Mathematical Methods and Mathematical Methods Computer Algebra System (CAS) 2006 - Concurrent Implementation with a Common Technology Free Examination

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Analyses and commentary for 2002-2005 Mathematical Methods (CAS) pilot examinations in Victoria, on student performance with respect to common items with the standard course have been reported at previous MERGA conferences. In 2006, both Mathematical Methods and Mathematical Methods (CAS) were available to all Victorian schools as equivalent subjects with a new examination structure that comprised a 1-hour *common technology-free* examination and a 2-hour approved technology active examination. This paper provides some analysis of student performance on the technology free examination, and also with respect to common items in both the multiple choice and extended response components of the technology-active examination.

Mathematical Methods and Mathematical Methods (CAS) are equivalent (in terms of curriculum and assessment) but alternative mainstream function, algebra, calculus and probability courses accredited 2006-2009 (Victorian Curriculum and Assessment Authority (VCAA), 2006a). Units 1 and 2 are typically studied at Year 11, and Units 3 and 4 are typically studied at Year 12 with corresponding end-of-final-year external examinations. Mathematical Methods was first accredited in 1993 and has been re-accredited several times, most recently in 2005. Student access to an approved graphics calculator (with stored material in calculator memory such as notes and supplementary programs allowed) both for learning and assessment, including examinations, has been assumed since 1998 (the use of graphics calculators was permitted but not assumed for the 1997 examinations). Mathematical Methods (CAS) was an accredited pilot study of the VCAA 2001-2005 and is now a fully accredited study available to all Victorian schools. Mathematical Methods (CAS) assumes student access to an approved CAS (calculator or software). For the first time in Australia it is now possible to carry out comparative analysis of student performance on two such studies with respect to a common technology-free examination.

During the most recent review of the Victorian Certificate of Education (VCE) Mathematics studies, the areas of study (content) and outcomes (expectations) for Mathematical Methods effectively converged to those for Mathematical Methods (CAS) – the latter essentially a progressive development from its parent study. In part this process was due to Mathematical Methods (CAS) being a more recently developed study of the mainstream function, algebra, calculus, and probability kind, but also it acknowledged the convergence between graphics calculator plus supplementary program and CAS functionality in several key regards. Thus, Mathematical Methods (CAS) encompasses

Mathematical Methods, and includes some additional curriculum content related principally to the use of matrices with respect to the solution of systems of simultaneous linear equations, transformations of the plane, two state Markov sequences, and an elementary introduction to functional relations. Mathematical Methods (CAS) also involves a more general treatment of families of functions defined using parameters and related algebra, and a greater emphasis on exact value representations. The VCAA has foreshadowed that the two studies will be merged into a single CAS-enabled study from 2010.

Aspects of research related to the use of CAS in senior secondary mathematics from Australia and around the world has been noted in Evans, Norton, and Leigh-Lancaster (2005). This included a summary of those systems and jurisdictions that have some CAS permitted or assumed components of examination assessment. In particular, by 2008, Denmark will have moved from several years of a situation similar to that which now applies in Victoria, to a technology-free and CAS-assumed examination structure for its *Baccalaureat Mathematics* examination.

The emergence over the past few years of hand-held enabling technologies (at comparable cost to graphics calculators) such as the *Classpad* 300 and TI-*n*spire (with corresponding software versions) that readily support integrated numerical, graphical, statistical, dynamic geometry, symbolic, and text functionality in a single platform, provides an opportunity for the related research agenda to move beyond the context (senior secondary, function, algebra, calculus, and probability) in which much of this, and earlier, work of the authors has been predicated. That is, it is now possible to go beyond a conceptualisation of CAS calculators as essentially graphics calculator devices with symbolic manipulation capability, to one where the relevant enabling technology is understood to provide a selection of mathematical functionalities that may be deployed, and of which symbolic manipulation is just one such functionality.

The Common Technology Free Examination

Mathematical Methods (denoted MM) and Mathematical Methods (CAS) (denoted MM CAS) Examination 1 is a common 1-hour technology-free examination comprising short answer questions and some extended-answer questions worth a total of 40 marks (see VCAA, 2006b). It is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways without the use of technology.

A comparison of the mean performance of the two groups on the technology-free paper showed that the MM CAS group (M = 21.22, n = 538) performed at an almost identical level to the MM non-CAS group (M = 21.12, n = 16 057). This is also evident from Figure 1, which displays for each group the mean mark obtained for each question part on the examination. A non-significant result obtained by applying a sign test to these data is consistent with this conclusion (n = 22, x = 11, $p > 0.05$).

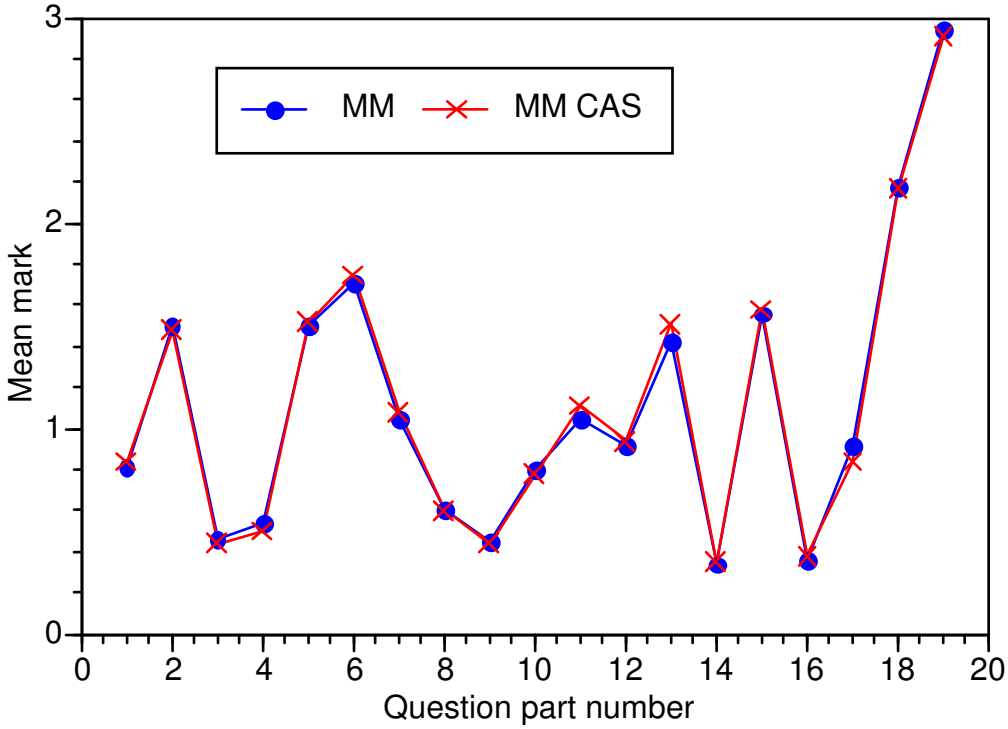

Figure 1. Mean question part marks by group (MM CAS and MM).

The virtually identical performance of the two groups on the technology-free examination does not appear to support the concern that students learning with the aid of CAS would potentially not develop the same level of symbolic facility as those learning without the support of a CAS. It should, however, be recognised that the group of students taking Mathematical Methods CAS in 2006 is not necessarily a representative sample of all students undertaking the Mathematical Methods study in 2006. The Victorian Tertiary Admissions Committee (VTAC, 2007) scaling report, which compares the performance of all students in a given study with the rest of the student cohort across studies, indicates that the overall level of ability of the two Mathematical Methods cohorts (the standard and CAS studies) is effectively the same. It would seem likely that the common curriculum requirements for *both* studies (in terms of key knowledge and key skills specified in the study designs) with respect to mental and by-hands skills of the type tested on the common examination 1, provides a robust basis for very similar levels of performance when students from either cohort do not have access to the relevant enabling technology. Indeed, given the slightly greater curriculum content for Mathematical Methods (CAS), it could be argued that these students have achieved very similar performance to the Mathematical Methods students, with slightly less available time.

Common Multiple Choice Items on the Technology Active Examinations

Mathematical Methods Examination 2 and Mathematical Methods (CAS) Examination 2 are separate two-hour approved technology-assumed access examinations worth a total of 80 marks each (VCAA 2006c, 2006d). They are designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse, and solve both routine and non-routine problems. Examination 2 comprises 22 multiple choice questions,

worth a total of 22 marks, and several extended-answer questions (four in 2006) worth a total of 58 marks. Although there are some distinctive questions and/or parts of questions between the two examinations, much is common or very similar (roughly 70 - 80 % of material). Here we only look at the 17 *common* multiple choice items.

Discussion of Multiple Choice Questions

A comparison of the mean performance of the two groups on the common multiple-choice questions showed that the MM CAS group (M = 12.13, n = 538) out-performed the MM group (M =11.50, n = 16 057). This is also evident from Figure 2 which displays, for each group, the percentage of students correctly answering each multiple choice question. The superior performance of the MM CAS group is confirmed by a sign test (n = 15, x = 13, p = 0.004).

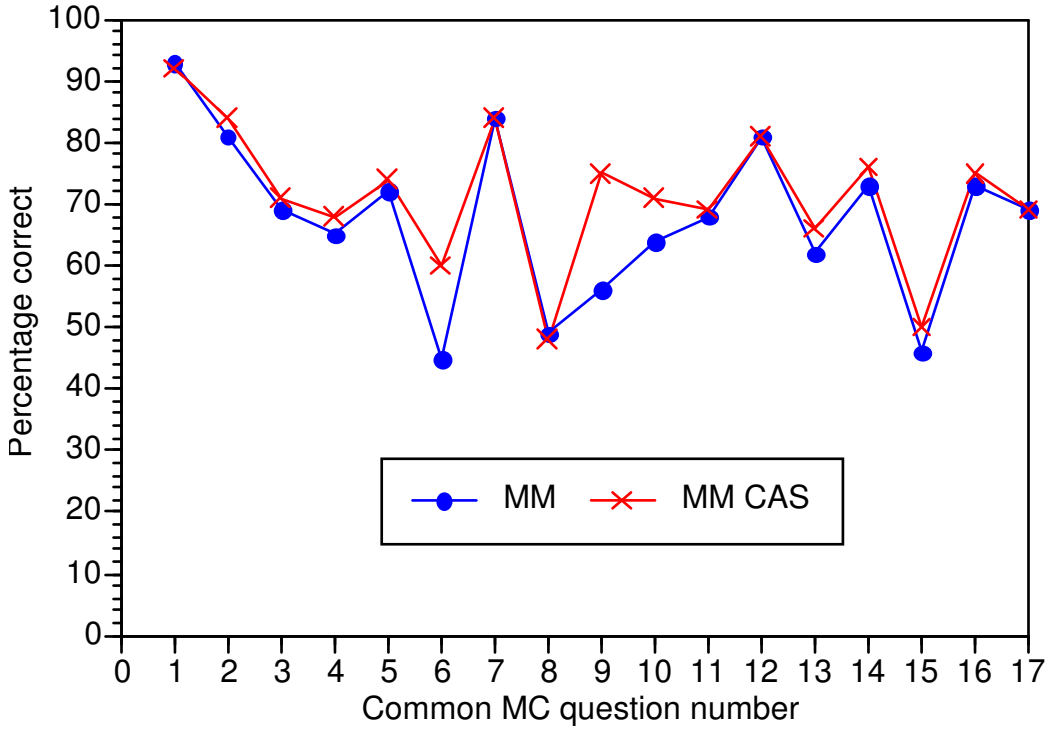

Figure 2. Percentage of students correctly answering each common multiple choice question by group (MM CAS and MM).

A comparison of the group mark profiles suggests that the MM CAS group outperformed the MM group on common questions 6 (by 15%), 9 (by 19%) and 10 (by 7%). A statistical test of these differences, conservatively corrected for the effects of repeated testing, shows all of these differences to be statistically significant (p < 0.001). There were no multiple-choice questions on which the MM group statistically outperformed the MM CAS group.

The questions have again been classified as technology independent (I); technology of assistance but neutral with respect to graphics calculators or CAS (N); or use of CAS likely to be advantageous (C). This classification scheme has now been used for several years in previous reports (Evans, Leigh-Lancaster, & Norton, 2005) and is similar to other schemes

used by researchers. Table 1 lists the stems of the multiple choice questions for which the MM CAS group outperformed the MM group and the classification of the questions.

Table 1
Classification of Multiple Choice Questions for Which the MM CAS Group Clearly Outperformed the MM Group

Question number (% difference)	Question stem	Classification
6 (15)	The function g has rule $g = \log_e \lvert x - b \rvert$, where b is a real constant. The maximal domain is	I
9 (19)	The value(s) of k for which $\lvert 2k + 1 \rvert = k + 1$ are	C
10 (7)	A fair coin is tossed 10 times. The probability, correct to four decimal places, of getting 8 or more heads is	N

Question 6 is a classic pencil-and-paper problem. Computational technology has no direct role to play in its solution, although an intelligent student could look at one or more graphs with technology where the b was replaced by a number to assist in answering the question. Both the absolute value function and the term "maximal domain" appeared for the first time in the MM curriculum, but had been in the MM CAS curriculum for the previous four years. Question 9 also uses the absolute value function, the equation can be directly solved by a student with a CAS by simply entering a command like "solve (abs(2k + 1) = k + 1, k)". The two required solutions, 0 and $-\frac{2}{3}$ are then automatically generated. In contrast, a non-algebraic graphics calculator only has a numerically-based equation solver that generates one solution at a time. This could potentially mislead a student into thinking that there is only a single solution. However, by drawing the graphs of either $y = \text{abs}(2x + 1) - x - 1$ or both $y = \text{abs}(2x + 1)$ and $y = x + 1$ a MM student could have arrived at the correct alternative. Moreover, this is an example of a question for which the correct answer could be obtained by substituting each of the given alternatives into the equation to determine the correct selection. In answering question 10, the use of computational technology is highly advantageous. However, a CAS offers no advantage over a non-CAS enabled graphics calculator in this situation.

Extended Answer Questions

Twenty-two question parts on the extended answer section of the MM CAS Examination 2 and the MM Examination 2 paper were both common in content and equally weighted in terms of marks. In terms of the marks obtained on these common questions, the MM CAS group ($M = 21.99$, $n = 538$) out performed the MM group ($M = 19.91$, $n = 16\,057$). This is also evident from Figure 3, which displays, for each group, the mean mark obtained for each question part. The superior performance of the MM CAS group is confirmed by a sign test ($n = 21$, $x = 19$, $p = 0.0007$).

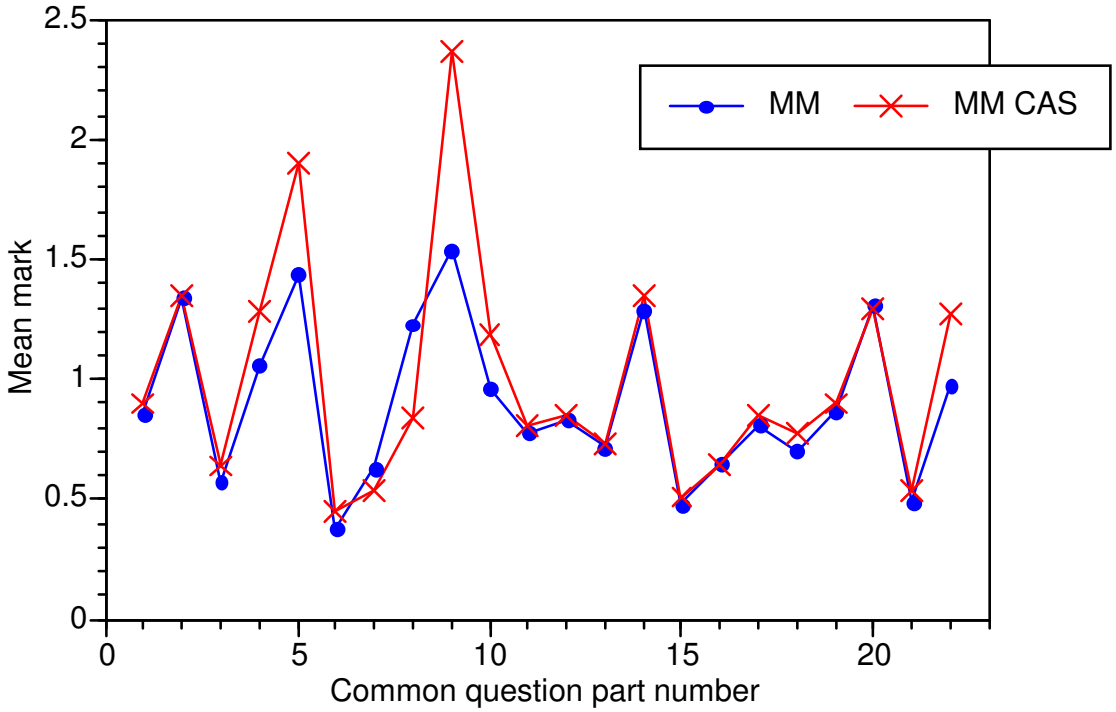

Figure 3. Mean question part marks by group (MM CAS and MM).

A comparison of the group mark profiles coupled with a statistical test of the observed differences, conservatively corrected for the effects of repeated testing, showed 11 questions on which the mean question marks differed between the two groups. All of these differences were found to be to be statistically significant ($p < 0.001$). For each of these questions, the mean difference in percentage terms (positive if in favour of the MM CAS group) and their classification in terms of technology independent (I), neutral (N) or CAS active (C) are displayed in Table 2. In addition, those items for which technology is of assistance but that are likely to be answered efficiently by conceptual understanding, pattern recognition or mental and/or by hand approaches have been indicated by an asterisk.

On nine of these questions the MM CAS group outperformed the MM group. On the remaining two questions, the situation was reversed.

Questions 7 and 8, where the MM group outperformed the CAS group, are clearly technology neutral (and asterisked), in that technology may be required to multiply and add fractions. There is evidence to suggest that the observed differences reflect the influences of curricula differences. These questions involved condition probabilities and their solution was best facilitated through the use of tree diagrams. This was consistent with the MM curriculum. In contrast, in the MM CAS curriculum, conditional probability is also introduced in the context of Markov chains in which problems are formulated in matrix terms. Using a matrix formulation to answer Questions 7 and 8 increases their difficulty level.

The other two questions appearing in Table 2 that are technology neutral, Questions 9 and 10, require a sketch of a density function and the calculation of an integral of a density function numerically, respectively. This area of continuous probability distributions is new to the Mathematical Methods curriculum.

Table 2
Classification of Extended Answer Question Parts for Which the MM CAS Group Clearly Outperformed the MM Group on Nine

Question	Mean difference (%)	Classification	Notes
3	12	C*	Solve $f'(x) = 0$
4	21	C	Equation of tangent
5	32	C*	Find axis intercepts of line
6	18	C*	Analysis skills required
7	–17	N*	Probability calculation based on conditional probabilities
8	–46	N*	Same as 7
9	54	N	Sketch of continuous density function
10	24	N	Numerical integral
18	11	I	Substitution of $x = 0$ into polynomial equation
21	10	C	Solve $f'(x) = 0$; find value of f at this point
22	10	C	Solve simultaneous equations, one arising from a derivative

All but one of the other questions mentioned are classified as being CAS-advantaged. Question 3 asked for the exact value of the other solution to $2\cos(x) = 1$ over the domain $[0, 2\pi]$. (The solution $\frac{\pi}{3}$ had already been given.) It should be noted that not all CAS will find this answer. Questions 21 and 22 were easily done using CAS. For question 22, students would simply define the function $g(x) = \frac{a}{1-bx}$, and then simply issue a command such as "solve $\{g(0) = 7,\ g'(0) = 4.25\}$ for $\{a, b\}$".

Conclusions

The virtually identical performance of the two groups on the technology-free examination does not appear to support the concern that students learning with the aid of CAS would potentially not develop the same level of algebraic skills as those learning with an ordinary graphing calculator. This is the first time that such a comparison has been able to be made. Follow up studies will be possible for the next few years while the Mathematical Methods and Mathematical Methods (CAS) examinations continue in their present form, with a technology-free examination.

As has been observed in previous studies of Evans et al. (2005) MM CAS students generally perform better overall than MM students on common multiple choice items and on common parts of extended response questions. One advantage of using CAS is that once a solution method has been formulated, it is often simple to carry out the method using CAS thus avoiding trivial algebraic errors. This then allows the student with CAS to engage easily with further parts of the question.

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