VCE
2006 Mathematical Methods
Trial Examination 2

Suggested Solutions

Kilbaha Multimedia Publishing ABN 47 065 111 373
PO Box 2227
Kew Vic 3101
Australia
Tel: 03 9817 5374
Fax: 03 9817 4334
chemas@chemas.com
www.chemas.com

PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by multiple-choice questions and extended answer questions involving multi-stage solutions of increasing complexity.

Question 1 D

Sketch the graphs on the calculator. There are three solutions at x = -0.77 , 2 and 4.

Question 2 C

$g\{f(x)\} = \log_e(x^2 - 9)$

$x^2 - 9 > 0$

$\Rightarrow x^2 > 9$

$\Rightarrow \pm x > 3$

$\Rightarrow x < -3$ or $x > 3$

Domain $(-\infty, -3) \cup (3, \infty)$

Question 3 E

The inverse only exists for a one-to-one function. The graph is one-to-one for

$-\frac{\pi}{2} \le 3x \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{6} \le x \le \frac{\pi}{6}$

Question 4 A

x^2	1	4	9	16	25
$\Pr(X = x)$	k^2	$4k^2$	$9k^2$	$16k^2$	$25k^2$

$\sum \Pr(X = x) = 1$

$\Rightarrow 55k^2 = 1$

$\Rightarrow k^2 = \frac{1}{55}$

$\Rightarrow k = \pm\frac{1}{\sqrt{55}} \times \frac{\sqrt{55}}{\sqrt{55}} = \pm\frac{\sqrt{55}}{55}$

Question 5 E

$2(1 - \cos^2\theta) - 7\cos\theta + 2 = 0$

$\Rightarrow 2 - 2\cos^2\theta - 7\cos\theta + 2 = 0$

$\Rightarrow -2\cos^2\theta - 7\cos\theta + 4 = 0$

$\Rightarrow 2\cos^2\theta + 7\cos\theta - 4 = 0$

$\Rightarrow (2\cos\theta - 1)(\cos\theta + 4) = 0$

$\Rightarrow 2\cos\theta - 1 = 0$ or $\cos\theta + 4 = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -4$

But $-1 \le \cos\theta \le 1$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}$

Sum $= \frac{\pi}{3} + \frac{5\pi}{3} = 2\pi$

Question 6 A

$f(x) = a(x+1)^2(x-3)$

When $x = 0$, $f(x) = 6$

$\Rightarrow 6 = a \times 1 \times -3$

$\Rightarrow a = -2$

$f(x) = -2(x+1)^2(x-3)$

$\Rightarrow f(x) = 2(x+1)^2(3-x)$

Question 7 B	Question 8 C
$y = 4 - \frac{1}{x-3}$ Interchange x and y $\Rightarrow x = 4 - \frac{1}{y-3}$ $\Rightarrow \frac{1}{y-3} = 4 - x$ $\Rightarrow y - 3 = \frac{1}{4-x}$ $\Rightarrow y = \frac{1}{4-x} + 3 = 3 + \frac{-1}{x-4}$ $\Rightarrow y = 3 - \frac{1}{x-4}$	$V = \frac{4}{3}\pi r^3$ $\frac{dV}{dr} = 4\pi r^2$ $\frac{dr}{dt} = 0.1$ $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = 4\pi r^2 \times 0.1 = 0.4\pi r^2$ When $r = 7$, $\frac{dV}{dt} = 0.4\pi \times 49 = 61.6 \text{cm}^3 \sec^{-1}$
Question 9 B	**Question 10 C**
Use a graphics calculator to sketch the graphs and find that the points of intersection are $x = 1.57$ and $x = 2.2$ Use the calculator to find the area between the curve and the X axis for $y = \lvert 6x - 11 \rvert$ from $x = 1.57$ to $x = 2.2$ **Area = 1.188** Use the calculator to find the area between the curve and the X axis for $y = \lvert x \rvert$ from $x = 1.57$ to $x = 2.2$ **Area = 0.576** Area between these graphs = 1.188 – 0.576 = 0.612	$\int \frac{1}{(3x+2)^{1/2}} dx$ $= \int (3x+2)^{-1/2} dx$ $= \frac{2}{3}(3x+2)^{1/2} + c$ $= \frac{2\sqrt{3x+2}}{3} + c$ $= \frac{2\sqrt{3x+2}}{3}$

Question 11 E

When $t = 0, A = A_0 = 200$

$A = 200e^{-kt}$

When $t = 20, A = 100$

$0.5 = e^{-20k}$

$\log_e 0.5 = -20k$

$k = 0.035$

$A = 200e^{-0.035t}$

$\frac{dA}{dt} = 200 \times (-0.035)e^{-0.035t}$

When $t = 30, \frac{dA}{dt} = -2.45$

The isotope is decaying at 2.45 g/day.

Question 12 E

The area in answer B can be checked as incorrect by using a graphics calculator.

$\text{gradient} = \frac{dy}{dx} = xe^x + e^x \times 1$

$= e^x(x+1) = 0$ when $x = -1$

When $x < -1, \frac{dy}{dx} < 0$

Therefore, negative gradient when $x < -1$

Question 13 A

$k\int_0^9 x^2 dx = 1$

$\Rightarrow \left.\frac{kx^3}{3}\right]_0^9 = 1$

$\Rightarrow \frac{729k}{3} - 0 = 1$

$\Rightarrow 243k = 1$

$\Rightarrow k = \frac{1}{243} = 0.004$

Question 14 B

$\sqrt{f(x)}$ only exists when $f(x) \geq 0$. That is, when y on the original graph is above or on the X axis.

<table>
<tr>
<td>

Question 15 B

The gradient is positive everywhere except when $x \le -1$ and when $x = 0$. $x = -1$ is a minimum as the graph goes from a negative to a positive gradient. $x = 0$ is a stationary point of inflexion as the graph goes from a positive to a positive gradient.

</td>
<td>

Question 16 D

$\frac{dy}{dx} = \frac{2}{2x-1} + 4 = \frac{2+8x-4}{2x-1} = \frac{8x-2}{2x-1}$

When $x = 1, \frac{dy}{dx} = 6$

So, gradient of normal $= -\frac{1}{6}$

Equation of normal is $y = -\frac{1}{6}x + c$

When $x = 1$ on curve $y = \log_e 1 + 4 = 4$

So point $(1, 4)$ lies on the normal.

Therefore, $4 = -\frac{1}{6} + c \Rightarrow \frac{25}{6} = c$

$\Rightarrow y = -\frac{1}{6}x + \frac{25}{6} \Rightarrow 6y + x - 25 = 0$

</td>
</tr>
</table>

Question 17 E

$A = \int_1^2 \frac{12}{12-5x}dx$

$= \frac{2}{-5}\int_1^2 \frac{-5}{12-5x}dx$

$= -\frac{2}{5}\log_e(12-5x)\Big]_1^2$

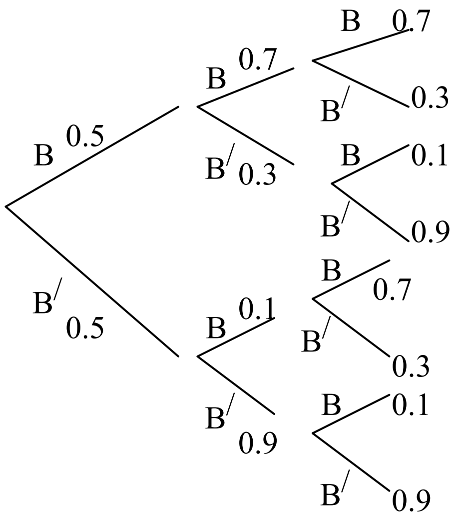
$= -\frac{2}{5}\log_e(2) + \frac{2}{5}\log_e(7)$

$= \frac{2}{5}\left[\log_e(7) - \log_e(2)\right]$

$= 0.4\log_e\left(\frac{7}{2}\right)$

$= 0.4\log_e 3.5$

Question 18 B



B B B = 0.5 × 0.7 × 0.7 = 0.245

B B/ B = 0.5 × 0.3 × 0.1 = 0.015

B/ B B = 0.5 × 0.1 × 0.7 = 0.035

B/ B/ B = 0.5 × 0.9 × 0.1 = 0.045

Total = 0.34

Question 19 C

$E(\cos x)$

$= \int_0^{\pi/2} \frac{2}{\pi}\cos x\,dx$

$= \left[\frac{2}{\pi}\sin x\right]_0^{\pi/2}$

$= \frac{2}{\pi}\left[\sin\frac{\pi}{2} - \sin 0\right]$

$= \frac{2}{\pi}\left[1-0\right] = \frac{2}{\pi} = 0.637$

Question 20 E

$\Pr(A) = \frac{1}{6}$

$\Pr(B) = \frac{1}{2}$

$\Pr(A \cap B) = \frac{3}{36} = \frac{1}{12}$

$\Pr(A \cap B) = \Pr(A) \times \Pr(B) = \frac{1}{12}$

Therefore, independent events.

$\Pr(A \cup B)$

$= \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$= \frac{1}{6} + \frac{1}{2} - \frac{1}{12} = \frac{7}{12}$

Therefore, not mutually exclusive.

Question 21 A	**Question 22 D**
Use a graphics calculator. Enter data in the **list** menu. Use **binompdf** to get probabilities. Then use **graph** to get A.	$\Pr(B \mid GG)$ $= \dfrac{\Pr(B \cap GG)}{\Pr(B \cap GG) + \Pr(Y \cap GG)}$ $= \dfrac{\frac{3}{6} \times \frac{2}{5}}{\frac{3}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{1}{5}}$ $= 0.75$

Suggested Solutions Part II

Question 1

a.

$f(x) = \dfrac{x^2}{1+x^2}$

$f'(x) = \dfrac{(1+x^2)1 - x \times 2x}{(1+x^2)^2}$

$= \dfrac{1+x^2-2x^2}{(1+x^2)^2}$

$= \dfrac{1-x^2}{(1+x^2)^2} = 0$ for turning point (1 mark)

$\Rightarrow 1 - x^2 = 0$

$\Rightarrow x = \pm 1$ (1 mark)

When $x = 1,\ f(x) = \dfrac{1}{2}$

When $x = -1,\ f(x) = \dfrac{1}{2}$

When $x < -1,\ f'(x) < 0$

When $-1 < x < 1,\ f'(x) > 0$

When $x > 1,\ f'(x) < 0$

Maximum $\left(1, \dfrac{1}{2}\right)$ Minimum $\left(-1, \dfrac{1}{2}\right)$ (1 mark)

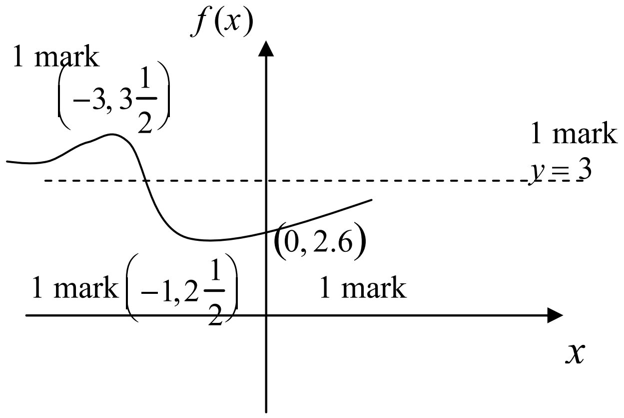
b.

- Graph is reflected in the X axis.
- Graph is translated 2 units to the left parallel to the X axis.
- Graph is translated 3 units up parallel to the Y axis.

(1 mark for each)

Question 1 (continued)

c.



d.

(-1 , 1) (1 mark)

e.

$$y = \frac{x}{1+x^2}$$

Interchange x and y.

$$x = \frac{y}{1+y^2}$$

$x(1+y^2) = y$ (1 mark)

$$x + xy^2 - y = 0$$

$$xy^2 - y + x = 0$$

$$y = \frac{+1 \pm \sqrt{1-4x^2}}{2x}$$

But $y > 0 \therefore y = \dfrac{+1 + \sqrt{1-4x^2}}{2x}$ (1 mark)

Domain $\left(0, \frac{1}{2}\right)$ (1 mark)

Question 2

a.

Maximum value of $\sin\left(\frac{2\pi t}{3}\right) = 1$

Max length of spring = $(a+b)$ cm (1 mark)

b.

Minimum value of $\sin\left(\frac{2\pi t}{3}\right) = -1$

Min length of spring = $(a-b)$ cm (1 mark)

c.

$8\sin\left(\frac{2\pi t}{3}\right) = 4$

$\Rightarrow \sin\left(\frac{2\pi t}{3}\right) = \frac{1}{2}$ (1 mark)

$\Rightarrow \frac{2\pi t}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ (1 mark)

$\Rightarrow t = \frac{\pi}{6}\times\frac{3}{2\pi}, \frac{5\pi}{6}\times\frac{3}{2\pi}, \frac{13\pi}{6}\times\frac{3}{2\pi}, \frac{17\pi}{6}\times\frac{3}{2\pi}$

$\Rightarrow t = \frac{1}{4}, 1\frac{1}{4}, 3\frac{1}{4}, 4\frac{1}{4}$ (1 mark)

d.

Period $= 2\pi \div \frac{2\pi}{3} = 3$ seconds (1 mark)

e.

$64 + 8\sin\left(\frac{2\pi t}{3}\right) > 60$

This is true when

$8\sin\left(\frac{2\pi t}{3}\right) > -4 \Rightarrow \sin\left(\frac{2\pi t}{3}\right) > -\frac{1}{2}$

Let $\sin\left(\frac{2\pi t}{3}\right) = \frac{1}{2}$

$\Rightarrow \frac{2\pi t}{3} = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\Rightarrow t = \frac{7\pi}{6}\times\frac{3}{2\pi}, \frac{11\pi}{6}\times\frac{3}{2\pi}$

$\Rightarrow t = \frac{7}{4}, \frac{11}{4}$ (1 mark)

$64 + 8\sin\left(\frac{2\pi t}{3}\right) > 60$ from 0 to 1.75 seconds.

Between 1.75 seconds and 2.75 seconds, the function is less than 60.

f.

When $t = 0$, length $(x) = 64$

When $t = 0.25$, length $(x) = 68$ (1 mark)

Average rate of change

$= \frac{\Delta x}{\Delta t} = \frac{4}{0.25} = 16$ cm/sec (1 mark)

g.

$\frac{dx}{dt} = \frac{16\pi}{3}\cos\left(\frac{2\pi t}{3}\right)$ (1 mark)

When $t = 2.75$, $\frac{dx}{dt} = \frac{16\pi}{3}\cos\left(\frac{2\pi \times 2.75}{3}\right) = \frac{8\pi\sqrt{3}}{3}$ cm/sec (1 mark)

Question 3

a.

$y = a(x-b)^2 + c \quad (b = 0)$

$y = ax^2 + c$

When $x = 0, y = 1$ so $y = ax^2 + 1$ (1 mark)

When $x = -3, y = 3.25$ so $3.25 = 9a + 1$

$\Rightarrow 2.25 = 9a$

$\Rightarrow \frac{2.25}{9} = a$

$\Rightarrow a = 0.25$ (1 mark)

b.

Diameter = distance from –10 to +10 = 20

(1 mark)

c.

Express x in terms of y.

$y = \frac{x^2}{4} + 1$

$\Rightarrow y - 1 = \frac{x^2}{4}$

$\Rightarrow x^2 = 4(y-1)$

$\Rightarrow x = \pm\sqrt{4(y-1)}$ (1 mark)

d.

$\int_a^b 4\pi(y-1)dy$ (1 mark)

a and b are the y values.

$a = 1$ (1 mark)

b is the value of y when $x = 10$ (1 mark)

When $x = 10, y = \frac{1}{4} \times 100 + 1 = 26$

$\Rightarrow b = 26$

$V = 4\pi\left[\frac{y^2}{2} - y\right]_1^{26} = 3927$ cubic units (1 mark)

e.

$r = 3 \Rightarrow x = 3$

When $x = 3$, $y = \frac{1}{4} \times 9 + 1 = \frac{13}{4}$

$V = 4\pi \int_1^{13/4} (y-1)dy$ (1 mark)

$V = 4\pi\left[\frac{y^2}{2} - y\right]_1^{13/4}$

$V = 4\pi\left[\frac{169}{32} - \frac{13}{4} - \frac{1}{2} + 1\right]$

$V = 4\pi \times \frac{81}{32} = \frac{81\pi}{8}$ cubic units (1 mark)

f.

$\frac{dy}{dx} = \frac{1}{9}\left[e^{3x} \times 3 + (3x+2) \times 3e^{3x}\right]$ (1 mark)

$= \frac{1}{3}e^{3x} \times [1 + 3x + 2]$

$= \frac{1}{3}e^{3x} \times [3x + 3]$

$= e^{3x}(x+1)$ (1 mark)

Question 3 (continued)

g. i.

$\frac{dh}{dt} = 4te^{3t} \Rightarrow h = \int_0^1 4te^{3t}\,dt$

Now $\int te^{3t} + \int e^{3t} = \frac{1}{9}e^{3t}(3t+2)+c$ (1 mark)

$\int te^{3t} = \frac{1}{9}e^{3t}(3t+2) - \frac{1}{3}e^{3t} + c$

$h = 4\int_0^1 te^{3t}dt = 4\left[\frac{1}{9}e^{3t}(3t+2) - \frac{1}{3}e^{3t}\right]_0^1$ (1 mark)

$h = 4\left[\frac{5}{9}e^3 - \frac{1}{3}e^3 - \frac{2}{9} + \frac{1}{3}\right]$

$h = 18.2983$ units (1 mark)

g. ii.

$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$

$h = y$

$\frac{dV}{dt} = \frac{dV}{dy}\frac{dy}{dt}$

$\frac{dV}{dy} = 4\pi(y-1)$ (1 mark)

$\frac{dh}{dt} = \frac{dy}{dt} = 4te^{3t} = 4e^3$ when $t = 1$

$\frac{dV}{dt} = 4\pi(18.2983 - 1) \times 4e^3$

$= 17464.52$ cubic units/time unit (1 mark)

Question 4

a. i.

$E(X) = \sum x\Pr(X = x)$

$= 0 + 0.01 + 0.1 + 0.6 + 1.6 + 1.5$

$= 3.81$ (1 mark)

a. ii.

$\sigma^2 = E(X^2) - \mu^2$

$E(X^2) = 0.01 + 0.2 + 1.8 + 6.4 + 7.5$

$= 15.91$ (1 mark)

$\sigma^2 = 15.91 - (3.81)^2$

$\sigma = \sqrt{15.91 - (3.81)^2} = 1.18$ (1 mark)

b. i.

$1 - 0.85 = 0.15$ (1 mark)

b. ii.

Probability(will **not** flower in 2008)

2006	2007	2008		
F	F	F/	=	$0.8 \times 0.8 \times 0.2$
F	F/	F/	=	$0.8 \times 0.2 \times 0.85$
F/	F/	F/	=	$0.2 \times 0.15 \times 0.2$
F/	F	F/	=	$0.2 \times 0.85 \times 0.85$

(1 mark)

$= 0.128 + 0.136 + 0.006 + 0.1445$

$= 0.4145$ (1 mark)

c. i.

$a\int_0^1 (t^2 - t^3)dt = 1$

$\Rightarrow a\left[\frac{t^3}{3} - \frac{t^4}{4}\right]_0^1 = 1$

$\Rightarrow a\left(\frac{1}{3} - \frac{1}{4}\right) = 1$

$\Rightarrow \frac{a}{12} = 1 \Rightarrow a = 12$ (1 mark)

c. ii.

$12\int_0^{0.25} (t^2 - t^3)dt$

$= 12\left[\frac{t^3}{3} - \frac{t^4}{4}\right]_0^{0.25}$

$= 0.05$ (1 mark)

Question 4 (continued)

c. iii.

$$\Pr(X \le 3) \mid \Pr(X \le 6) = \frac{\Pr(X \le 3) \cap \Pr(X \le 6)}{\Pr(X \le 6)}$$

$$= \frac{\Pr(X \le 3)}{\Pr(X \le 6)}$$

$$\Pr(X \le 6) = 12\left[\frac{t^3}{3} - \frac{t^4}{4}\right]_0^{0.5} = 0.3125 \quad \text{(1 mark)}$$

$$\Pr(X \le 3) \mid \Pr(X \le 6) = \frac{0.05078125}{0.3125}$$

$= 0.1625$

$= 0.16$ to 2 decimal places (1 mark)

Question 4 (continued)

d. i.

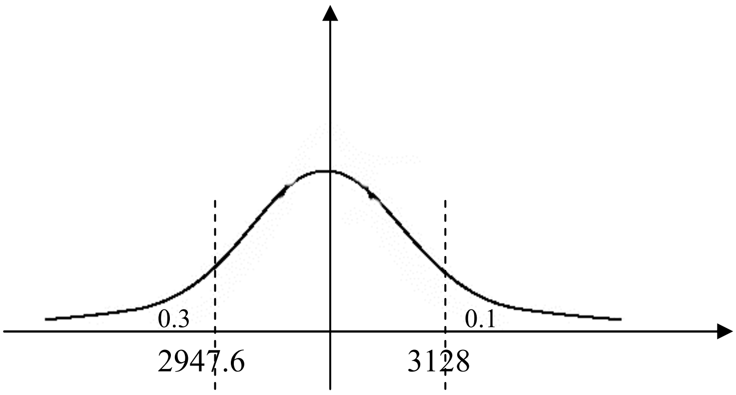
Binomial $n = 10$, $p = 0.3$

binompdf(10,0.3,1) = 0.1493

Probability = 0.1493

(1 mark)

d. ii.



$\Pr(X < 3128) = \Pr(Z < a) = 0.9$

$a = 1.2816$ (1 mark)

$\Pr(X < 2947.6) = \Pr(Z < -b) = 0.3$

$\Pr(Z > b) = 0.3 \Rightarrow \Pr(Z < b) = 0.7$

$b = 0.5244$

$-b = -0.5244$ (1 mark)

$Z = \dfrac{x - \mu}{\sigma}$

$1.2816 = \dfrac{3128 - \mu}{\sigma} \ldots (1)$

$-0.5244 = \dfrac{2947.6 - \mu}{\sigma} \ldots (2)$

$\mu + 1.2816\sigma = 3128 \ldots (1a)$

$\mu - 0.5244\sigma = 2947.6 \ldots (2a)$ (1 mark)

$(1a) - (2a)$ gives $1.806\sigma = 180.4$

$\sigma = 99.98 = 100$ g to the nearest g (1 mark)

Substituting in (1a) gives

$\mu = 3128 - 128.018 = 2999.98 = 3000$ g to the nearest g (1 mark)

END OF SUGGESTED SOLUTIONS
2006 Mathematical Methods Trial Examination 2

KILBAHA MULTIMEDIA PUBLISHING	**TEL: (03) 9817 5374**
PO BOX 2227	**FAX: (03) 9817 4334**
KEW VIC 3101	**chemas@chemas.com**
AUSTRALIA	www.chemas.com