THE HEFFERNAN GROUP

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Student Name……………………………………

MATHEMATICAL METHODS UNITS 3 & 4

WRITTEN TRIAL EXAMINATION 1

2006

Reading Time: 15 minutes
Writing time: 60 minutes

Instructions to students

This exam consists of 11 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the eleven questions are indicated throughout.
Where more than one mark is allocated to a question, appropriate working must be shown.
Unless otherwise stated, diagrams in this exam are not drawn to scale.
Students may not bring any notes or calculators into the exam.
A formula sheet can be found on page 12 of this exam.

Question 1

The probability distribution for the discrete random variable X is shown in the table below.

x	0	1	2	3	4
$\Pr(X = x)$	0.27	0.13	0.19	0.34	a

a. Find the value of a.

b. Find $\Pr(X < 3)$.

c. Find the mean of X.

d. Find the median of X.

$1 + 1 + 1 + 1 = 4$ marks

Question 2

a. Find, in simplest form, $\frac{d}{dx}\left(e^{2x}\tan(2x)\right)$.

b. Find the antiderivative of the function $y = (2x+3)^5$.

$2 + 1 = 3$ marks

Question 3

For $f:[0,\infty)\to R,\ f(x)=x^2+2$ and $g:R^+\to R,\ g(x)=e^{-x}$,

a. explain whether or not $g(f(x))$ exists.

b. explain whether or not the inverse function g^{-1} exists.

c. find the rule and the domain of the inverse function f^{-1}.

2 + 1 + 3 = 6 marks

Question 4

Solve the equation $2\sin(2x) = -\sqrt{3}$ for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Express your answers as exact values in terms of π .

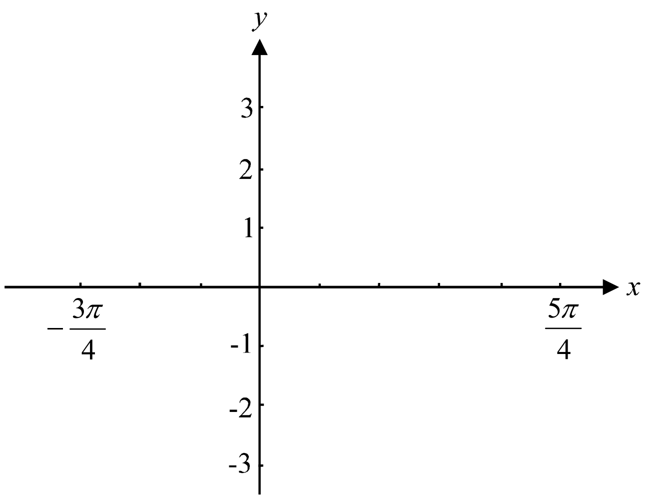
3 marks

Question 5

For the function $f : \left(-\frac{3\pi}{4}, \frac{5\pi}{4}\right) \to R,\ f(x) = 2\tan\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right)$

a. write down the period of the function.

b. Sketch the graph of the function *f* on the set of axes below. Given that $\tan\left(-\frac{\pi}{8}\right) = -0 \cdot 41$ (to 2 decimal places) label any intercepts with the axes and give the equation of any asymptotes.



1 + 3 = 4 marks

Question 6

The random variable X has a normal distribution with a mean of 16 and standard deviation of 4. The random variable Z has a standard normal distribution.

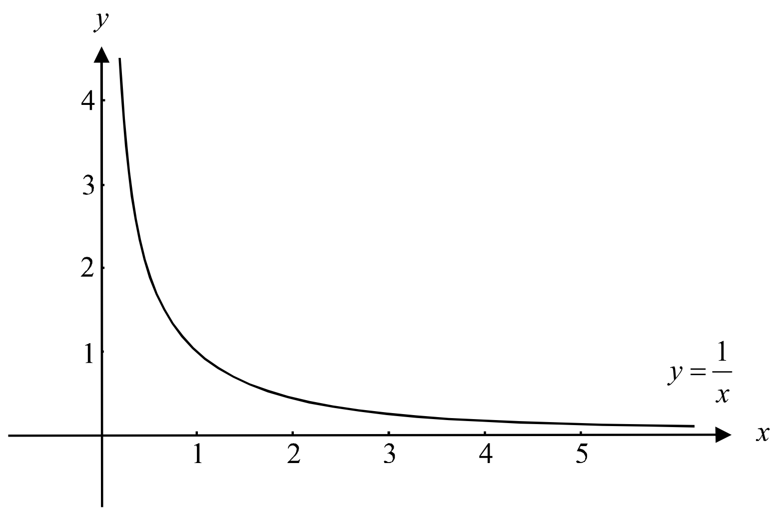
a. Find the value of a such that $\Pr(X > 22) = \Pr(X < a)$.

b. Find the value of b such that $\Pr(X < 7) = \Pr(Z > b)$

$1 + 1 = 2$ marks

Question 7

Consider the function $f:(0,\infty)\to R,\ f(x)=\dfrac{1}{x}$. The graph of $y=f(x)$ is shown on the graph below.



The graph of $y=f(x)$ is to be transformed to become the graph of $y=f\left(\dfrac{x}{4}\right)$.

a. On the graph above, sketch this transformed function. Clearly mark the coordinates of any two points on your graph.

b. Describe the transformation that has taken place.

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c. Write down a rule for this transformed function.

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$2+1+1=4$ marks

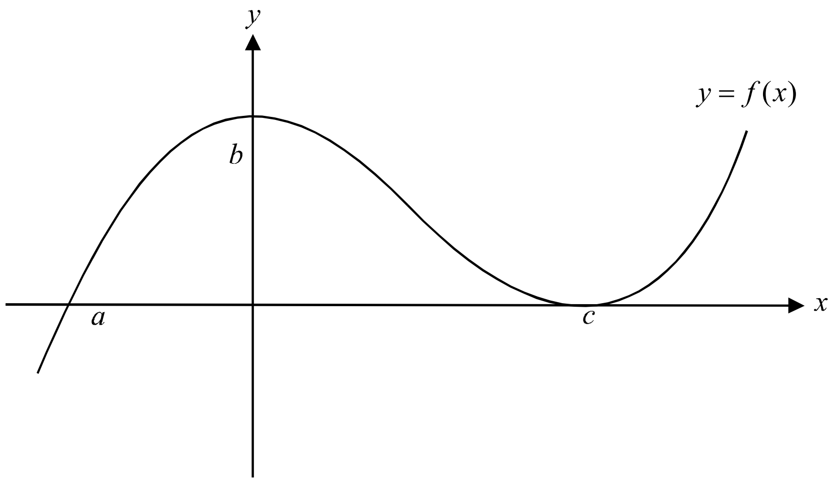
Question 8

The tangent to the curve $y = -x^2 + 5x$ at the point where $x = 1$, crosses the x-axis at the point $(a, 0)$. Find the value of a.

4 marks

Question 9

The graph of a cubic function with rule $y = f(x)$ is shown below. There are stationary points located at $(0, b)$ and $(c, 0)$.



a. Write down the three linear factors of the function *f*.

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b. Find the values of *x* for which $f(x) > 0$.

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c. Find the values of *x* for which $f'(x) < 0$.

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$1 + 1 + 1 = 3$ marks

Question 10

A different team of students is assigned to clean up the school yard each day. The probability that a team does a good or bad job is dependent on whether a good or bad job was done by the team on the previous day. The probability that a team does a good job when the previous day's team did a good job is 0.8. The probability that a team does a good job when the previous day's team did a bad job is 0.6.

a. What is the probability that a team does a bad job when the previous day's team did a good job?

b. What is the probability that Friday's team did a good job given that the team on the previous Wednesday did a bad job?

1 + 3 = 4 marks

Question 11

Find the exact area enclosed by the graph of $y = e^{\frac{x}{2}}$, the line $x = 1$ and the positive x and y axes.

3marks

END OF EXAM

Mathematical Methods Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$$

$$\int \frac{1}{x}\,dx=\log_e|x|+c$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$$

$$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$$

$$\int \cos\ (ax)\,dx=\frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h)\approx f(x)+hf'(x)$

Probability

$\Pr(A)=1-\Pr(A')$

$\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A/B)=\frac{\Pr(A\cap B)}{\Pr(B)}$

mean: $\mu=E(X)$

variance: $\mathrm{var}(X)=\sigma^2=E((X-\mu)^2)=E(X^2)-\mu^2$

probability distribution		mean	variance
discrete	$\Pr(X=x)=p(x)$	$\mu=\Sigma x\,p(x)$	$\sigma^2=\Sigma(x-\mu)^2\,p(x)$
continuous	$\Pr(a<X<b)=\int_a^b f(x)dx$	$\mu=\int_{-\infty}^{\infty} x\,f(x)dx$	$\sigma^2=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx$