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MATHS METHODS 3 & 4 TRIAL EXAMINATION 2 SOLUTIONS 2006

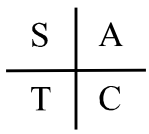
SECTION 1 – Answers

1. B	9. C	17. B
2. E	10. E	18. C
3. A	11. B	19. D
4. E	12. E	20. A
5. C	13. C	21. D
6. E	14. C	22. B
7. B	15. D	
8. D	16. D	

SECTION 1– Multiple-choice solutions

Question 1

$$\sqrt{3}\tan(2x)=1$$
$$\tan(2x)=\frac{1}{\sqrt{3}}$$
$$2x=\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}$$
$$x=\frac{\pi}{12},\frac{7\pi}{12},\frac{13\pi}{12}$$



The answer is B.

Question 2

$$3\log_5(2)+\log_5(7)$$
$$=\log_5(2^3)+\log_5(7)$$
$$=\log_5(8\times 7)$$
$$=\log_5(56)$$
$$=\frac{\log_{10}(56)}{\log_{10}(5)} \quad OR \quad \frac{\log_e(56)}{\log_e(5)}$$
$$=2\cdot 5010... \qquad =2\cdot 5010....$$

The answer is E.

Question 3

$$2e^{2x} = e^x$$
$$2e^{2x} - e^x = 0$$
$$e^x\left(2e^x - 1\right) = 0$$
$$e^x = 0 \quad \text{or} \quad 2e^x - 1 = 0$$
$$\text{no real solution} \qquad e^x = \frac{1}{2}$$
$$\log_e\left(\frac{1}{2}\right) = x$$
$$\log_e\left(2^{-1}\right) = x$$
$$x = -\log_e(2)$$

The answer is A.

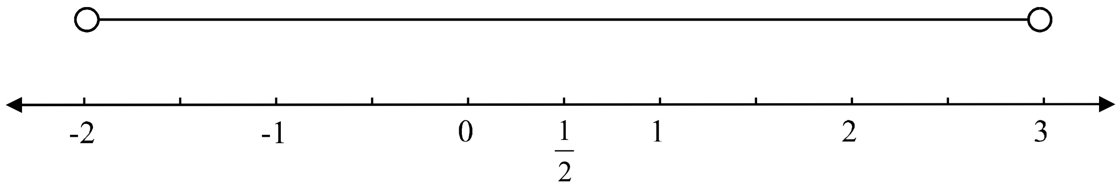
Question 4

Method 1

Using the number line.

$$|2x - 1| < 5$$
$$\left|x - \frac{1}{2}\right| < \frac{5}{2}$$

The distance from x to $\frac{1}{2}$ is less than $\frac{5}{2}$.



So $-2 < x < 3$

Method 2

$$|2x - 1| < 5$$

So $-5 < 2x - 1 < 5$

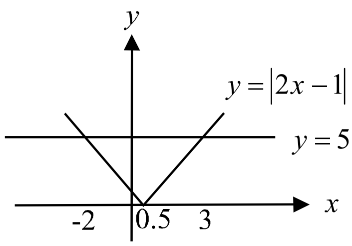
$$-4 < 2x < 6$$

So $-2 < x < 3$

Method 3

Sketch the graph of $y = |2x - 1|$.

Find the values of x for which for which $y < 5$.



So $-2 < x < 3$

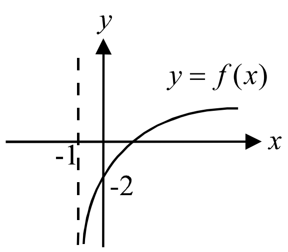
The answer is E.

Question 5

Let $p(x) = x^4 + 4x^3 - 7x^2 - 10x$
$= x(x^3 + 4x^2 - 7x - 10)$
$p(1) = 1(1 + 4 - 7 - 10) \neq 0$
$p(-1) = -1(-1 + 4 + 7 - 10) = 0$
$(x + 1)$ is a factor.
$p(x) = x\{x^2(x + 1) + 3x(x + 1) - 10(x + 1)\}$
$= x(x + 1)(x^2 + 3x - 10)$
$= x(x + 1)(x + 5)(x - 2)$
The linear factors are x, $x - 2$, $x + 1$ and $x + 5$.
The answer is C.

Question 6

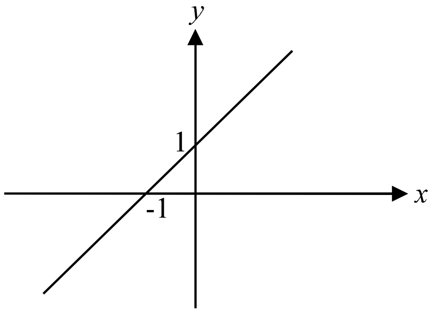
$f(x) = \log_e(x + 1) - 2$
Let $y = \log_e(x + 1) - 2$
Swap x and y.
$x = \log_e(y + 1) - 2$
$x + 2 = \log_e(y + 1)$
$e^{x+2} = y + 1$
$y = e^{x+2} - 1$
So $f^{-1}(x) = e^{x+2} - 1$
$d_f = (-1, \infty)$, $r_f = R$
So $d_{f^{-1}} = R$, $r_{f^{-1}} = (-1, \infty)$
So we have $f^{-1} : R \rightarrow R$, $f^{-1}(x) = e^{x+2} - 1$.



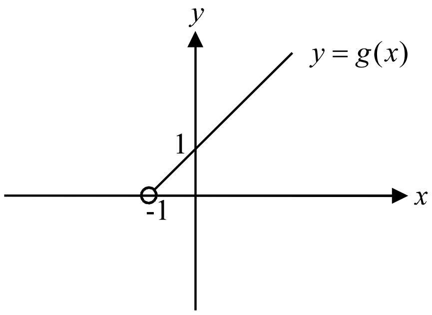
The answer is E.

Question 7

$f(g(x))$ exists iff $r_g \subseteq d_f$
Now,
$d_f = (0, \infty)$
Sketch the graph of g, that is $g(x) = x + 1$.



If $r_g = (0, \infty)$ then $d_g = (-1, \infty)$ as shown in the diagram below.



So $a = -1$.
The answer is B.

Question 8

The part of the graph shown could form part of a sin graph, or a graph of a cubic function or a log function or a quadratic function if there are appropriate transformations involved.

It could not form part of the graph of the function $y = \frac{a}{x^2}$ since there is an asymptote at $x = 0$ on this graph.
The answer is D.

Question 9

The 2 in the rule for g indicates that there has been a dilation by a scale factor of 2 from the x-axis (or parallel to the y-axis).
There is a horizontal translation of π units to the left.
The answer is C.

Question 10

The rate of change of f is also the derivative of f or the gradient of f. It is positive for $(-\infty,-1)\cup(2,\infty)$
The answer is E.

Question 11

$$\text{Let } y = \sin\left(2x^2 + 5x\right)$$
$$= \sin(u) \text{ where } u = 2x^2 + 5x$$
$$\frac{dy}{du} = \cos(u) \qquad \frac{du}{dx} = 4x + 5$$
$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$
$$= \cos(u)\cdot(4x+5)$$
$$= (4x+5)\cos\left(2x^2 + 5x\right)$$

The answer is B.

Question 12

$$f(x) = e^{\sin(2x)}$$
$$= e^u \text{ where } u = \sin(2x)$$
$$\text{Let } y = e^u \qquad \frac{du}{dx} = 2\cos(2x)$$
$$\frac{dy}{du} = e^u$$
$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$
$$= e^u \cdot 2\cos(2x)$$
$$= 2e^{\sin(2x)}\cos(2x)$$

The answer is E.

Question 13

For the left branch $(x < 0)$ of the graph of $y = f(x)$ the gradient is negative.
This eliminates options B and E.
For the right branch $(x > 0)$ of the graph of $y = f(x)$ the gradient is positive.
This eliminates option A.
As $x \to 0$ from the left hand side, the gradient of the graph of $y = f(x)$ approaches $-\infty$.
Conversely, as $x \to 0$ from the right hand side, the gradient of the graph of $y = f(x)$ approaches ∞.
Only option C shows this.
The answer is C.

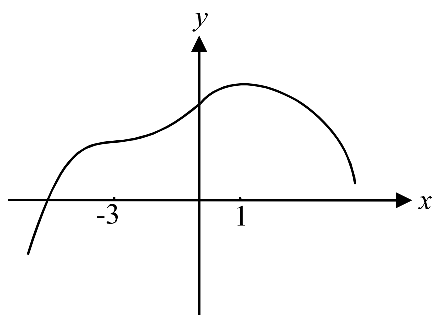
Question 14

$$A = \pi r^2$$
$$\frac{dA}{dr} = 2\pi r$$
$$\frac{dr}{dt} = 0.5$$
$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} \quad \text{Chain rule}$$
$$= 2\pi r \times 0 \cdot 5$$
$$= \pi r$$

When $r = 10\text{cm}$, $\frac{dA}{dt} = 10\pi \text{ cm}^2 / s$.
The answer is C.

Question 15

Sketch the graph of $y = f(x)$.



There is a local maximum at $x = 1$ and a stationary point of inflection at $x = -3$.
The answer is D.

Question 16

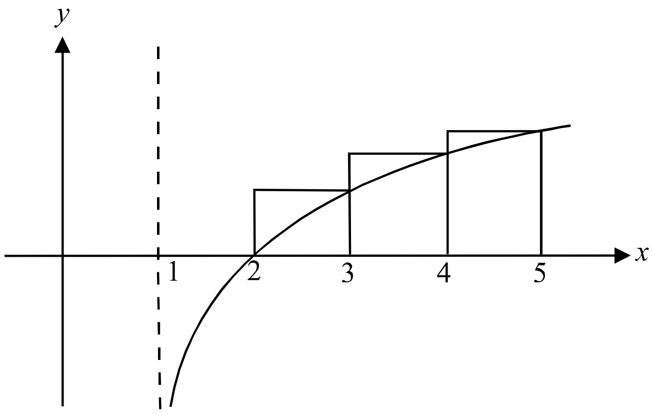
For $-5 < x < 0,\ g(x) > f(x)$.
For $0 < x < 8,\ f(x) > g(x)$.

$$\text{Area} = \int_{-5}^{0} \left(g(x) - f(x)\right)dx + \int_{0}^{8} \left(f(x) - g(x)\right)dx$$

The answer is D.

Question 17

Sketch the graph.

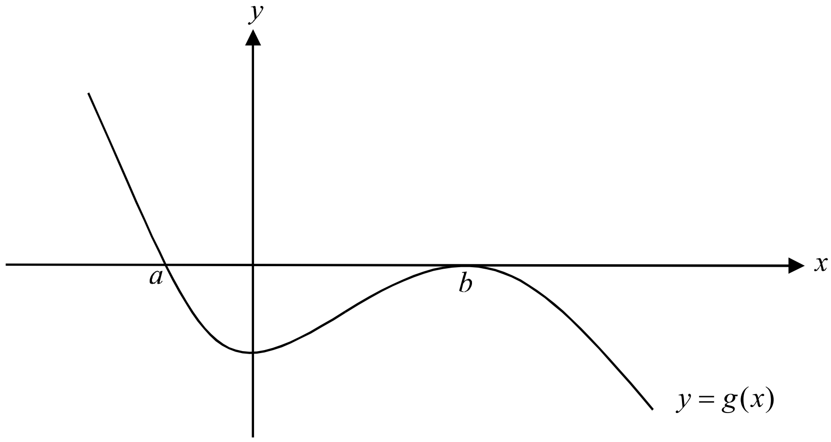


$$\begin{aligned}\text{Area approximation} &= 1 \times \log_e(3-1) + 1 \times \log_e(4-1) + 1 \times \log_e(5-1)\\ &= \log_e(2) + \log_e(3) + \log_e(4)\\ &= \log_e(24)\end{aligned}$$

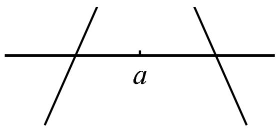
The answer is B.

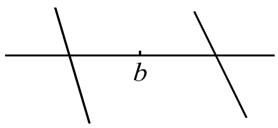
Question 18

The graph we are given can be considered as the gradient function.



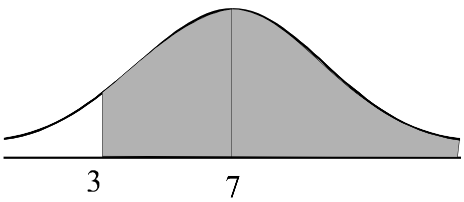
For the graph of the antiderivative function of g, for $x < a$, the gradient is positive.
At $x = a$, there is a stationary point and because the gradient to the left of $x = a$ is positive and to the right is negative, it is a local maximum.





Similarly at $x = b$ there is a stationary point.
To the left of $x = b$ the gradient is negative and to the right it is negative so there is a stationary point of inflection at $x = b$. Only option C shows these features.
The answer is C.

Question 19



Using a graphics calculator $\Pr(X > 3) \approx 0 \cdot 9962$
The answer is D.

Question 20

$$\Pr(X < 0\cdot 2) = \int_0^{0.2} e^{2x}\,dx$$
$$= \left[\frac{1}{2}e^{2x}\right]_0^{0.2}$$
$$= \frac{1}{2}\left(e^{0\cdot 4} - e^{0}\right)$$
$$= \frac{1}{2}\left(e^{0.4} - 1\right)$$
$$= 0\cdot 2459 \quad \text{(to 4 decimal places)}$$

The answer is A.

Question 21

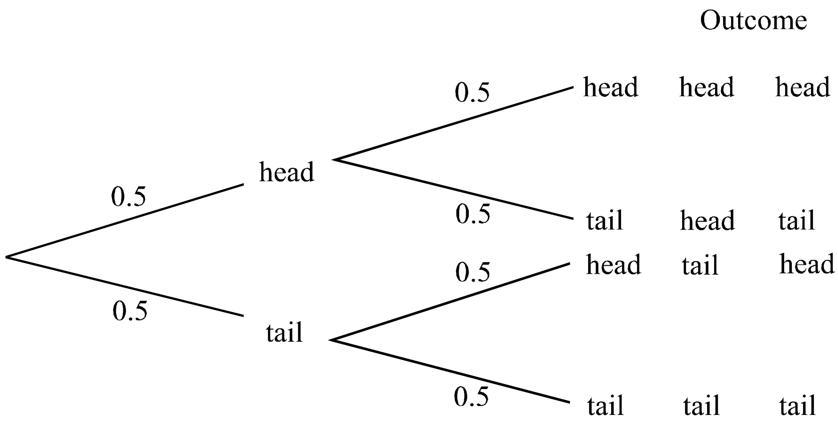
We have a binominal distribution since we have 3 trials and each attempt is independent of every other attempt.

$$\Pr(X = 1) = {}^{3}C_{1}(x)^{1}(1-x)^{2}$$
$$= 3x(1-x)^{2}$$
$$= 3x\left(1 - 2x + x^{2}\right)$$
$$= 3x^{3} - 6x^{2} + 3x$$

The answer is D.

Question 22

Use a tree diagram to create a probability distribution.



The probability distribution is

X	0	1	2
$\Pr(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$\begin{aligned}
\text{Now } E(X) &= \sum x \cdot \Pr(X = x) \\
&= 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} \\
&= 1 \\
E(X^2) &= \sum x^2 \cdot p(x) \\
&= 0^2 \times \frac{1}{4} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{4} \\
&= 1\frac{1}{2} \\
\text{Var}(X) &= E(X^2) - (E(X))^2 \\
&= 1\frac{1}{2} - (1)^2 \\
&= \frac{1}{2}
\end{aligned}$$

The answer is B.

SECTION 2

Question 1

a. Since the area under the pdf is equal to 1 we have,

$$\int_0^a \sin(\theta)d\theta = 1$$ **(1 mark)**

$$[-\cos\theta]_0^a = 1$$
$$\{-\cos(a) - (-\cos 0)\} = 1$$
$$-\cos(a) + 1 = 1$$
$$-\cos(a) = 0$$
$$\cos(a) = 0$$
$$a = \frac{\pi}{2}$$

as required

(1 mark)

(Note that a cannot equal $\frac{3\pi}{2}$ since $\sin\left(\frac{3\pi}{2}\right) = -1$ and for a pdf, the value of the function cannot be negative.)

b. $\Pr\left(\theta < \frac{\pi}{4}\right) = \int_0^{\frac{\pi}{4}} \sin(\theta)d\theta$ **(1 mark)**

$$= [-\cos(\theta)]_0^{\frac{\pi}{4}}$$
$$= \left\{-\cos\left(\frac{\pi}{4}\right) - (-\cos(0))\right\}$$
$$= -\frac{1}{\sqrt{2}} + 1$$
$$= 0 \cdot 293$$

(correct to 3 decimal places)

(1 mark)

c. The probability that the door is left ajar at an angle of less than $\frac{\pi}{4}$ on the next two occasions is

$0 \cdot 293 \times 0 \cdot 293$

$= 0 \cdot 086$ (correct to 3 decimal places)

(1 mark)

d. Let X = the outcome that the door is left open at an angle less than $\frac{\pi}{4}$.

There are 5 trials.
X has a binomial distribution.

Pr(the door is left open at an angle less than $\frac{\pi}{4}$ on 4 of the next 5 occasions)

$= P(X=4) + \Pr(X=5)$ **(1 mark)**

$= {}^5C_4(0\cdot 293)^4(0\cdot 707)^1 + {}^5C_5(0\cdot 293)^5$

$= 0\cdot 028$ correct to 3 decimal places

(1 mark)

e. The median value is the middle value, 50% of values are less than the median value.
Let n = the median value.

$\int_0^n \sin(\theta)\,d\theta = 0\cdot 5$ **(1 mark)**

$\left[-\cos(\theta)\right]_0^n = 0\cdot 5$ **(1 mark)**

$(-\cos(n) - -\cos(0)) = 0\cdot 5$

$-\cos(n) + 1 = 0\cdot 5$

$\cos(n) = 0\cdot 5$ **(1 mark)**

$n = \frac{\pi}{3} \qquad 0 \le \theta \le \frac{\pi}{2}$

The median value is $\frac{\pi}{3}$.

(1 mark)
Total 11 marks

Question 2

a. The amplitude of the function is 20. So the length of the track is 40cm.

(1 mark)

b. At $t = 0$,

$$p(0) = -20\cos(0)$$
$$= -20$$

Since $p < 0$, the meat holder starts at the left end of the track.

(1 mark)

c. The period of the motion of the meat holder is given by $\frac{2\pi}{n}$ where $n = \frac{2\pi}{3}$.

So $2\pi \div \frac{2\pi}{3} = 3$.

The period is 3 seconds.

(1 mark)

d. From part **c.**, the period is 3 secs, so 20 slices of meat are cut in 1 minute.
So $20 \times 15 = 300$ slices have been cut by 10.15am on Tuesday.

(1 mark)

e. The meat first hits the rotating blade when

$$p(t) = -20\cos\left(\frac{2\pi t}{3}\right) = -15$$ **(1 mark)**

$$\cos\left(\frac{2\pi t}{3}\right) = 0 \cdot 75$$

$$\frac{2\pi t}{3} = 0 \cdot 7227...$$

$$t = 0 \cdot 345 \text{ (correct to 3 decimal places)}$$

(1 mark)

f. **i.** The blade is 15cm in radius. The width of the meat is 17cm. So the trailing edge of the meat will be cut when the leading edge is 2cm to the right of *C*; that is when

$$-20\cos\left(\frac{2\pi t}{3}\right) = 2$$ **(1 mark)**

$$\cos\left(\frac{2\pi t}{3}\right) = -0 \cdot 1$$

$$t = 0 \cdot 798 \text{ seconds (correct to 3 decimal places)}$$

(1 mark)

ii. Using your answer to parts **e.** and **f. i.,** it takes $0 \cdot 798 - 0 \cdot 345 = 0 \cdot 453$ seconds to cut one slice of meat.

(1 mark)

g. **i.**

$$p(t) = -20\cos\left(\frac{2\pi t}{3}\right)$$

$$p'(t) = -20 \times -\frac{2\pi}{3}\sin\left(\frac{2\pi t}{3}\right)$$

$$= \frac{40\pi}{3}\sin\left(\frac{2\pi t}{3}\right)$$

gives the rate of change of p with respect to time.

(1 mark)

ii. The rate of change of p is a maximum when

$$\sin\left(\frac{2\pi t}{3}\right) = 1$$

$$\frac{2\pi t}{3} = \frac{\pi}{2}$$

$$4t = 3$$

$$t = \frac{3}{4} \text{ seconds}$$

(1 mark)

iii.

$$p\left(\frac{3}{4}\right) = -20\cos\left(\frac{2\pi}{3} \times \frac{3}{4}\right)$$

$$= -20\cos\left(\frac{\pi}{2}\right)$$

$$= 0$$

The leading edge of the meat slicer is located at point C when the rate of change is a maximum.

(1 mark)

iv. At 10.30 on Tuesday,

$$t = 60 \times 30 = 1800$$

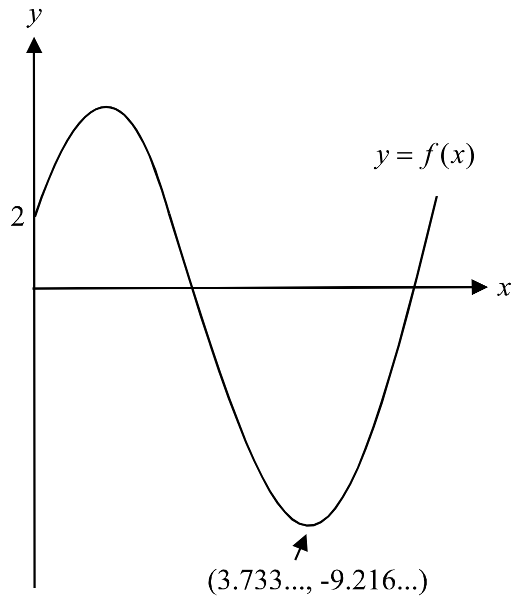
$$p'(1800) = \frac{40\pi}{3}\sin\left(\frac{2\pi}{3} \times 1800\right)$$

$$= 0 \text{ cm/sec}$$

(1 mark)
Total 13 marks

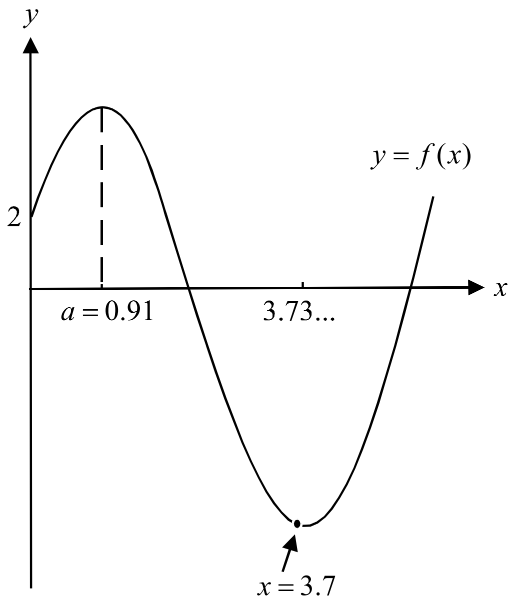
Question 3

a. Sketch the graph of $y = e^x - x^3 + 1$.
Note that $d_f = [0, \infty)$.
The minimum value of *f* occurs at the turning point located at $x = 3 \cdot 733...$

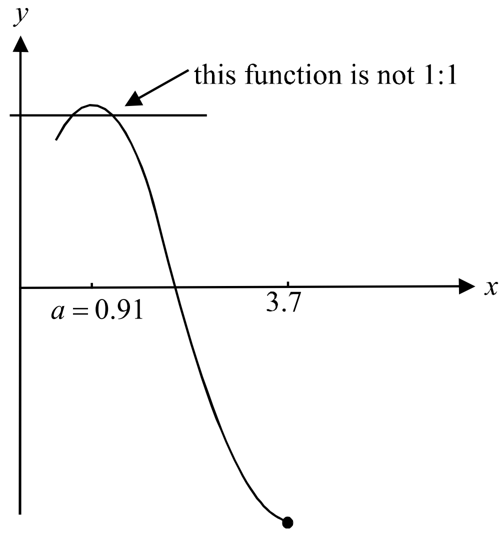


$r_f = [-9.216, \infty)$, where –9.216 is expressed correct to 4 significant figures.
(1 mark) for –9.216
(1 mark) for ∞

b. **i.** Look at the graph of $y = e^x - x^3 + 1$ again since f_1 has the same rule as *f*.
The right endpoint of the function f_1 occurs at $x = 3 \cdot 7$ which is just before the minimum turning point that occurs at $x = 3 \cdot 733...$ from part **i.**
Since $a < 3 \cdot 7$, *a* must be the *x*-coordinate of the local maximum that occurs between $x = 0$ and $x = 2$. So $a = 0 \cdot 91...$ (correct to 2 decimal places).



The value of *a* cannot be less than 0.91 because then the function f_1 is not a 1:1 function and the inverse $f^{-1}(x)$ will not exist. **(1 mark)**
The value of *a* cannot be greater than 0.91 because then the function f_1 will not have a maximal domain.
So the value of *a* is 0.91(correct to 2 decimal places).
(1 mark)



ii. The endpoints of the function $f_1(x)$ are $(0 \cdot 91, 2 \cdot 73)$ and $(3 \cdot 7, -9 \cdot 21)$ where the *y*-coordinates are expressed correct to 2 decimal places. **(1 mark)**
The endpoints of the inverse function $f_1^{-1}(x)$ are therefore $(2 \cdot 73, 0 \cdot 91)$ and $(-9 \cdot 21, 3 \cdot 7)$.

(1 mark)

iii. The graphs of the functions $y = f_1(x)$ and $y = f_1^{-1}(x)$ will intersect along the line $y = x$. Sketch the graph of $y = f_1(x)$ i.e. $y = e^x - x^3 + 1$ and $y = x$. They intersect at the point $(1 \cdot 67, 1 \cdot 67)$.(each correct to 2 decimal places)**(1 mark)**

c. **i.** Since $y=g(x)$ is a reflection in the x-axis of the graph of $y=f(x)$, then

$$\begin{aligned}g(x)&=-f(x)\\&=-\left(e^x-x^3+1\right)\\&=-e^x+x^3-1\end{aligned}$$

(1 mark)

ii. Method 1

Look at the graphs of $y=f(x)$ and $y=g(x)$ for $x\in[0,4\cdot5]$. The difference between the values of the functions is a maximum where the graph of $y=f(x)$ has a local minimum (and therefore where the graph of $y=g(x)$ has a local maximum).

From part **a**. we saw that the local minimum for the graph of $y=f(x)$ occurred at $(3\cdot733...,-9\cdot216...)$. **(1 mark)**

Therefore the maximum value of $|f(x)-g(x)|$ is $2\times9\cdot216...=18\cdot43$ correct to 2 decimal places. **(1 mark)**

Method 2

Consider the graph of

$$\begin{aligned}y&=f(x)-g(x)\\&=e^x-x^3+1-(-e^x+x^3-1)\\&=2e^x-2x^3+2\end{aligned}$$

(1 mark)

The maximum value of $\left|2e^x-2x^3+2\right|$ for $x\in[0,4.5]$ is 18.43 correct to 2 decimal places.

(1 mark)

iii. Method 1

Again look at the graphs of $y=f(x)$ and $y=g(x)$ for $x\in[0,4\cdot5]$. The difference between the values of the functions is a maximum where the graph of $y=f(x)$ has a local minimum (and therefore where the graph of $y=g(x)$ has a local maximum).

From part **a**. we saw that the local minimum for the graph of $y=f(x)$ occurred at $(3\cdot733...,-9\cdot216...)$. Therefore the maximum value occurs when $x=3.73$ correct to 2 decimal places. **(1 mark)**

Method 2

Again, consider the graph of

$$\begin{aligned}y&=f(x)-g(x)\\&=e^x-x^3+1-(-e^x+x^3-1)\\&=2e^x-2x^3+2\end{aligned}$$

The maximum value of $\left|2e^x-2x^3+2\right|$ for $x\in[0,4.5]$ occurs when $x=3.73$ correct to 2 decimal places.

(1 mark)

iv. Method 1

Again, looking at the graphs of $y = f(x)$ and $y = g(x)$ for $x \in [0, 4 \cdot 5]$ the minimum difference is zero and this occurs when the two graphs intersect which occurs when they cross the *x*-axis. This occurs at $x = 2 \cdot 08$ (correct to two decimal places). **(1 mark)**

Method 2

Again, consider the graph of

$$\begin{aligned} y &= f(x) - g(x) \\ &= e^x - x^3 + 1 - (-e^x + x^3 - 1) \\ &= 2e^x - 2x^3 + 2 \end{aligned}$$

The value of $\left|2e^x - 2x^3 + 2\right|$ is a minimum when $x = 2.08$ correct to 2 decimal places. **(1 mark)**

d. The graph of $y = f(x)$ has been moved *h* units to the left. **(1 mark)**

e. i. Look at the graph of $y = f(x)$. It crosses the *x*-axis at $x = 2 \cdot 08$ and at $x = 4 \cdot 50$; each correct to two decimal places. (from part **c. iv**) **(1 mark)**

ii. Using our answers from part **e.i.**, if the graph of $y = f(x)$ is moved 2.07 units to the left, it will cross the *x*-axis at $x = 0 \cdot 01$ and at $x = 2 \cdot 43$. This means that there will be two positive solutions to the equation $f(x) = 0$.

If the graph of $y = f(x)$ is moved 2.08 units to the left, it will cross the *x*-axis at $x = 0$ and at $x = 2 \cdot 42$. In this case the equation $f(x) = 0$ will have only 1 positive solution.

So the graph can go left up to but not including 2.08 units.

The graph can move any amount to the right and always have two positive *x*-intercepts and hence $f(x) = 0$ will always have 2 positive solutions. So *h* can have any negative value.

So $h \in (-\infty, 2 \cdot 08)$.

(1 mark) for $-\infty$
(1 mark) for 2.08
(1 mark) for the round bracket around 2.08
Total 17 marks

Question 4

a. When $x = 0$, $y = a$.

$$f(0) = \frac{1 - \log_e(0+1)}{(0+1)^2}$$
$$= \frac{1 - \log_e 1}{1}$$
$$= \frac{1-0}{1} \qquad \text{since } \log_e 1 = 0$$
$$= 1$$

So, $a = 1$. **(1 mark)**

b. The function *f* crosses the *x*-axis when $f(x) = 0$.

$$\frac{1 - \log_e(x+1)}{(x+1)^2} = 0$$

This can only happen if

$1 - \log_e(x+1) = 0$ **(1 mark)** since if $(x+1)^2 = 0$ the function *f* is undefined

$$\log_e(x+1) = 1$$
$$e^1 = x + 1$$
$$x = e - 1$$

as required

(1 mark)

c. Look at the diagram. The eastern boundary extends from the point $(5,1)$, since $(a = 1)$ to the point $(5, f(5))$

Now $f(5) = \dfrac{1 - \log_e(5+1)}{(5+1)^2}$

$$= \frac{1 - \log_e(6)}{36}$$
$$= -0 \cdot 02199...$$

(1 mark)

So the length of the eastern boundary is $1 + 0 \cdot 02199... = 1 \cdot 0220\text{km}$ correct to 4 decimal places.

(1 mark)

d. Sketch the graph.
Be careful to use appropriate window settings since the minimum is a very "shallow" one.
The furthest point south has coordinates of $(3 \cdot 48, -0 \cdot 02)$ where each coordinate is expressed correct to 2 decimal places.

(1 mark) for *x* coordinate
(1 mark) for *y* coordinate

e. **i.** $$\frac{d}{dx}\left(\frac{\log_e(x+1)}{x+1}\right)=\frac{(x+1)\times\frac{1}{x+1}-1\times\log_e(x+1)}{(x+1)^2}$$ (quotient rule)

$$=\frac{1-\log_e(x+1)}{(x+1)^2}$$ **(1 mark)**

So, $a=1$, $b=1$ and $n=2$.

(1 mark)

ii. Since $$\frac{d}{dx}\left(\frac{\log_e(x+1)}{x+1}\right)=\frac{1-\log_e(x+1)}{(x+1)^2}$$

$$\frac{\log_e(x+1)}{x+1}+c=\int\frac{1-\log_e(x+1)}{(x+1)^2}dx$$

$$\int\frac{1-\log_e(x+1)}{(x+1)^2}dx=\frac{\log_e(x+1)}{x+1}+c$$

(1 mark)

f. **i.** $$\int_{e-1}^{5}f(x)dx=\int_{e-1}^{5}\frac{1-\log_e(x+1)}{(x+1)^2}\,dx$$

$$=\left[\frac{\log_e(x+1)}{x+1}\right]_{e-1}^{5}$$ **(1 mark)**

$$=\frac{\log_e(6)}{6}-\frac{\log_e(e-1+1)}{e-1+1}$$

$$=\frac{\log_e(6)}{6}-\frac{\log_e(e)}{e}$$

$$=\frac{\log_e(6)}{6}-\frac{1}{e}\text{ since }\log_e(e)=1$$

as required **(1 mark)**

ii. The value is negative because the area enclosed by the graph of the function $y=f(x)$, the x-axis, and the lines $x=5$ and $x=e-1$ lies below the x-axis.

(1 mark)

iii. First find the area between the graph of the function $y = f(x)$ and the x and y-axis. Let this area be called area A.

That is, find

$$\int_0^{e-1} f(x)\,dx = \int_0^{e-1} \frac{1-\log_e(x+1)}{(x+1)^2}\,dx$$

$$= \left[\frac{\log_e(x+1)}{x+1}\right]_0^{e-1}$$ **(1 mark)**

$$= \frac{\log_e(e-1+1)}{e-1+1} - \frac{\log_e(1)}{1}$$

$$= \frac{\log_e(e)}{e}$$

$$= \frac{1}{e}$$

So area A is $\frac{1}{e}$ square kilometres.

(1 mark)

Total area of golf course

$= \text{area of rectangle}\,(5\text{km} \times 1\text{km}) - \text{area } A + \text{area } B$ (from diagram below)

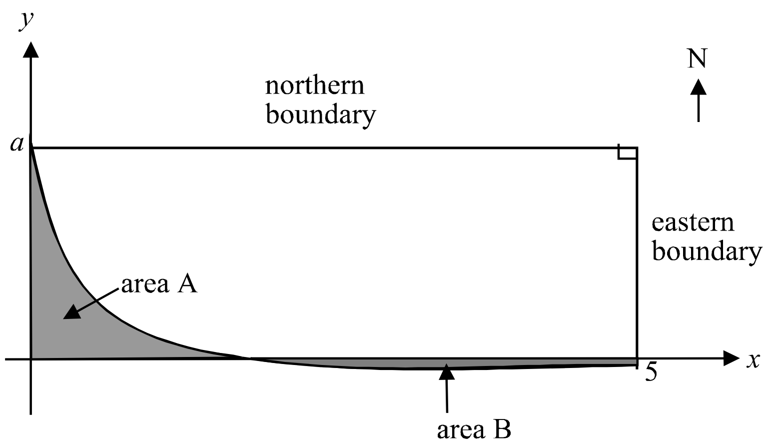
$= 5 \times 1 - \frac{1}{e} + \left(\frac{\log_e(6)}{6} - \frac{1}{e}\right) \times -1$ **(1 mark)**

$= 5 - \frac{1}{e} - \frac{\log_e(6)}{6} + \frac{1}{e}$

$= 5 - \frac{\log_e(6)}{6}$ square kilometres

(Note that area B is equal to $\left(\frac{\log_e(6)}{6} - \frac{1}{e}\right) \times -1$ because area B lies below the x-axis and the value of the integral in part **f. i.** is therefore negative.)

(1 mark)



Total 17 marks

Total for Section 2 - 58 marks