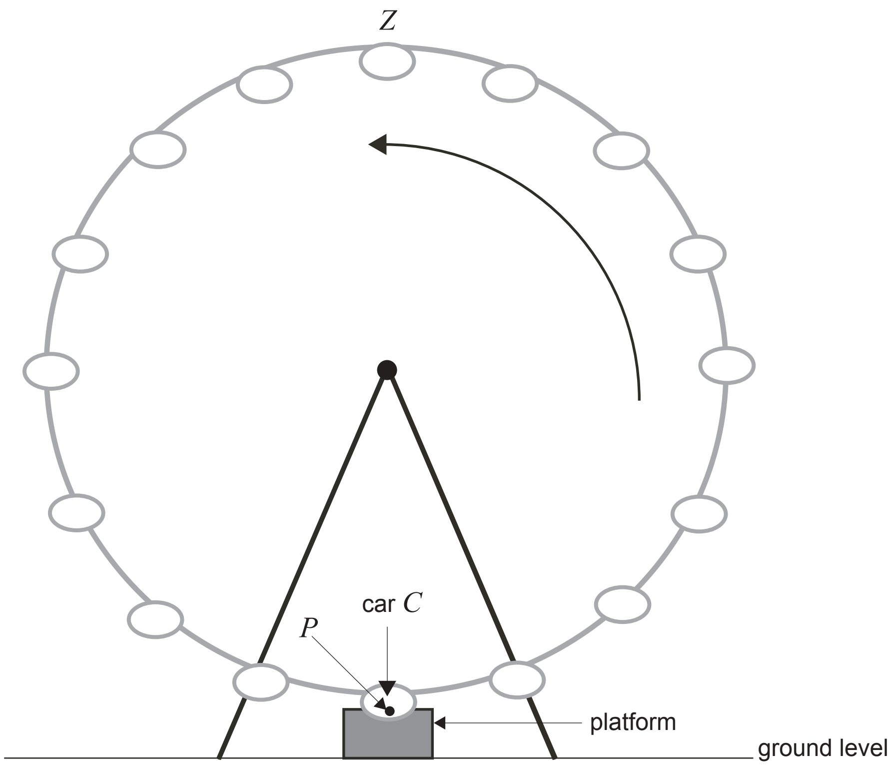
4 marks

Total 14 marks

Question 4

A Ferris wheel at a theme park rotates in an anticlockwise direction at a constant rate. People enter the cars of the Ferris wheel from a platform which is above ground level. The Ferris wheel does not stop at any time.

The Ferris wheel has 16 cars, spaced evenly around the circular structure.



A spider attached itself to the point P on the side of car C when the point P was at its lowest point at 1.00 pm.

The height, h metres, of the point P above ground level, at time t hours after 1.00 pm, is given by

$$h(t) = 62 + 60 \sin\left(\frac{(5t - 1)\pi}{2}\right)$$

a. Write down the maximum height, in metres, of the point P above ground level.

1 mark

b. Write down the minimum height, in metres, of the point P above ground level.

1 mark

c. At what time, after 1.00 pm, does point P first return to its lowest point?

1 mark

d. **i.** Find the time, after 1.00 pm, when P first reaches a height of 92 metres above ground level.

ii. Find the **number of minutes** during one rotation that P is at least 92 metres above ground level.

2 + 1 = 3 marks

e. **i.** Write down an expression, in terms of t, for the rate of change of h with respect to time.

ii. At what rate (in m/h), correct to one decimal place, is h changing when $t = 1$?

1 + 1 = 2 marks