VCE
2006 Mathematical Methods
Trial Examination 1

Suggested Solutions

PURPOSE OF THIS TRIAL EXAMINATION

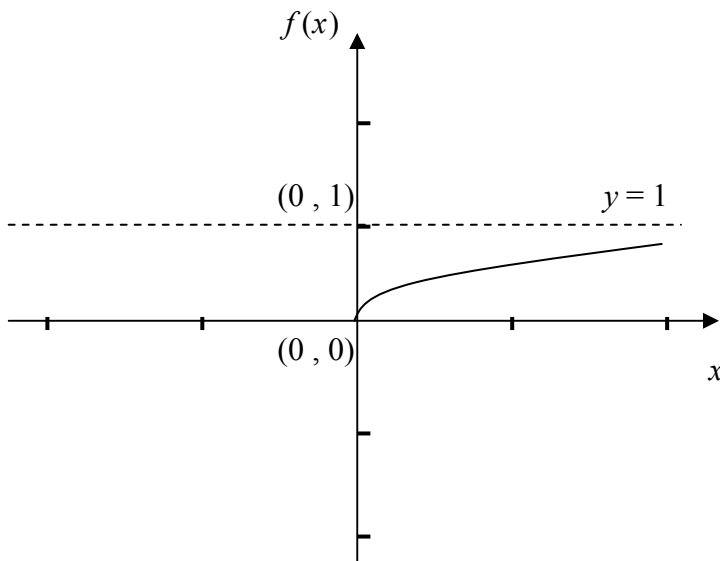This Mathematics Methods Trial Examination is designed to assess
- knowledge of mathematical concepts
- skills in carrying out mathematical algorithms
- ability to apply concepts and skills in standard ways

Question 1

a.

(1 mark) for point (0 , 0).
(1 mark) for the equation of the asymptote.
(1 mark) for the shape of the graph.

b.

Range is $[0 , 1)$ (1 mark)

Question 2

a.

At the point of intersection:
$e^x = 6e^{-x} - 1$

Multiply both sides by e^x

$\Rightarrow e^{2x} = 6 - e^x$

$\Rightarrow e^{2x} + e^{-x} - 6 = 0$

Let $y = e^x$

$y^2 + y - 6 = 0$

$(y + 3)(y - 2) = 0$

$\Rightarrow y = -3 \; or \; y = 2$ (1 mark)

$\Rightarrow e^x = -3 \; or \; e^x = 2$

But $e^x > 0 \therefore e^x = 2$ only   (1 mark)

$y = 2, x = \log_e 2$

$(\log_e 2, 2)$ (1 mark)

Question 3

a.	b.
$f'(x) = \dfrac{1}{x^2 + 5} \times 2x$ (1 mark) $f'(x) = \dfrac{2x}{x^2 + 5}$ (1 mark)	$f(x) = \displaystyle\int (1 - 6\sin 3x)dx$ $f(x) = x + 2\cos 3x + c$ (1 mark) $f\left(\dfrac{\pi}{6}\right) = \dfrac{\pi}{6} + 0 + c = \dfrac{\pi}{3}$ $\Rightarrow c = \dfrac{\pi}{3} - \dfrac{\pi}{6} = \dfrac{\pi}{6}$ $f(x) = x + 2\cos 3x + \dfrac{\pi}{6}$ (1 mark)

Question 4

a.	b.
$x = \dfrac{1}{3y - 1}$ $\Rightarrow 3y - 1 = \dfrac{1}{x}$ (1 mark) $\Rightarrow 3y = \dfrac{1}{x} + 1 = \dfrac{1 + x}{x}$ $\Rightarrow y = \dfrac{1 + x}{3x}$ (1 mark)	Domain is $(0, \infty)$ (1 mark) Range is $\left(\dfrac{1}{3}, \infty\right)$ (1 mark)

Question 5

$2\sin 3\left(\theta + \dfrac{\pi}{4}\right) - 1 = 0, \;\; 0 \le \theta \le \dfrac{\pi}{2}$

$\Rightarrow 2\sin 3\left(\theta + \dfrac{\pi}{4}\right) = 1, \;\; \dfrac{3\pi}{4} \le 3\left(\theta + \dfrac{\pi}{4}\right) \le \dfrac{9\pi}{4}$ (1 mark)

$\Rightarrow \sin 3\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{1}{2}$

$\Rightarrow 3\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{\pi}{6}, \pi - \dfrac{\pi}{6}, 2\pi + \dfrac{\pi}{6}$

$\Rightarrow 3\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}$

$\Rightarrow \theta + \dfrac{\pi}{4} = \dfrac{\pi}{18}, \dfrac{5\pi}{18}, \dfrac{13\pi}{18}$ (1 mark)

$\Rightarrow \theta = \dfrac{\pi}{18} - \dfrac{\pi}{4}, \dfrac{5\pi}{18} - \dfrac{\pi}{4}, \dfrac{13\pi}{18} - \dfrac{\pi}{4} \Rightarrow \theta = \dfrac{-7\pi}{36}, \dfrac{\pi}{36}, \dfrac{17\pi}{36}$

But $0 \le \theta \le \dfrac{\pi}{2} \Rightarrow \theta = \dfrac{\pi}{36}, \dfrac{17\pi}{36}$ (1 mark)

Question 6
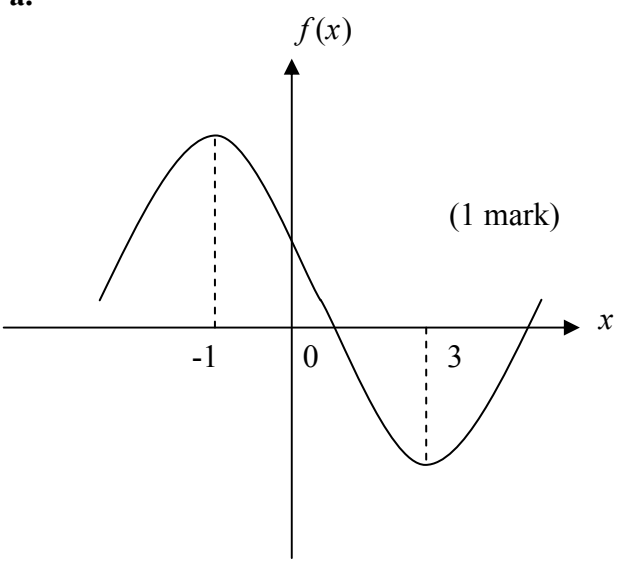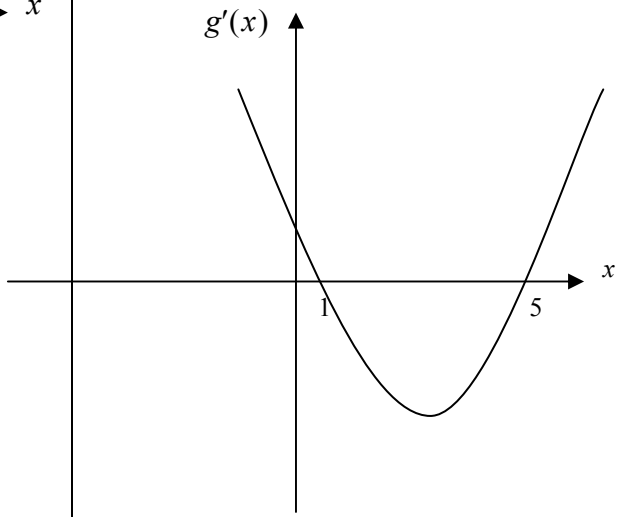
a.

$$\int_0^a 4x^3\,dx = x^4\Big]_0^a = \frac{1}{4} \quad \text{(1 mark)}$$

$$a^4 = \frac{1}{4}$$

$$\Rightarrow a^2 = \pm\frac{1}{2}$$

$$\Rightarrow a = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

But $a > 0$

$$\Rightarrow a = \frac{\sqrt{2}}{2} \quad \text{(1 mark)}$$

b.

$$\int_{1/2}^{3/4} 4x^3\,dx = x^4\Big]_{1/2}^{3/4} \quad \text{(1 mark)}$$

$$= \frac{81}{256} - \frac{1}{16}$$

$$= \frac{81}{256} - \frac{16}{256}$$

$$= \frac{65}{256} \quad \text{(1 mark)}$$

Question 7

a.

$$\Pr(A\,|\,B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B)$$

$$= 0.3 + 0.6 - 0.8$$

$$= 0.1 \quad \text{(1 mark)}$$

$$\Pr(A\,|\,B) = \frac{0.1}{0.6} = \frac{1}{6} \quad \text{(1 mark)}$$

b.

$$\Pr(X < 12\,|\,X > 8)\frac{\Pr(X < 12 \cap X > 8)}{\Pr(X > 8)}$$

$$= \frac{\Pr(8 < X < 12)}{\Pr(X > 8)}$$

$$Z = \frac{12 - 8}{2} = 2$$

95% lie within 2 standard deviations of mean

5% lie outside 2 standard deviations of mean   (1 mark)

2.5% are greater than $Z = 2$

$$\therefore = \frac{\Pr(8 < X < 12)}{\Pr(X > 8)} = \frac{0.5 - 0.025}{0.5} \quad \text{(1 mark)}$$

$$= \frac{0.475}{0.5}$$

$$= 0.475 \times 2$$

$$= 0.95 \quad \text{(1 mark)}$$

Question 8

a.

(1 mark)

b.

Increasing when $g'(x) > 0$

$g'(x) = 3x^2 - 18x + 15$

$g'(x) = 3(x^2 - 6x + 5) = 0$

$g'(x) = 3(x - 5)(x - 1) = 0$ (1 mark)

$x = 5$ or $x = 1$
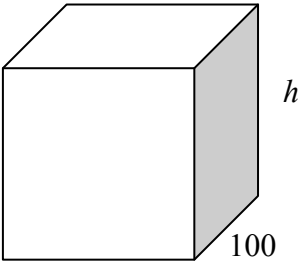
$g'(x) > 0$ when $x < 1$ or $x > 5$ (1 mark)

Question 9

a.

$\sigma^2 = np(1 - p)$

$2.25 = 9p(1 - p)$

$\Rightarrow \dfrac{2.25}{9} = p(1 - p)$

$\Rightarrow 0.25 = p - p^2$ (1 mark)

$\Rightarrow p^2 - p + 0.25 = 0$

$\Rightarrow 4p^2 - 4p + 1 = 0$

$\Rightarrow (2p - 1)(2p - 1) = 0$

$\Rightarrow 2p - 1 = 0$

$\Rightarrow p = \dfrac{1}{2}$ (1 mark)

b.

$\Pr(X \geq 1) = 1 - \Pr(X = 0)$

$= 1 - \dbinom{9}{0}\left(\dfrac{1}{2}\right)^0\left(\dfrac{1}{2}\right)^9$

$= 1 - \left(\dfrac{1}{2}\right)^9$

$= 1 - \dfrac{1}{512} = \dfrac{511}{512}$ (1 mark)

Question 10

a.	b.
$\frac{dy}{dx} = 3e^{3x}\cos 3x + 3e^{3x}\sin 3x$ (1 mark)	$\int (3\sin 3x(e^{3x} - 1) + 3e^{3x}\cos 3x)dx$
	$= \int (3\sin 3x e^{3x} - 3\sin 3x + 3e^{3x}\cos 3x)dx$
	$= \int (3e^{3x}\cos 3x + 3e^{3x}\sin 3x)dx - \int 3\sin 3x dx$ (1 mark)
	$= e^{3x}\sin 3x + \cos 3x + c$ where c is a constant (1 mark)

Question 11

a.

$\frac{dV}{dt} = -50 \text{ ml sec}^{-1} = -50 \text{ cm}^3 \text{ sec}^{-1}$ (1 mark)

$\frac{dh}{dt} = \frac{dh}{dV}\frac{dV}{dt}$ where h = depth of water.

$V = 100 \times 100 \times h = 10000h$

$\frac{dV}{dh} = 10000$ (1 mark)

$\frac{dh}{dt} = \frac{1}{10000} \times \frac{-50}{1} = -0.005$

Height of water is falling at 0.005 cm sec^{-1}

(1 mark)

b.

$\frac{60}{0.005} = \frac{60000}{5} = 12000$ seconds

12000 seconds $= \frac{12000}{60}$ minutes $= 200$ minutes

200 minutes $= \frac{200}{60} = 3\frac{2}{3}$ hours (1 mark)

END OF SUGGESTED SOLUTIONS
2006 Mathematical Methods Trial Examination 1