The Mathematical Association of Victoria

MATHEMATICAL METHODS

Trial written examination 2

2006

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2006 Mathematical Methods Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

This Trial Examination is licensed to the purchasing school or educational organisation with permission for copying within that school or educational organisation. No part of this publication may be reproduced, transmitted or distributed, in any form or by any means, outside purchasing schools or educational organisations or by individual purchasers, without permission.

Published by The Mathematical Association of Victoria
"Cliveden", 61 Blyth Street, Brunswick, 3056
Phone: (03) 9380 2399 Fax: (03) 9389 0399
E-mail: office@mav.vic.edu.au Website: http://www.mav.vic.edu.au

© MATHEMATICAL ASSOCIATION OF VICTORIA 2006

MULTIPLE CHOICE ANSWER SHEET

Student Name: ....

Circle the letter that corresponds to each correct answer

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**
11.	**A**	**B**	**C**	**D**	**E**
12.	**A**	**B**	**C**	**D**	**E**
13.	**A**	**B**	**C**	**D**	**E**
14.	**A**	**B**	**C**	**D**	**E**
15.	**A**	**B**	**C**	**D**	**E**
16.	**A**	**B**	**C**	**D**	**E**
17.	**A**	**B**	**C**	**D**	**E**
18.	**A**	**B**	**C**	**D**	**E**
19.	**A**	**B**	**C**	**D**	**E**
20.	**A**	**B**	**C**	**D**	**E**
21.	**A**	**B**	**C**	**D**	**E**
22.	**A**	**B**	**C**	**D**	**E**

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The graph of $y = \dfrac{2}{\sqrt[3]{(x-1)}} + 3$ has asymptotes with equations

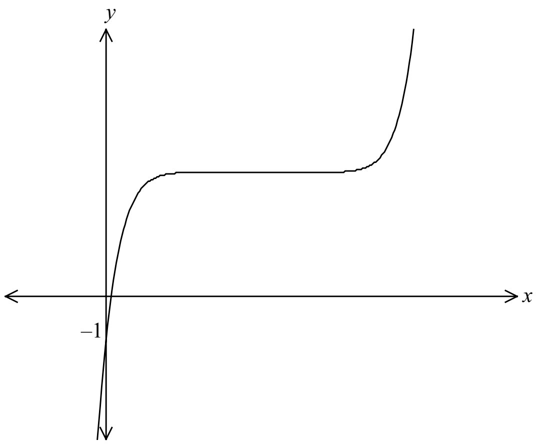
A. $x = 1$ and $y = -3$
B. $x = 1$ and $y = 3$
C. $x = -1$ and $y = 3$
D. $x = -1$ and $y = -3$
E. $x = 0$ and $y = 3$

Question 2

The number of distinct real solutions to the equation $(x-a)^3(x+b)^2(x^2-c) = 0$, where a, b, and c are **negative** real constants and $a \neq b \neq c$ is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 3



A possible equation for the graph above could be

A. $y = (x-1)^9 + 3$
B. $y = (x-1)^{10} + 3$
C. $y = -(x-1)^9 + 3$
D. $y = -4(x-1)^{10} + 3$
E. $y = 4(x-1)^9 + 3$

Question 4

If $h: R\backslash\{1\} \to R$, where $h(x) = \dfrac{1}{x+1}$ and $k: R \to R$, where $k(x) = |2x|$ then the **range** of $h(k(x))$ is

A. $(0, 1]$

B. $[0, \infty)$

C. $R\backslash\{0\}$

D. $(0, \infty)$

E. R

Question 5

Consider $f: R \to R$, where $f(x) = 2e^x$ and $g: R \to R$, where $g(x) = -x + 1$. The graph of $f(g(x))$ is obtained from the graph of $f(x)$ by which of the following transformations?

A. A reflection in the x-axis, followed by a translation of 1 unit parallel to the x-axis.

B. A dilation by a factor of 2 from the x-axis, followed by a reflection in the y-axis and then a translation of 1 unit parallel to the x-axis.

C. A reflection in the y-axis, followed by a translation of 1 unit parallel to the x-axis.

D. A reflection in the y-axis, followed by a translation of –1 units parallel to the x-axis.

E. A reflection in the y-axis, followed by a translation of 1 unit parallel to the y-axis.

Question 6

If $f: R \to R$, where $f(x) = 3e^{-2x} + 1$ then the inverse function, f^{-1}, is defined as

A. $f^{-1}: R \to R$, where $f^{-1}(x) = \log_e\left(\sqrt{\dfrac{3}{x-1}}\right)$

B. $f^{-1}: (-\infty, 1) \to R$, where $f^{-1}(x) = \log_e\left(\sqrt{\dfrac{3}{x-1}}\right)$

C. $f^{-1}: (1, \infty) \to R$, where $f^{-1}(x) = \log_e\left(\dfrac{1}{3\sqrt{x-1}}\right)$

D. $f^{-1}: (1, \infty) \to R$, where $f^{-1}(x) = \log_e\left(\sqrt{\dfrac{x-1}{3}}\right)$

E. $f^{-1}: (1, \infty) \to R$, where $f^{-1}(x) = \log_e\left(\sqrt{\dfrac{3}{x-1}}\right)$

Question 7

The solutions to the equation $2^{2x} - 9 \times 2^x + 8 = 0$ are

A. $x = 1$ or $x = 8$

B. $x = 0$ or $x = 3$

C. $x = 0$ or $x = 4$

D. $x = 1$ or $x = 3$

E. $x = 1$ or $x = 4$

Question 8

If $2\log_e(|x-1|)+\log_e(9)=\log_e(a^2)$, where a is a positive real constant, then x equals

A. $\frac{a}{3}-1$ only

B. $-\frac{a}{3}-1$ only

C. $1+\frac{a}{3}$ or $1-\frac{a}{3}$

D. $\frac{a}{3}+1$ only

E. $1-\frac{a}{3}$ only

Question 9

The sum of the solutions to the equation $\sqrt{3}\tan(2\theta)+1=0$ for $0\le\theta\le 2\pi$ is

A. $\frac{4\pi}{3}$

B. $\frac{14\pi}{3}$

C. $\frac{5\pi}{12}$

D. $\frac{10\pi}{3}$

E. $\frac{2\pi}{3}$

Question 10

The range and period, respectively, of the function with the rule $f(t)=1-5\cos\left(\frac{\pi t}{12}\right)$ are

A. $[-4,6]$ and $\frac{\pi}{12}$

B. $[-5,5]$ and $\frac{\pi}{12}$

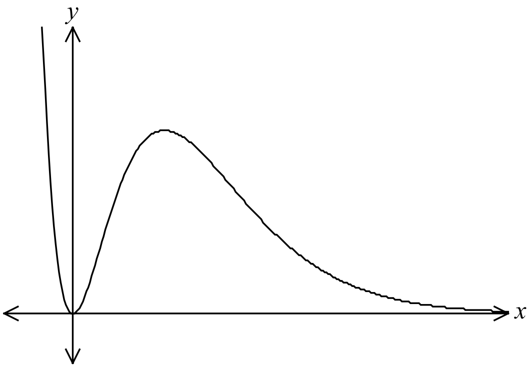
C. $[-4,4]$ and 24

D. $[-4,6]$ and 12

E. $[-4,6]$ and 24

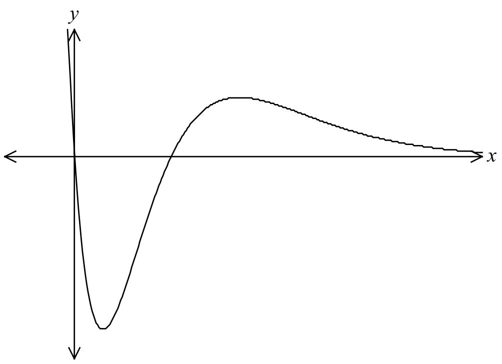
Question 11

Part of the graph of a function, f, is shown below.

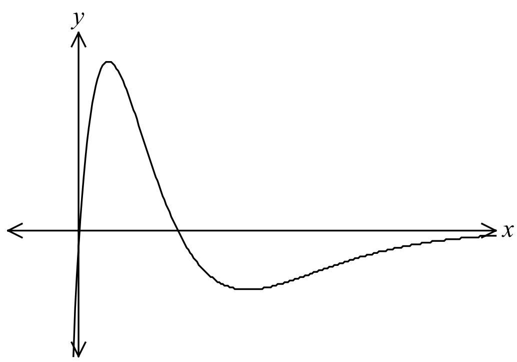


Which one of the following could be the graph of the gradient function, f'?

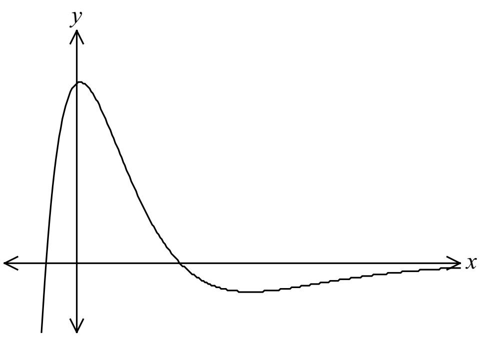
A.



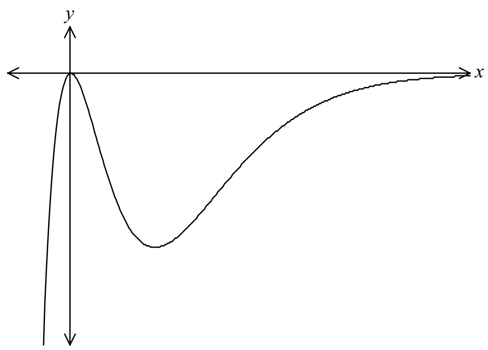
B.



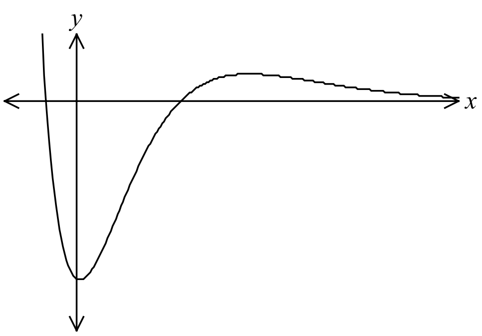
C.



D.



E.



Question 12

For $f:[0,\pi] \to R,\ f(x) = 4\cos\left(\frac{x}{2}\right)$, the x-coordinate of the point where the graph of f has a gradient of -1 is

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

E. π

Question 13

The approximate value for $\frac{1}{\sqrt[3]{1.1}}$, using the linear approximation $f(x+h) \approx f(x) + hf'(x)$ is

A. $-\frac{1}{30}$

B. $\frac{1}{30}$

C. 0.969

D. $\frac{29}{30}$

E. $1\frac{1}{30}$

Question 14

Let $f(x) = \sqrt{(-x+2)}$ and $g(x) = \sqrt{(x+3)}$. If $h(x) = f(x)g(x)$ then the maximal domain of the derivative of h is

A. $(-3,-2)$

B. $[-3,2]$

C. $(-3,2)$

D. $[-2,3]$

E. $(-2,3)$

Question 15

Consider $y = \left(\frac{x}{2} - 3\right)^8$ where x and y are functions of t. If $\frac{dx}{dt} = 3$, then $\frac{dy}{dt}$, in terms of x, equals

A. $12\left(\frac{x}{2} - 3\right)^7$

B. $24\left(\frac{x}{2} - 3\right)^7$

C. $4\left(\frac{x}{2} - 3\right)^7$

D. $12\left(\frac{x}{2} - 3\right)^8$

E. $\frac{4}{3}\left(\frac{x}{2} - 3\right)^7$

Question 16

If $x \in R\backslash\left\{\frac{1}{2}\right\}$, an antiderivative of $\dfrac{1}{1-2x}$ is

A. $-\dfrac{1}{2}\log_e(|2x-1|)$

B. $-2\log_e(|2x-1|)$

C. $\dfrac{1}{2}\log_e(1-2x)$

D. $-\dfrac{1}{2}\log_e\left(\dfrac{1}{1-2x}\right)$

E. $-2\log_e\left(\dfrac{1}{1-2x}\right)$

Question 17

The area bounded by the x-axis, the lines with equations $x = 0$ and $x = 2$ and the curve with equation $y = x^{\frac{1}{4}}$, is approximated using left-end rectangles of width $\dfrac{1}{2}$ unit. The area, in square units, of these rectangles can be found by evaluating

A. $\dfrac{1}{2}\left(\left(\dfrac{1}{2}\right)^{\frac{1}{4}} + 1 + \left(\dfrac{3}{2}\right)^{\frac{1}{4}}\right)$

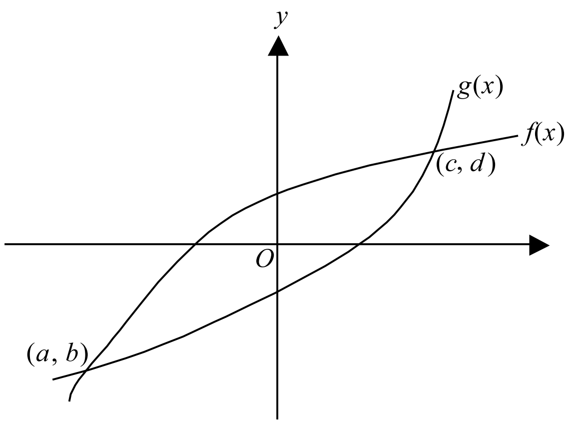
B. $\dfrac{1}{2}\left(\left(\dfrac{1}{2}\right)^{\frac{1}{4}} + 1 + \left(\dfrac{3}{2}\right)^{\frac{1}{4}} + (2)^{\frac{1}{4}}\right)$

C. $\dfrac{2^{\frac{13}{4}}}{5}$

D. $\left(\dfrac{1}{2}\right)^{\frac{1}{4}} + 1 + \left(\dfrac{3}{2}\right)^{\frac{1}{4}}$

E. $\left(\dfrac{1}{2}\right)^{\frac{1}{4}} + 1 + \left(\dfrac{3}{2}\right)^{\frac{1}{4}} + (2)^{\frac{1}{4}}$

Question 18



The area bounded by the graphs of $f(x)$ and $g(x)$ shown above, where (a,b) and (c,d) are the coordinates of the points of intersection of the two graphs, is equal to

A. $\int_b^d (f(x) - g(x))dx$

B. $\int_{-a}^c (f(x) - g(x))dx$

C. $\int_a^c (g(x) - f(x))dx$

D. $\int_a^c (f(x) - g(x))dx$

E. $\int_b^d (g(x) - f(x))dx$

Question 19

A discrete random variable, X, has the probability distribution shown below, where t is a real constant.

x	-2	-1	0	1	2
$\Pr(X = x)$	$0.2t$	t^2	$0.5t$	t^2	$0.3t$

For this distribution, t can equal

A. $\frac{1}{2}$ only

B. $\frac{1}{2}$ or -1

C. 0 only

D. 1 only

E. 1 or -1

Question 20

$f: R \rightarrow R$ where $f(x) = ke^{-(x^2/2)}$, where k is a real constant, is the probability density function of a continuous random variable, X. Given that $\int_{-\infty}^{\infty} e^{-(x^2/2)}dx = \sqrt{2\pi}$ and $\int_{-\infty}^{\infty} xe^{-(x^2/2)}dx = 0$, the value of k and the expected value of X, respectively, are

A. $\sqrt{2\pi}$ and 1

B. $\sqrt{2\pi}$ and 0

C. $\sqrt{2\pi}$ and 0.5

D. $\frac{1}{\sqrt{2\pi}}$ and 0

E. $\frac{1}{\sqrt{2\pi}}$ and 0.5

Question 21

The length of bamboo stakes cut by a particular machine is a normally distributed random variable, X, with a mean of 250 mm. Z is the standard normal random variable. If $\Pr(X > 241) = 1 - \Pr(Z < -1.5)$ then the standard deviation of the length of the stakes, in mm, is

A. 12

B. 9

C. 6

D. 3

E. 1.5

Question 22

Cassandra passed three sets of traffic lights while driving to school to sit a CAS Trial Exam. The lights at each intersection operate independently of each other. The probability of having to stop for a red light is $\frac{1}{3}$ and the probability of passing through a set of green lights is $\frac{2}{3}$. If Cassandra encountered a red light at least once, the probability that she encountered exactly two red lights is

A. $\frac{3}{4}$

B. $\frac{6}{19}$

C. $\frac{8}{27}$

D. $\frac{2}{27}$

E. $\frac{2}{9}$

END OF SECTION 1

Working space

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

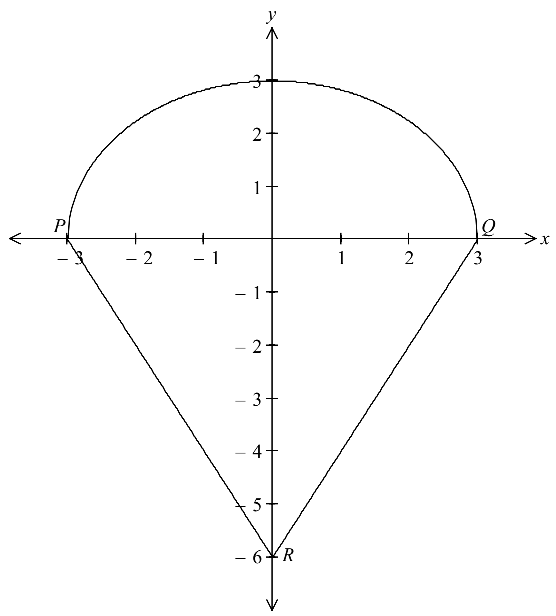
In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

An ice cream is cut in half along its length. The cross-section of the ice cream and its cone are shown in the diagram below. The cone is very thin. The original cone was filled with ice cream and there was ice cream at the top of the cone in the shape of a hemisphere. The radius of the cone is 3 cm.



a. **i.** Let $f:[-3,3] \rightarrow R$, where $f(x) = |ax| + b$, and a is a positive real constant and b is a negative real constant, represent the edge of the cone in the diagram above. Determine the valus of a and b.

ii. The rule for the semicircle, in the above diagram, is in the form $y = \sqrt{c - x^2}$ where c is a constant. Find the value of c.

2 + 1 = 3 marks

b. Use calculus to show that

i. the area of the triangle, PQR, shown above, is 18 cm^2.

ii. the area of the semicircle shown above is $\frac{9\pi}{2}$ cm^2, given that

$$\frac{d}{dx}\left(\frac{c\sin^{-1}\left(\frac{x}{\sqrt{c}}\right)}{2}+\frac{x\sqrt{c-x^2}}{2}\right)=\sqrt{c-x^2}$$

2 + 3 = 5 marks

Mike buys one of these ice creams. He licks the ice cream off the top of the cone so that only the cone filled with ice cream remains. Unfortunately, an ant then bites a hole in the bottom of his cone. Ice cream starts to leak from the cone at a rate of π cm^3/s.

c. **i.** At what rate is the radius of the ice cream decreasing, with respect to time, when the radius is 2 cm?

ii. When will there be no ice cream left in the cone?

4 + 2 = 6 marks

Total 14 marks

Question 2

Let $f:[-5,5] \to R$, where $f(x) = x^2 - 10$ and $g(x) = |x|$.

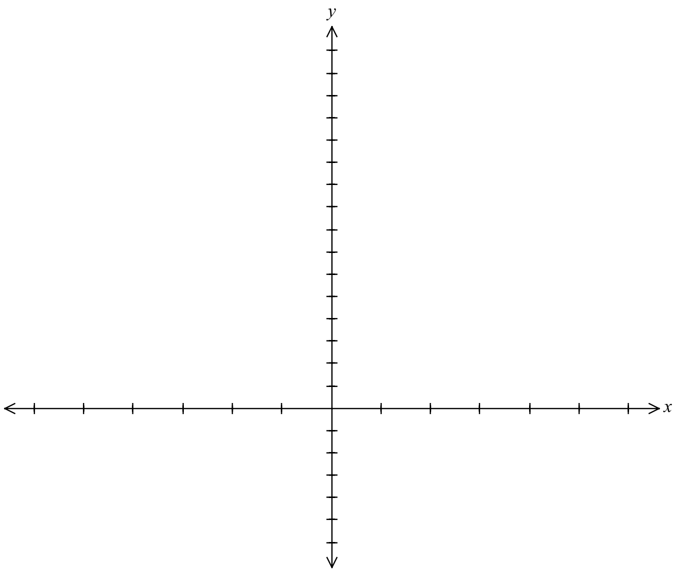
a. Consider the composite function $g(f(x))$.

i. Find the rule for $g(f(x))$.

ii. Find the domain of $g(f(x))$.

1 + 1 = 2 marks

b. Sketch the graph of $y = g(f(x))$ on the set of axes below, labelling the coordinates of the endpoints and intercepts.



3 marks

c. For what values of *k*, where *k* is a real constant, does $g(f(x)) = k$ have

i. 4 solutions?

ii. 2 solutions?

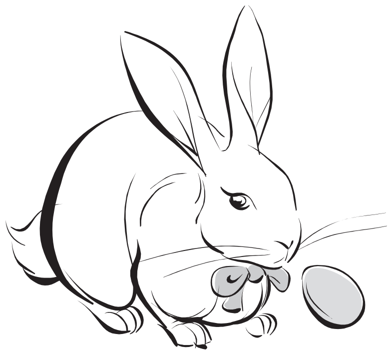
1 + 2 = 3 marks

The region bounded by the line with equation $y = 15$ and the curve of $g(f(x))$, forms the side view of a bridge over a stream. This side of the bridge needs to be painted.

d. Find the cost of the paint, to the nearest dollar, which will be needed if paint costs \$50 per square unit.

2 marks

Before the bridge had been built, rabbits fed on one side of the stream only. There were no rabbits on the other side of the stream. Due to the drought and overgrazing, pasture was becoming scarce. With the introduction of the bridge, rabbits were able to cross the waterway to better pastures. The population, r, of rabbits, t days after the opening of the bridge, can be modelled by the function $r:[0,20] \rightarrow R$, where $r(t) = 5e^{Bt} + C$, and B and C are real constants. On day 14 the population reached 100 rabbits.



Show that

e. **i.** $C = -5$

ii. $B = \dfrac{\log_e(21)}{14}$

1 + 2 = 3 marks

f. According to the model, during what day did the rabbit population first reach 200?

1 mark

At the same time, the number of foxes started to increase in the area. The model for the number of foxes is given by the inverse function, r^{-1}.

g. Determine the rule for r^{-1}.

2 marks

h. How many foxes were there on day 20?

1 mark

Total 17 marks

Question 3

A child's toy consists of a length of smooth track and two gliders that move on the track, as illustrated in Figure 1. The black glider is free to slide down the ramped section of track. The striped glider is attached, by a spring, to a solid barrier at point *O*. When the carriages collide they become coupled and oscillate (move backwards and forwards) about the point *M*, as illustrated in Figure 2.

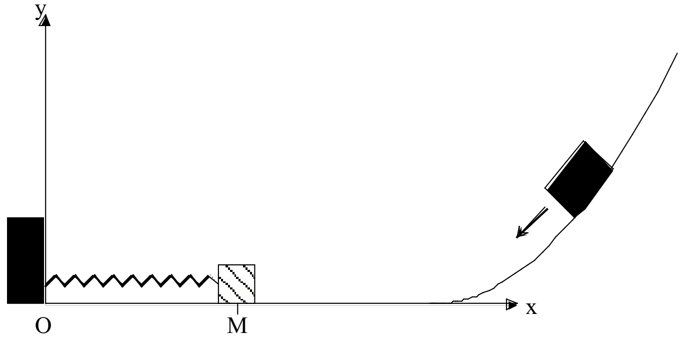


Figure 1

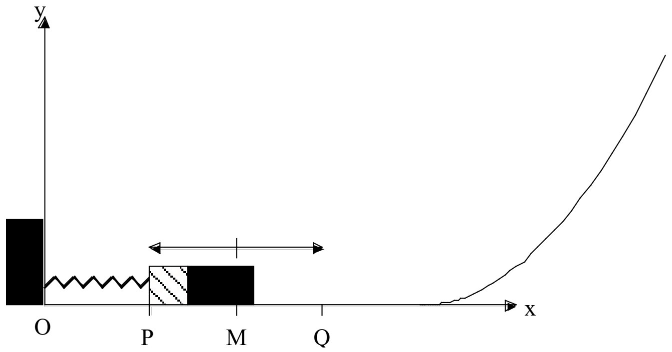


Figure 2

When the coupled gliders are oscillating, the distance, *x* cm, from *O* to the near end of the glider, *t* seconds after the gliders first moved in the direction of the positive *x*-axis, is modelled by the function

$x(t) = 30 - 20\sin\left(\frac{5\pi}{4}\left(t + \frac{2}{5}\right)\right),\ 0 \le t \le 6.4.$

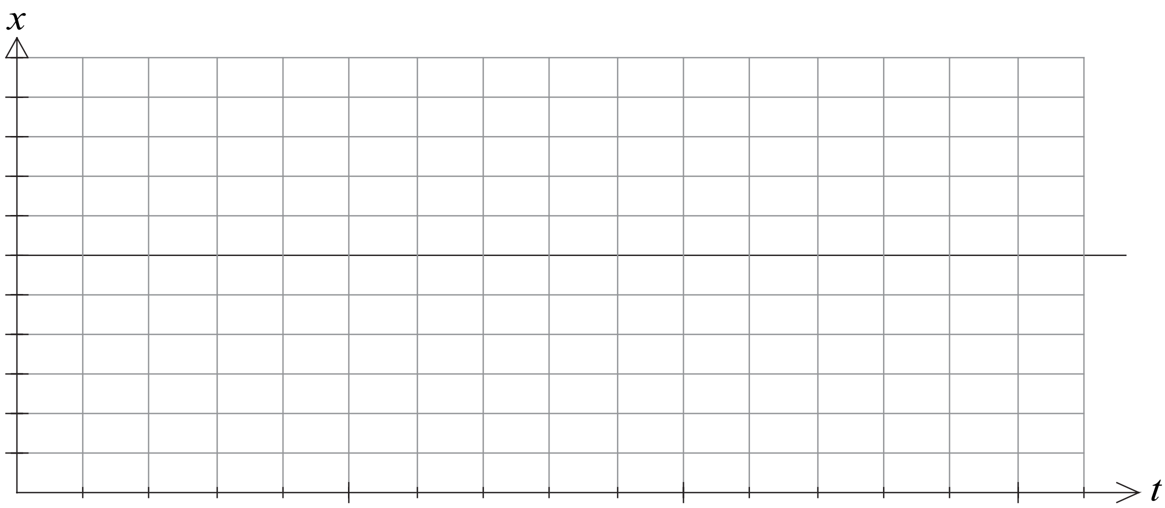
a. What are the maximum and minimum values of *x*?

2 marks

b. What is the period of oscillation of the gliders?

1 mark

c. Sketch the graph of x for the first two oscillations of the gliders. Label the coordinates of maximum and minimum points and end points.



3 marks

d. In the first oscillation, use your calculator to find the length of time that the gliders spend at a distance of more than 20 cm from O.

2 marks

e. Find the average rate of change of x with respect to time for $t = 0$ to $t = \frac{4}{5}$.

2 marks

One of the gliders became damaged and created much greater friction with the track. The oscillating motion of the gliders was quickly reduced. This motion was modelled by the function
$s(t) = 30 - 20e^{\frac{-t}{10}}\cos\left(\frac{5\pi}{4}t\right)$, $t \geq 0$, where s is the distance in cm from O to the near end of the glider.

f. Write an expression, in terms of t, for the rate of change of s with respect to t.

2 marks

Total 12 marks

Question 4

A social researcher carried out a lifestyle survey on people in two different areas, *A* and *B*. Mathematical models were then developed from the results.

a. The time spent each week on sport and recreation by people from area *A* is a continuous random variable, *T*, with probability density function

$$f(t) = \begin{cases} -kt^2(t-5) & 0 \le t \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

where k is a positive real constant.

i. Show that $k = 0.0192$.

ii. Find the **mode** of the distribution, correct to 2 decimal places.

iii. A person is chosen at random from area *A*. Write an expression for the probability that he or she spends between two and four hours per week on sport and recreation. Hence, find the probability.

iv. Suppose that 20% of the population is classified as belonging to area *A*. The researcher surveyed six randomly selected people. What is the probability, correct to four decimal places, that no more than two of the six people were from area *A*?

2 + 2 +2 + 2 = 8 marks

b. The time spent on sport and recreation each week by people from area *B* is a normally distributed random variable, *X*, with a mean of 6 hours and a standard deviation of 90 minutes.
A person from area *B* is selected at random. If he or she spends at least 5 hours per week on sport and recreation, what is the probability, correct to four decimal places, that he or she spends less than 8 hours?

3 marks

c. The researcher also studied the dietary habits of 100 office workers who buy their lunch every working day at a particular food outlet. The workers choose their lunches from either the Fast menu or from the new Healthy menu. The researcher found that the menu choice each day depended only on the choice made the previous day. If a worker chose from the Fast menu one day, there was a probability of 0.4 of choosing from the Healthy menu the following day. However, if a worker chose from the Healthy menu one day the probability of choosing from the Fast menu the following day was 0.2.
On Monday, 35 workers chose from the Healthy menu and 65 chose from the Fast menu. How many workers will choose from the Healthy menu on Wednesday of the same week? Give the answer correct to the nearest integer.

4 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

© VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY 2006

Version 2 – March 2006

REPRODUCED WITH PERMISSION MATHEMATICAL ASSOCIATION OF VICTORIA 2006

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET

Version 2 – March 2006

© VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY 2006
REPRODUCED WITH PERMISSION MATHEMATICAL ASSOCIATION OF VICTORIA 2006