2006

VCE

Mathematical Methods

Trial Examination 1

This Mathematics Methods Trial Examination must be attempted without the use of technology. This is a common examination for Mathematical Methods and Mathematical Methods CAS.

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PURPOSE OF THIS TRIAL EXAMINATION

This Mathematics Methods Trial Examination is designed to assess

- knowledge of mathematical concepts
- skills in carrying out mathematical algorithms
- ability to apply concepts and skills in standard ways

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VICTORIAN CERTIFICATE OF EDUCATION
2006

	STUDENT NUMBER								Letter
Figures									
Words									

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape. Calculators are not permitted.

Materials supplied
- Question and answer book of 11 pages, with a detachable sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your student number in the space provided above on this page.
- All written responses must be in English

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin A$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax) \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax) \qquad \int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

mean: $\mu = E(X)$

variance: $\operatorname{var}(X) = \sigma^2 = E((X-\mu)^2) = E(x^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum xp(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} xf(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

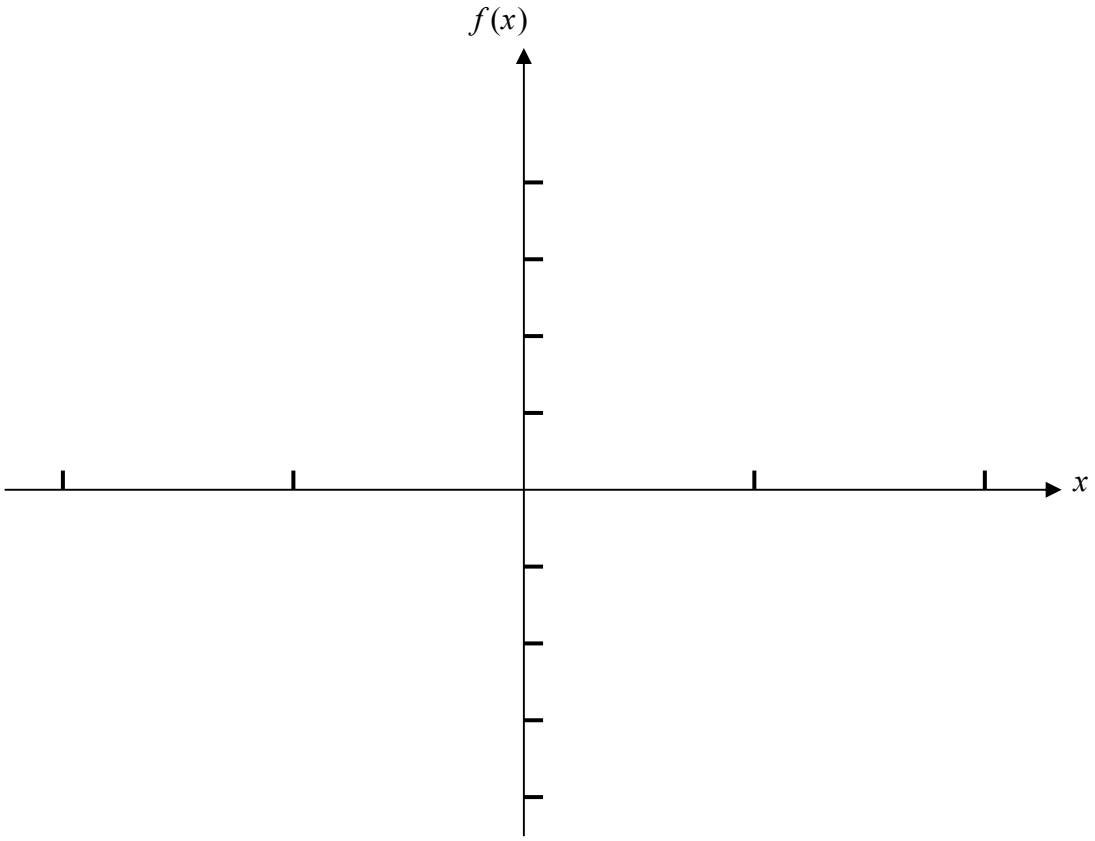
In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For the function $f : [0, \infty) \to R, f(x) = 1 - e^{-x}$,

a. Sketch the graph of the function $f(x)$ on the set of axes below. Label any intercepts with the axes and give the equation of any asymptotes.



b. What is the range of this function?

_____

_____

3 + 1 = 4 marks

Question 2

Find the exact values of the point(s) of intersection of the graphs $y = e^x$ and $y = 6e^{-x} - 1$.

3 marks

Question 3

a. Find $f'(x)$ if $f(x) = \log_e(x^2 + 5)$.

b. Find the rule of $f(x)$ if $f'(x) = 1 - 6\sin 3x$ and $f\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$.

2 + 2 = 4 marks

Question 4

a. $f:\left\{x:x>\frac{1}{3}\right\}\to R$, where $f(x)=\frac{1}{3x-1}$.

Find the inverse function $f^{-1}(x)$.

b. Give the domain and range of $f^{-1}(x)$.

2 + 2 = 4 marks

Question 5

Find the solutions of the equation: $2\sin 3\left(\theta+\frac{\pi}{4}\right)-1=0,\ \ 0\le\theta\le\frac{\pi}{2}$

3 marks

Question 6

A continuous random variable X has a probability density function:

$$f(x) = \begin{cases} 4x^3 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the exact value of a so that $\Pr(X < a) = \frac{1}{4}$.

b. Find the probability that $\frac{1}{2} \le x \le \frac{3}{4}$.

2 + 2 = 4 marks

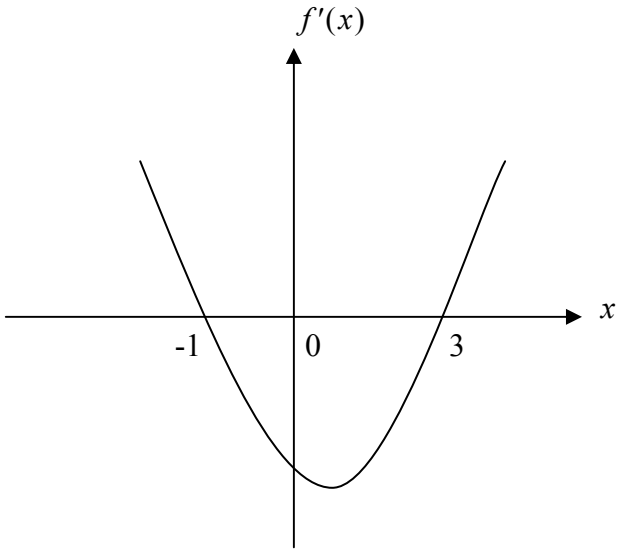
Question 7

a. *If* $\Pr(A) = 0.3, \Pr(B) = 0.6$ and $\Pr(A \cup B) = 0.8$, find $\Pr(A \mid B)$.

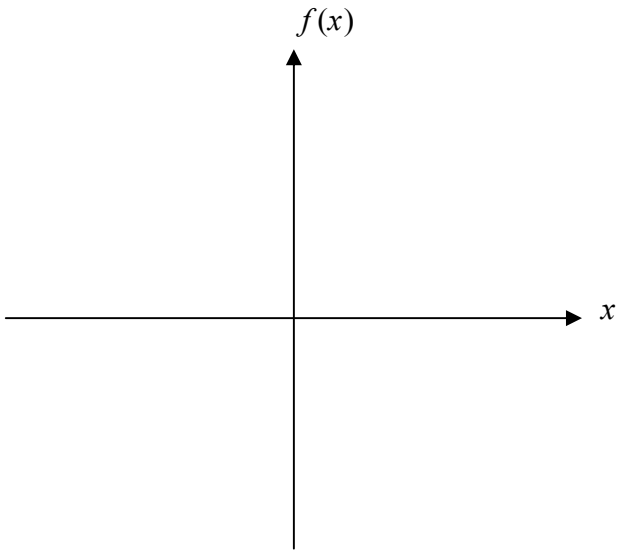
b. A normal random variable has a mean of 8.0 and a standard deviation of 2.0
Find the probability $X < 12 \mid X > 8$.

2 + 3 = 5 marks

Question 8



a. The above is a graph of $f'(x)$. On the axes below, sketch a graph of $f(x)$



b. If $g(x) = x^3 - 9x^2 + 15x + 13$, find the values of x for which $g(x)$ is an increasing function.

1 + 2 = 3 marks

Question 9

A random variable X has a binomial distribution with $n = 9$ and a standard deviation of 1.5

a. Find the probability of a success p.

b. Find the probability that X is at least one.

2 + 1 = 3 marks

Question 10

a. Find the derivative of $e^{3x}\sin 3x + 3$.

b. Hence, find an antiderivative of $3\sin 3x(e^{3x} - 1) + 3e^{3x}\cos 3x$.

1 + 2 = 3 marks

Question 11

Water is dripping at a rate of 50 millilitres per second from a cuboidal tank which has a square base of side 100 cm.

a. Find the rate at which the depth of the water is falling in the tank.

b. How long will it take for the depth of the water to decrease by 60 cm?
Give your answer in hours.

3 + 1 = 4 marks

End of 2006 Mathematical Methods Trial Examination 1
Question and Answer Book

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