Year 2006

VCE

Mathematical Methods

Examination 1

Suggested Solutions

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Solutions Mathematical Methods Examination 1 2006

Question 1

a. $f(x) = x^2 + 1$ and $g(x) = 2x + 1$

$f(g(x)) = f(2x+1) = (2x+1)^2 + 1 = 4x^2 + 4x + 2$

Question 2

a. $f : R \to R$, $f(x) = 3e^{2x} - 1$ so the function is f: $y = 3e^{2x} - 1$

the inverse f^{-1}: $x = 3e^{2y} - 1$ transposing $x + 1 = 3e^{2y}$

$\frac{x+1}{3} = e^{2y}$ $2y = \frac{1}{2}\log_e\left(\frac{x+1}{3}\right)$

$f^{-1}(x) = \frac{1}{2}\log_e\left(\frac{x+1}{3}\right)$

b. dom f^{-1} = ran $f = (-1, \infty)$

Question 3

a. $f(x) = e^{\cos(x)}$ using the chain rule

$y = e^u$ where $u = \cos(x)$

$\frac{dy}{du} = e^u$ $\frac{du}{dx} = -\sin(x)$

$f'(x) = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -\sin(x)e^{\cos(x)}$

b. $y = x\tan(x)$ product rule

$\frac{dy}{dx} = \tan(x) + x\sec^2(x) = \tan(x) + \frac{x}{\cos^2(x)}$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = \tan\left(\frac{\pi}{6}\right) + \frac{\frac{\pi}{6}}{\cos^2\left(\frac{\pi}{6}\right)} = \frac{1}{\sqrt{3}} + \frac{\frac{\pi}{6}}{\left(\frac{\sqrt{3}}{2}\right)^2}$

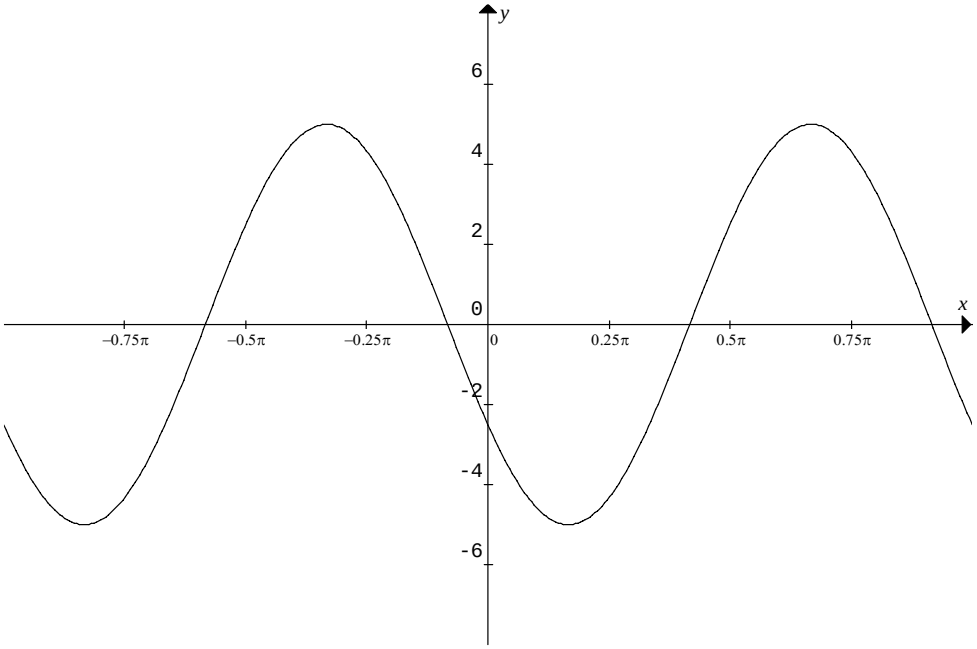
$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = \frac{\sqrt{3}}{3} + \frac{2\pi}{9} = \frac{3\sqrt{3} + 2\pi}{9}$

Question 4

a. $f : [-\pi, \pi] \to R$, $f(x) = 5\cos\left(2\left(x + \frac{\pi}{3}\right)\right)$

amplitude is 5 period is $\frac{2\pi}{2} = \pi$

b.



end-points $(-\pi, -2.5)$ $(\pi, -2.5)$ y-intercept $(0, -2.5)$

x-intercepts $\left(-\frac{7\pi}{12}, 0\right), \left(-\frac{\pi}{12}, 0\right), \left(\frac{5\pi}{12}, 0\right), \left(\frac{11\pi}{12}, 0\right)$ since the period is π

Question 5

$X \stackrel{d}{=} N\left(\mu = 150, \sigma^2 = 15^2\right)$

a. $\Pr(X > 80)$

$= \Pr\left(Z > \frac{80-72}{8}\right) = \Pr(Z > 1)$

$= 1 - \Pr(Z < 1) = 1 - 0.84$

$= 0.16$

b. $\Pr(64 < X < 72)$

$= \Pr\left(\frac{64-72}{8} < Z < \frac{72-72}{8}\right) = \Pr(-1 < Z < 0)$

$= \Pr(Z < 0) - \Pr(Z < -1.0) = 0.5 - (1 - 0.84)$

$= 0.34$

c. $\Pr(X < 64 / X < 72)$

$= \frac{\Pr(X < 64)}{\Pr(X < 72)} = \frac{\Pr(Z < -1)}{\Pr(Z < 0)}$

$= \frac{0.16}{0.5}$

$= 0.32$

Question 6

$$f(x)=\begin{cases}\dfrac{x}{12} & 1\le x\le 5\\ 0 & \text{otherwise}\end{cases}$$

a. $\Pr(X<3)=\displaystyle\int_1^3 \frac{x}{12}\,dx$

$$\Pr(X<3)=\left[\frac{x^2}{24}\right]_1^3=\frac{1}{24}(9-1)$$

$$\Pr(X<3)=\frac{1}{3}$$

b. $\Pr(X\ge a)=\displaystyle\int_a^5 \frac{x}{12}\,dx$

$$\Pr(X\ge a)=\left[\frac{x^2}{24}\right]_a^5=\frac{1}{24}\left(25-a^2\right)=\frac{5}{8}$$

$25-a^2=15$ $\quad a^2=10$

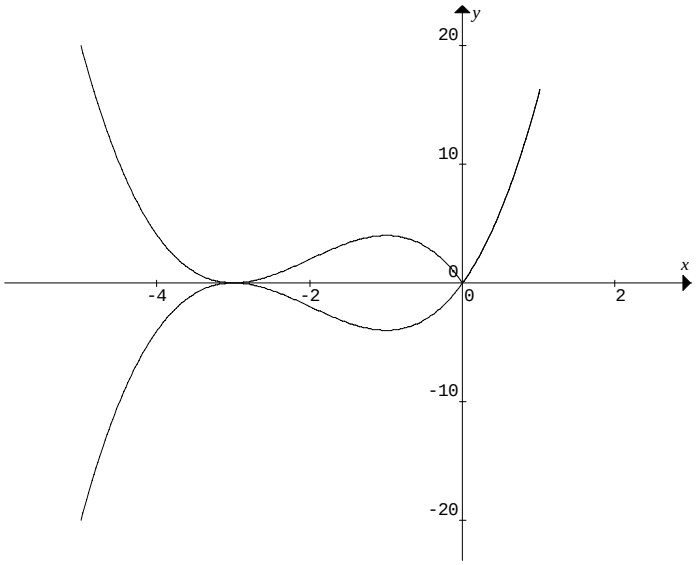
$a=\pm\sqrt{10}$ $\quad$ but $\quad 1<a<5$ $\quad$ take positive

$a=\sqrt{10}$

Question 7

$f:[-5,1]\to R$ where $f(x)=x^3+6x^2+9x$

a. graph of $y=|f(x)|$



b. $f(x)=x\left(x^2+6x+9\right)=x(x+3)^2$

$f(1)=16$ $\quad f(-5)=-5(3-5)^2=-20$

the range of $y=|f(x)|=[\,0\,,\,20]$

Question 8

$y = \sqrt{x} = x^{\frac{1}{2}} \quad \dfrac{dy}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}} = m_T$

$m_N = -2\sqrt{x} = -4$ so $\sqrt{x} = 2 \quad x = 4 \quad y = \sqrt{4} = 2$ point is $(4, 2)$ on the normal

$2 = -16 + a$

$a = 18$

Question 9

a. $A = 2ab$ but $b = 9 - 3a^2$

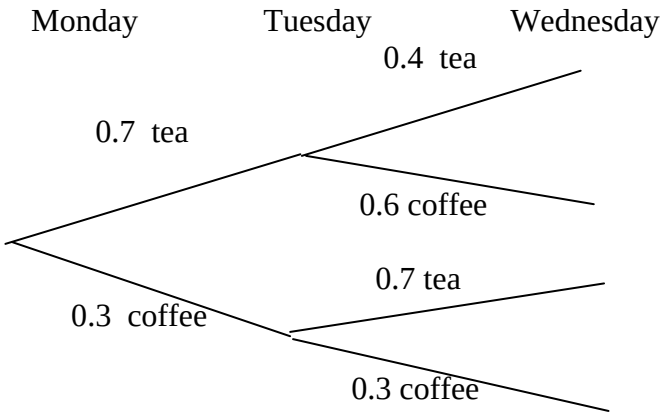
$A = 2a\left(9 - 3a^2\right)$

b. $\dfrac{dA}{da} = 18 - 18a^2 = 0$ for a maximum value of A

$18a^2 = 18 \qquad a^2 = 1 \qquad a = \pm 1$

when $a = 1 \quad A_{\max} = 12$

Question 10



$\Pr(\text{tea Wednesday}) = 0.7 \times 0.4 + 0.3 \times 0.7 = 0.28 + 0.21 = 0.49$

Question 11

$A = \int_0^3 \left(-x^2 + ax + 12\right) dx = 45$

$A = \left[-\dfrac{1}{3}x^3 + \dfrac{1}{2}ax^2 + 12\right]_0^3 = \left(-9 + \dfrac{9a}{2} + 36\right) = 45$

$\dfrac{9a}{2} = 18 \qquad a = 4$

$f(x) = -x^2 + 4x + 12 = -\left(x^2 - 4x - 12\right) = -(x + 2)(x - 6)$

$a = 4 \qquad m = 6 \qquad n = -2$

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Mathematics 2007

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