Student Name: ____________________

Mathematical Methods

Written examination 2

2006 Trial Examination

Reading Time: 15 minutes
Writing Time: 2 Hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, a protractor, set-squares, aids for curve sketching, one bound reference and an approved **graphics** calculator and if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 24 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Instructions for Section 1
A correct answer scores 1, an incorrect answer scores 0. Marks are not deducted for incorrect answers. If more than 1 answer is completed for any question, no mark will be given.

Question 1

The probability distribution for a discrete random variable X is given by

x	0	1	2	3	4
$\Pr(X = x)$	v	0.2	$2v$	0.3	0.1

The value of v and the expected value of x respectively are closest to

A. $\frac{1}{6}$, $2\frac{1}{6}$
B. $\frac{1}{6}$, $2\frac{1}{3}$
C. $\frac{1}{6}$, $5\frac{5}{6}$
D. $\frac{2}{15}$, $1\frac{2}{15}$
E. $\frac{1}{15}$, $2\frac{2}{15}$

Question 2

A hockey player scores 2 out of every 7 shots at goal. If she has 13 shots at goal, the probability that she scores on at most 3 occasions is closest to:

A. 0.5340
B. 0.4660
C. 0.2857
D. 0.2353
E. 0.2306

Question 3

A binomial random variable has $\sigma = 2.26$ and $\mu = 6$.
The values of n and p respectively are closest to

A. 7 and 0.85
B. 7 and 0.86
C. 10 and 0.6
D. 10 and 0.62
E. 40 and 0.15

Question 4

A detective only drinks black coffee or white coffee. He drinks coffee once a day and may switch the type of coffee he drinks each day. If he drinks black coffee one day, then the probability that he drinks white coffee the next day is 0.6. If he drinks white coffee one day, then the probability that he drinks black coffee the next day is 0.7. If he drinks black coffee on Thursday, then the probability that he will drink black coffee on Saturday is equal to:

A. 0.16
B. 0.18
C. 0.24
D. 0.42
E. 0.58

Question 5

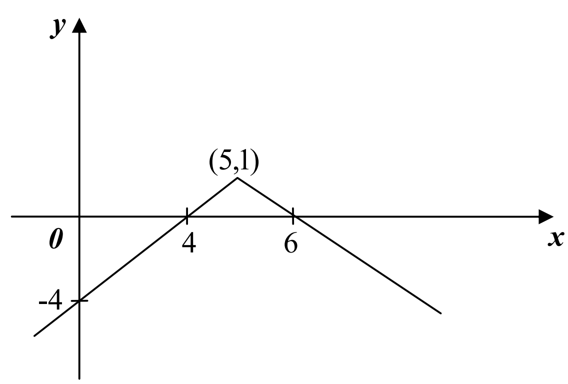
The height of students in a classroom is normally distributed with a mean of 164 cm and a standard deviation of 15.8 cm. The statistician collating the results wishes to make a claim that 5% of students are greater than X cm. The value of X, correct to two decimal places is equal to

A. 132.40 cm
B. 138.01 cm
C. 189.99 cm
D. 190.00 cm
E. 195.60 cm

Question 6

The equation of the curve at right is:

A. $y = -|x-5|+1$
B. $y = -|x+5|+1$
C. $y = x-4$
D. $y = -|x-5|-1$
E. $y = |x-5|+1$



SECTION 1- continued
TURN OVER

Question 7

The equation of the curve at right is best described by:

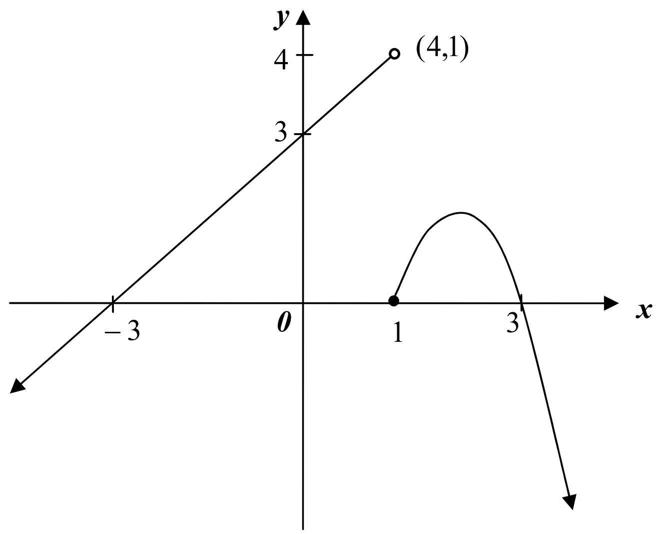
A. $y = \begin{cases} -(x-2)^2+1, & x \ge 1 \\ x+3, & x < 1 \end{cases}$

B. $y = \begin{cases} -(x-2)^2+1, & x \ge 1 \\ x-3, & x < 1 \end{cases}$

C. $y = \begin{cases} (x-2)^2+1, & x \ge 1 \\ x+3, & x < 1 \end{cases}$

D. $y = \begin{cases} -(x+2)^2+1, & x \ge 1 \\ x+3, & x < 1 \end{cases}$

E. $y = \begin{cases} -(x+2)^2+1, & x \ge 1 \\ x-3, & x < 1 \end{cases}$



Question 8

The maximal domain of the function $f(x) = -\frac{3}{\sqrt{4x-5}}$, is:

A. R^+
B. R
C. $\left\{x : x < \frac{5}{4}\right\}$
D. $\left\{x : x \ge \frac{5}{4}\right\}$
E. $\left\{x : x > \frac{5}{4}\right\}$

Question 9

At $x = -2$ the graph of the function f with rule $f(x) = (x+1)(x+2)^3 - 50$ has a

A. $y - axis$ intercept
B. $x - axis$ intercept
C. local maximum
D. local minimum
E. stationary point of inflection

SECTION 1- continued

Question 10

If $f(x)=\sqrt{3-\frac{x}{2}}$ for all $x \le 6$ and $g(x)=x-2$ for all $x \ge 2$ then $f \circ g$ is equal to

A. $(f \circ g)(x)=\sqrt{4-\frac{x}{2}},\ dom(f \circ g)=(-\infty,6]$

B. $(f \circ g)(x)=\sqrt{4-\frac{x}{2}},\ dom(f \circ g)=(-\infty,8]$

C. $(f \circ g)(x)=\sqrt{4-\frac{x}{2}},\ dom(f \circ g)=[2,\infty)$

D. $(f \circ g)(x)=\sqrt{4-\frac{x}{2}},\ dom(f \circ g)=[2,8]$

E. $(f \circ g)(x)=\sqrt{2-\frac{x}{2}},\ dom(f \circ g)=(-\infty,4]$

Question 11

Which one of the following is **not** a factor of $-2x^4+15x^3-28x^2+15x$?

A. $2x-3$
B. $5-x$
C. $x-1$
D. $x+1$
E. x

Question 12

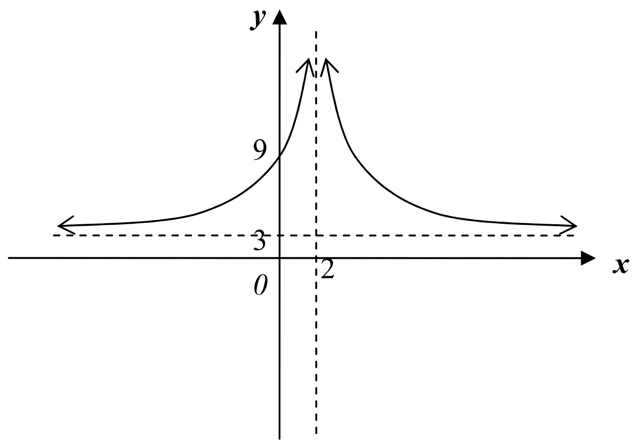
The function $f:[-3,b]\to R, f(x)=9-2x-x^2$ will have an inverse if

A. $b > -1$
B. $b=-1$
C. $b \ge -1$
D. $b=1$
E. $b=10$

SECTION 1- continued
TURN OVER

Question 13

Part of the graph of a function with rule $y = \frac{a}{(x-b)^2} + c$ is shown below.

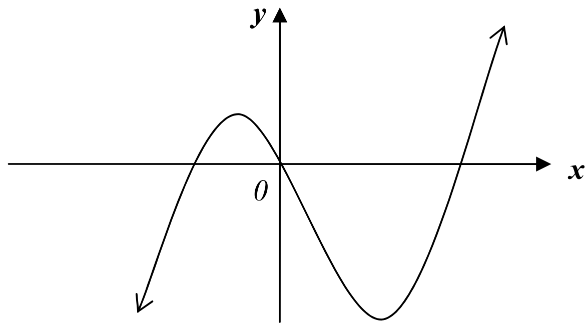


The values of *a*, *b* and *c* respectively are

	a	*b*	*c*
A.	6	2	3
B.	6	-2	3
C.	24	2	3
D.	24	-2	3
E.	63	3	2

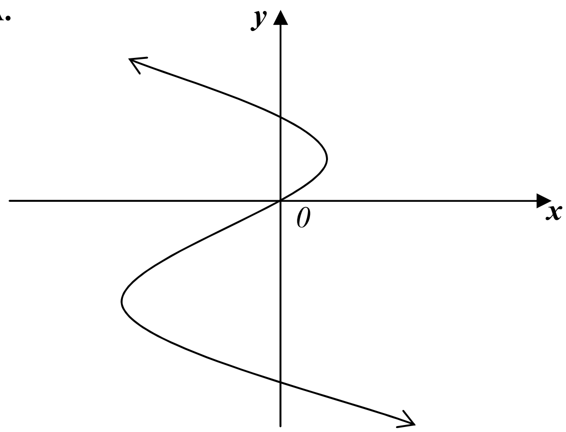
Question 14

The graph of the function whose rule is $y = f(x)$ is shown below:

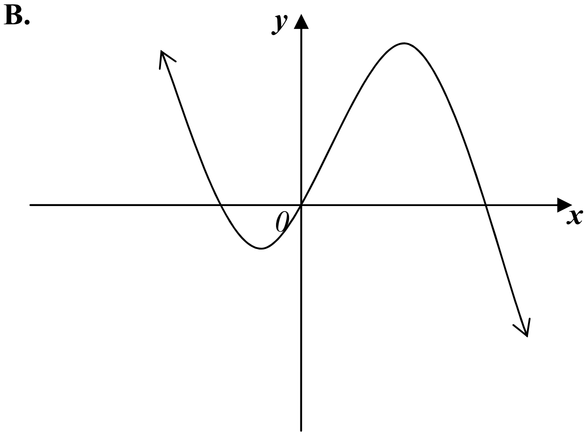


Which one of the following is most likely to be the graph of its inverse?

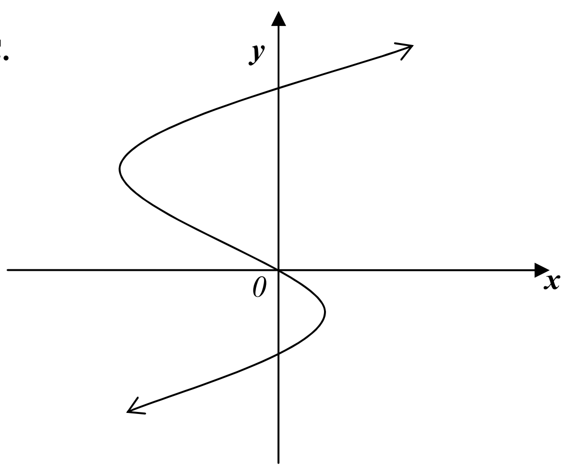
A.



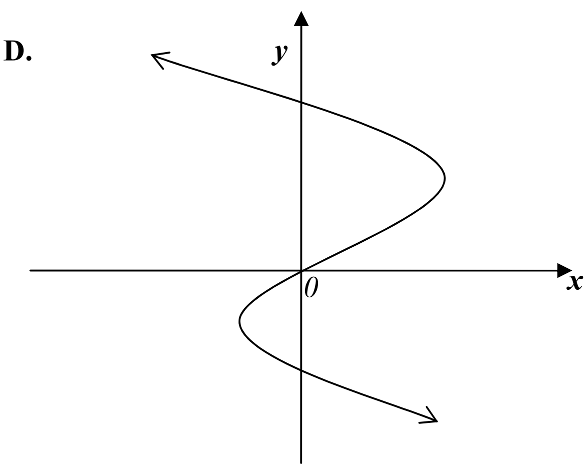
B.



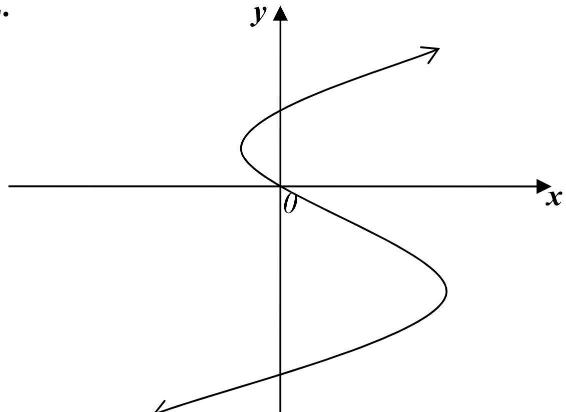
C.



D.



E.



SECTION 1- continued
TURN OVER

Question 15

If $\log_e(x-2)+\log_e 4-\log_e x=2$ then x is equal to:

A. $\log_e\left(\frac{4x-8}{x}\right)$

B. $\log_e x$

C. $\frac{2}{4-e^2}$

D. $\frac{8}{4-e}$

E. $\frac{8}{4-e^2}$

Question 16

The period of the function with rule $f(x)=2\sin\left(\frac{3\pi x}{2}\right)$ is

A. $\frac{3}{2}$

B. $\frac{3\pi}{2}$

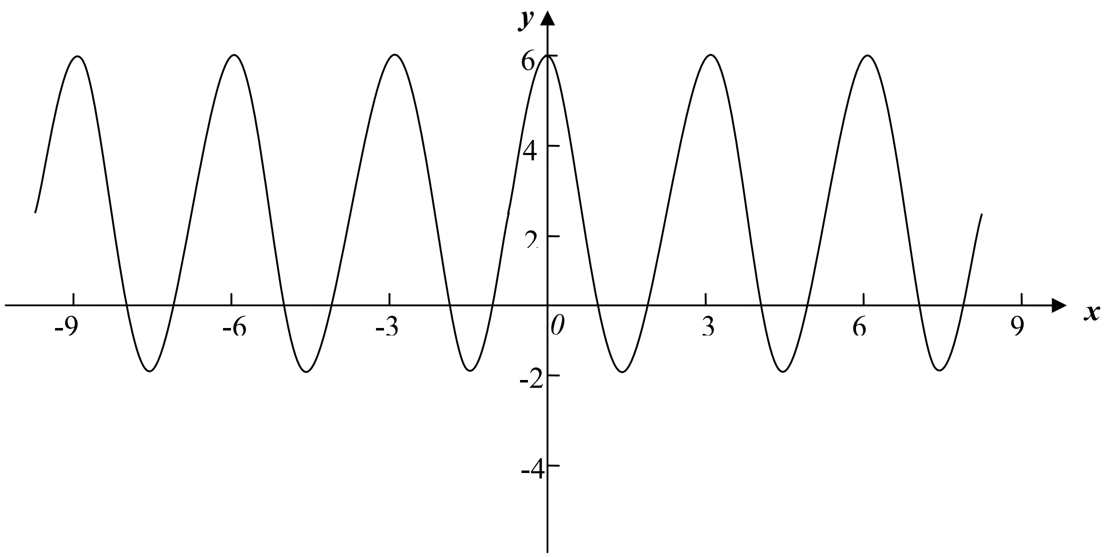
C. $\frac{4\pi}{3}$

D. $\frac{4}{3}$

E. 2

Question 17

Part of a graph of a function with rule $y = f(x)$ is shown below:



The rule for f could be

A. $f(x) = 4\cos(6\pi x) + 2$
B. $f(x) = 4\sin(6\pi x) + 2$
C. $f(x) = 4\sin\left(\frac{2}{3}\pi x + \frac{3}{4}\right) + 2$
D. $f(x) = 4\cos\left(\frac{2}{3}\pi x\right) + 2$
E. $f(x) = 4\sin\left(\frac{2}{3}\pi x\right) + 2$

Question 18

For the equation $\sqrt{3}\sin\frac{1}{2}x = -\cos\frac{1}{2}x$, the **largest** solution over the interval $-4\pi \le x \le 0$ is equal to

A. $-\frac{\pi}{6}$
B. $-\frac{\pi}{3}$
C. $-\frac{7\pi}{3}$
D. $\frac{5\pi}{6}$
E. $\frac{5\pi}{3}$

SECTION 1- continued
TURN OVER

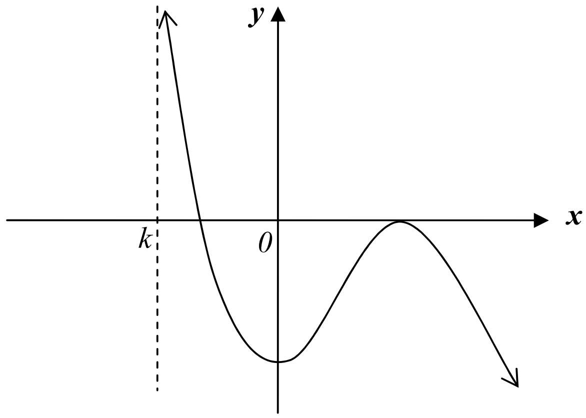
Question 19

The **smallest** solution to the equation $3-\log_e(4-x)=-(x^2-9)$ is closest to

A. -2.8
B. -2.9
C. 2.5
D. 2.6
E. - 3

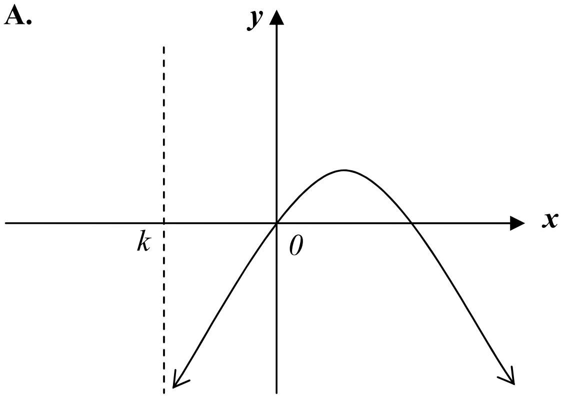
Question 20

The graph of the function f whose rule is $y = f(x)$ is shown below:

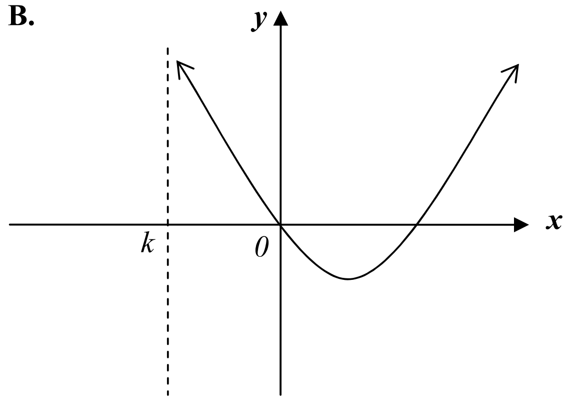


The graph of $f'(x)$ is:

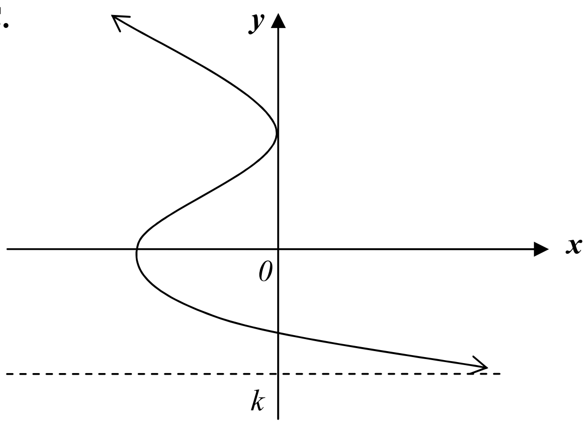
A.



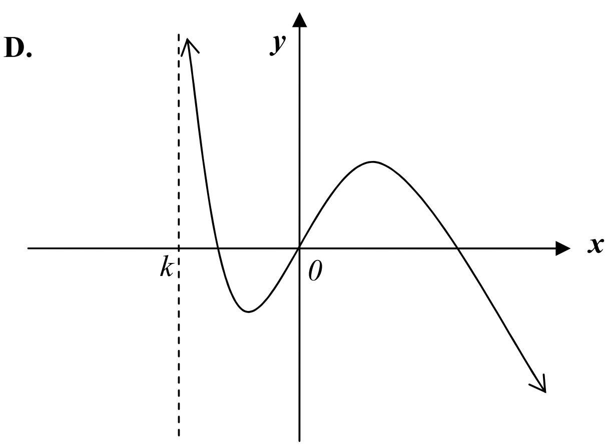
B.



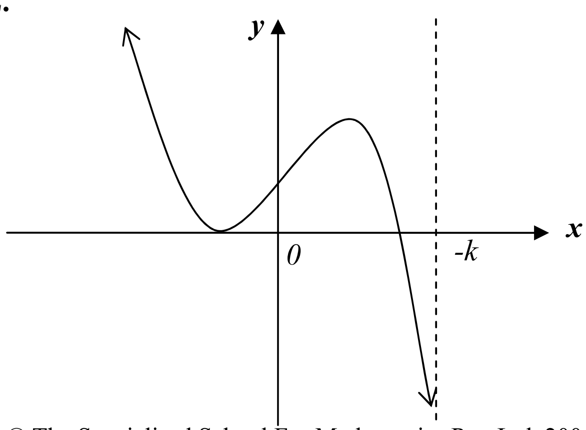
C.



D.



E.



SECTION 1- continued

TURN OVER

Question 21

Let $f : R \to R$ be a differentiable function. For all real values of x the derivative of $f\left(4\cos\left(\frac{x}{2}\right)\right)$ with respect to x will be equal to

A. $-2\sin x\, f'(2\cos x)$

B. $-4\ \sin\left(\frac{x}{2}\right) f'\left(4\cos\left(\frac{x}{2}\right)\right)$

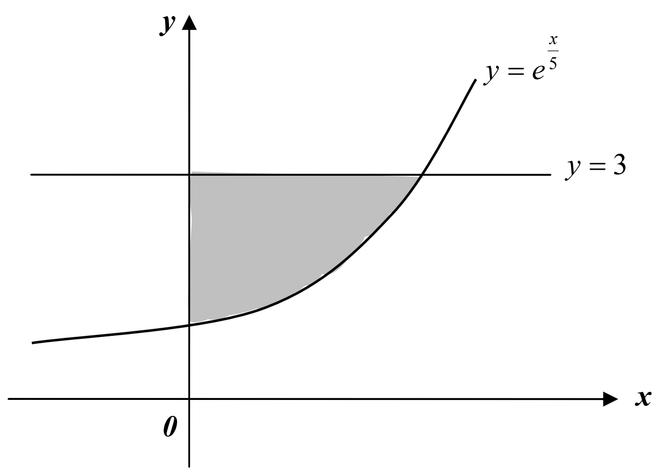
C. $-2\ \sin\left(\frac{x}{2}\right)$

D. $2\ \sin\left(\frac{x}{2}\right) f'\left(4\cos\left(\frac{x}{2}\right)\right)$

E. $-2\ \sin\left(\frac{x}{2}\right) f'\left(4\cos\left(\frac{x}{2}\right)\right)$

Question 22

Parts of the graphs with equations $y = e^{\frac{x}{5}}$ and $y = 3$ are shown below:



The total area bounded by the curves with equations $y = e^{\frac{x}{5}}$, y = 3 and $x = 0$ is given by

A. $\int_0^{5\log_e 3} \left(e^{\frac{x}{5}} - 3\right) dx$

B. $\int_0^{5\log_e 3} \left(3 - e^{\frac{x}{5}}\right) dx$

C. $\int_{5\log_e 3}^{0} \left(3 - e^{\frac{x}{5}}\right) dx$

D. $\int_0^{15} \left(3 - e^{\frac{x}{5}}\right) dx$

E. $\int_1^{3} \left(3 - e^{\frac{x}{5}}\right) dx$

END OF SECTION 1

Instructions for Section 2

A decimal approximation will not be accepted if the question specifically asks for an **exact** answer. In questions worth more than one mark, appropriate working **must** be shown.
The diagrams are not drawn to scale. Marks are given as specified for each question. Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Consider the function $f : (-2, \infty] \to R, f(x) = (1+x)^2(3-x) - 4$.

a. If $f'(x) = -(1+x)(ax+b)$, where a and b are constants, use calculus to find the values of a and b.

3 marks

b. Find the coordinates of the turning points of the graph of $y = f(x)$ and state their nature.

2 marks

SECTION 2- Question 1- continued
TURN OVER

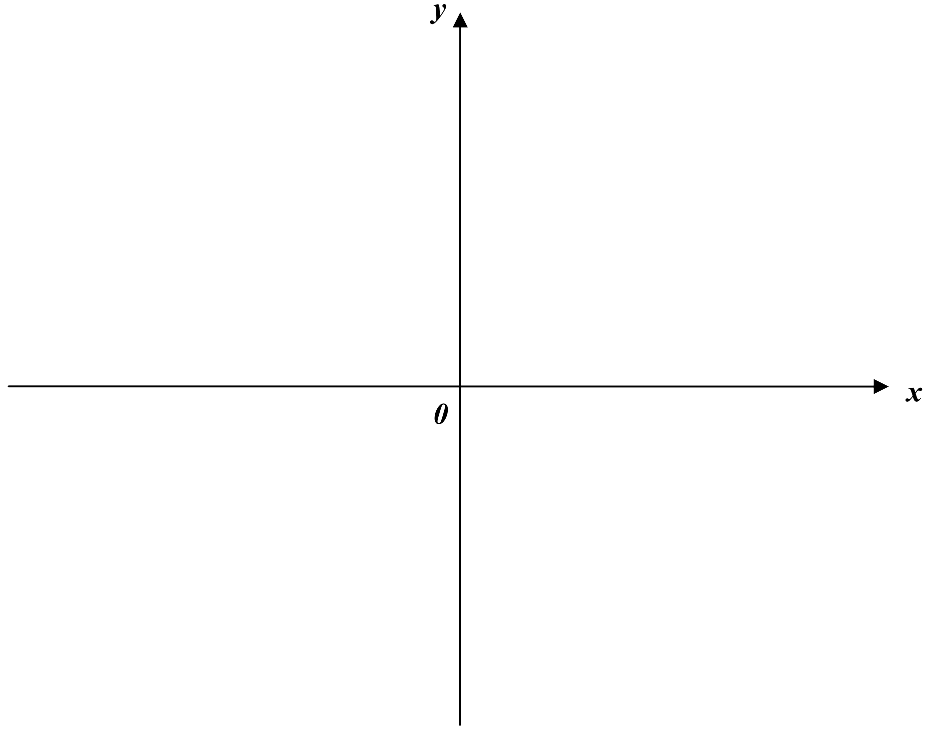
c. **i.** Find an equation of the form $g(x) = -x^2 + bx + c$ which joins the curve $f(x)$ smoothly at the point where $x = -2$.

ii. Find the coordinate/s of the turning point/s of the graph of $y = g(x)$.

4 + 2 = 6 marks

SECTION 2- Question 1- continued

d. On the axis below, sketch the graph of the function defined by $h(x) = \begin{cases} g(x) & \text{if } x \le -2 \\ f(x) & \text{if } x \ge -2 \end{cases}$. Clearly show any intercepts and label all stationary points.



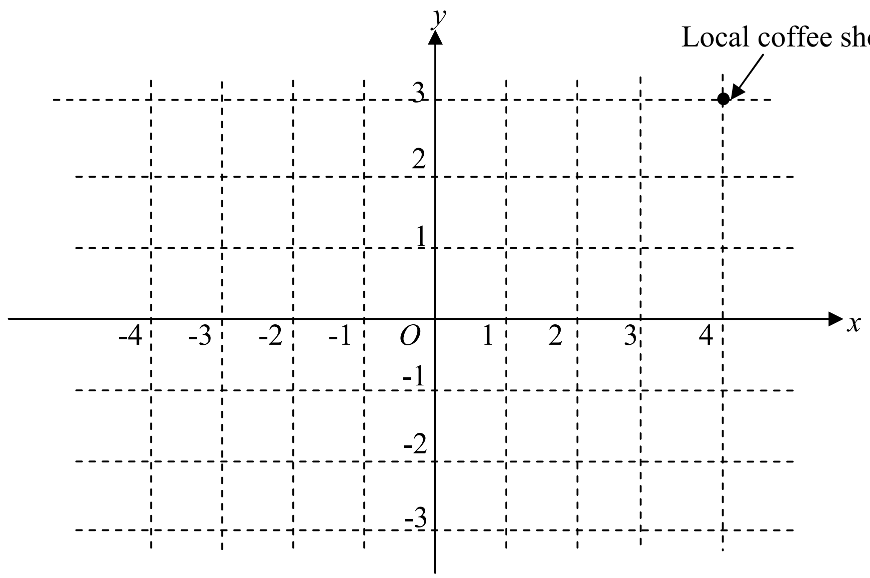
3 marks
Total 14 marks

Question 2

In a regional town, it is decided that a new bicycle track should be built. The council has specified that the road will pass through the local coffee shop and follow the path with equation

$$y = e^{bx}\left(x^2 - 3x\right)$$

The grid below illustrates the location of the local coffee shop. In each direction of the grid, 1 unit represents 500m.

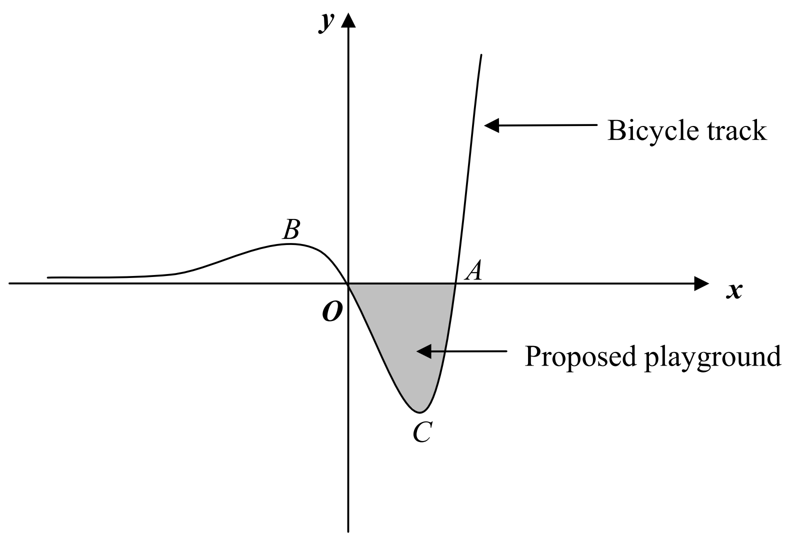


a. Find the **exact** value of *b*.

3 marks

SECTION 2- Question 2- continued

After consulting a civil engineering firm and refining their design, the council has decided to build the track for which $b = 1$ as shown in the diagram below.



b. Find the x-coordinate of the point A.

2 marks

c.

i. Find $\frac{dy}{dx}$ in simplest factorised form.

ii. Calculate the coordinates of the turning points B and C giving values correct to 2 decimal places.

4 + 2 = 6 marks

The council has decided to construct a playground on the shaded area by the bicycle track. This will be a handy playground for the young children of the town.

d. Find the values of p and q for which

$$\frac{d}{dx}\left[e^x\left(x^2+px+q\right)\right]=e^x\left(x^2-3x\right)$$

And hence find the exact area of the playground.

7 marks

Total 18 marks

Question 3

John studies mathematics each night, either at home or at the local library. The place he studies at each night depends only on the place that he studied at the night before. If John studies at home one night, then the probability of him studying at home the next night is 0.4. If John studies at the library one night, then the probability of him studying at the library the next night is 0.3.

a. Suppose John studies at home one Wednesday night.

i. What is the probability that John studies at home on exactly two of the next three nights?

ii. What is the probability that John studies at home on Friday night?

3 + 1 = 4 marks

The time, t, in hours that John spends studying each night is independent of the place that he studies in and is a random variable with probability density function

$$f(t) = \begin{cases} \frac{6}{125} t(5-t) & \text{if } 0 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$$

b. What is the probability that John spends longer than 3 hours studying on a night?

2 marks

SECTION 2- Question 3- continued

TURN OVER

c. What is the probability, correct to three decimal places, that John spends longer than 3 hours studying on at least two out of three nights?

2 marks

d. On 30% of nights John studies less than n minutes. Find the value of n.

4 marks
Total 12 marks

Question 4

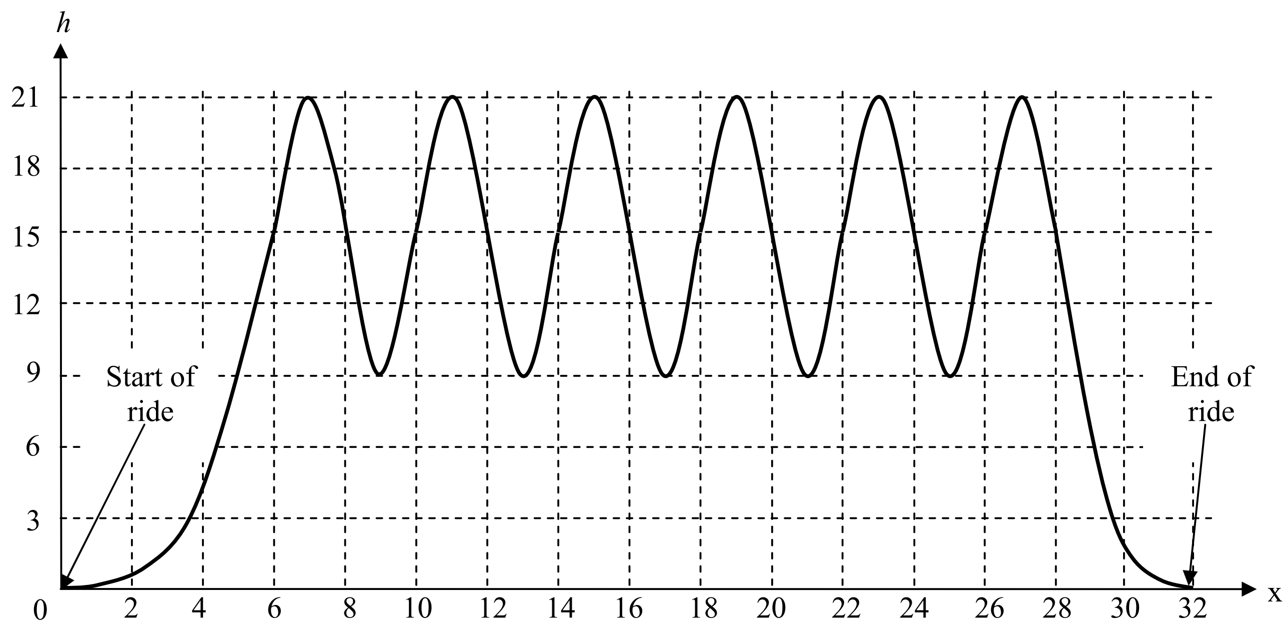
A new theme park is developing a rollercoaster ride, where the leading engineer, Peter is developing the path that the rollercoaster will travel. While brainstorming, Peter took down the following notes:

- The rollercoaster ride will start and finish on the ground.
- The rollercoaster ride will end at a different position to where it starts. It will deploy its passengers and then make its way back to start the ride again.

Peter, has suggested that **part** of the rollercoaster's path should follow the equation:

$$h(x) = 6\sin\frac{\pi}{2}(x-2)+15,\ 6 \le x \le 28$$

where h m is the height of the rollercoaster at any given time t seconds above the ground. A diagram illustrating the entire path of the rollercoaster's ride is shown below:



a. State the maximum height, in metres, of the rollercoaster above ground level.

1 mark

b. State the minimum height, in metres, of the rollercoaster above ground level where $t \in [6,28]$.

1 mark

SECTION 2- Question 4- continued
TURN OVER

c. Find x, where the height of the rollercoaster first reaches 17 m above the ground level. Give your answer correct to three decimal places.

2 marks

Peter has established that the incline path of the rollercoaster ride can be described by the general equation:

$$f(x) = ax^3 + bx^2 + cx + d \ , \ \ 0 \le x \le 6$$

Where f m is the height of the rollercoaster at any given distance x m from the starting point and a, b, c and d are constants.

d.

i. Find an expression, in terms of x, for the rate of change of h with respect to distance. **Hence** find the rate of change of the height of the rollercoaster above the ground level when $x = 6$.

ii. Considering that the rollercoaster must join **smoothly** at the point where $x = 6$, find an equation, in terms of a, b and c equating the rate of change of f and h at this point.

SECTION 2- Question 4- continued

iii. Considering that the rollercoaster begins at ground level with zero gradient, find the exact values of a, b, c and d.

2 + 2 + 4 = 8 marks

Following a similar construction, the decline path of the rollercoaster must joint smoothly at $x = 28$ and then come to a complete stop 32 metres after the commencement of the rollercoaster ride. This will give a symmetrical ride.

e. Use calculus to find the area bounded by the rollercoaster path and the x-axis on the interval $0 \le x \le 16$ (half the ride). Give your answer correct to three decimal places.

4 marks

Total 16 marks

END OF QUESTION AND ANSWER BOOK