MATHEMATICS METHODS EXAM 1: SOLUTIONS

Exam 1 Part I

Question 1 **E**

$\Pr(X \cup Y) = \Pr(X) + \Pr(Y) - \Pr(X \cap Y)$

But as events X and Y are independent,

$\Pr(X \cap Y) = \Pr(X) \times \Pr(Y)$
$= 0.7 \times 0.5$
$= 0.35$

$\therefore \Pr(X \cup Y) = 0.7 + 0.5 - 0.35$
$= 0.85$

Question 2 **D**

$\mu = np$
$= 12 \times 0.35$
$= 4.20$

$\sigma^2 = np(1-p)$
$= 12 \times 0.35 \times 0.65$
$= 2.73$

$\sigma = \sqrt{2.73} = 1.65$

Question 3 **E**

Pr (none failing) = $(0.99)^8$
Pr (one failing) = ${}^8C_1\,(0.01)\,(0.99)^7$
Pr (more than one failing)
= 1 – [Pr (none failing) + Pr(one failing)]

Question 4 **D**

Hypergeometric $N = 15$; $D = 3$; $n = 3$; $x = 1$

$$\Pr(X = 1) = 3 \times \frac{12}{15} \times \frac{11}{14} \times \frac{3}{13}$$

$$= \frac{1}{5} \times \frac{12}{14} \times \frac{11}{13} \times 3$$

Question 5 **E**

$$z = \frac{x-\mu}{\sigma} = \frac{6-11.2}{6.5} = -0.8$$

Question 6 **B**

$\Pr(X < 181) = 0.97$ is equivalent to
$\Pr(Z < 1.8808)$

$$z = \frac{x-\mu}{\sigma}$$

$$1.8808 = \frac{181-171}{\sigma}$$

$\sigma = 5.3$

Question 7 **D**

$$\tan^2(\theta) = \frac{1}{\cos^2(\theta)} - 1$$
$= 16 - 1$
$= 15$

$\tan(\theta) = \pm\sqrt{15}$ but θ is in second quadrant where $\tan < 0$ $\therefore \tan(\theta) = -\sqrt{15}$

Question 8 **A**

At 6 pm, $t = 18$

$$h = 3\sin\left(\frac{3\pi}{2}\right) + 4 = 3 \times -1 + 4$$
$= 1$

Question 9 **D**

Maximum = 5, minimum = 1 so median = 3 (vertical translation) and amplitude = 2.

Not sine curve A, horizontal shift not $\frac{\pi}{4}$,

horizontal shift $\frac{\pi}{2}$ to the right.

Question 10 **D**

Transformation in order give: $y = 2\sin(\theta)$
$y = 2\sin(-\theta)$
$y = 2\sin(-(\theta - 3))$
$y = 2\sin(3 - \theta)$

Question 11 **C**

Maximum when $\sin(x - \pi) = -1$,
so max = $a + b$; minimum when $\sin(x - \pi) = 1$,
so min = $a - b$

Question 12 **C**

Possible equations could be

$y = (x - 2)^3 + b$

or $y = -(x - 2)^3 + b$

Question 13 **D**

$ax^4 + 4x^3 + 2x^2 = x^2(ax^2 + 4x + 2)$

For $ax^2 + 4x + 2$, $b^2 - 4ac = 0$
$16 - 8a = 0$
$a = 2$

$$x = \frac{-b}{2a} = \frac{-4}{2 \times 2} = -1$$

Hence, $2x^4 + 4x^3 + 2x^2$ has local minimums at $x = 0$ and $x = -1$.

Question 14 **A**

The domain of $f(x) = \dfrac{2}{\sqrt{x-1}}$ is $(1, \infty)$.

The domain of $g(x) = \dfrac{-3}{(x+7)^2}$ is $R \backslash \{-7\}$.

Hence the domain of $f + g$ is $(1, \infty)$.

Question 15 **A**

f^{-1} has to have a vertical asymptote $x = 1$.

$y = \log_e(x-1)$ has an asymptote $x = 1$.

C, **D** and **E** each have a horizontal asymptote.
B has a vertical asymptote at $x = 0$.

Question 16 **B**

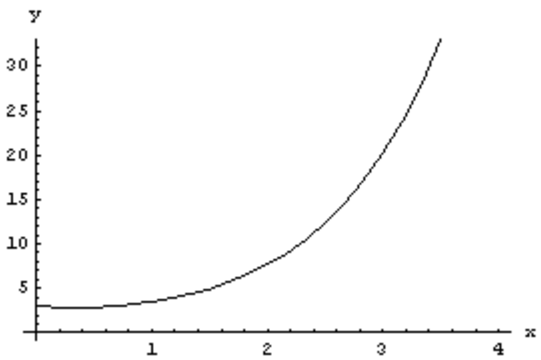
$$0.3^x > 0.09$$
$$x\log_{10}0.3 > \log_{10}0.09$$
$$(\log_{10}0.3 \text{ is negative})$$
$$x < \frac{\log_{10}0.09}{\log_{10}0.3} < \frac{\log_{10}(0.3)^2}{\log_{10}0.3}$$
$$x < 2$$

Question 17 **B**

$$\log_2(y) - \frac{\log_2(x^2-4x+4)}{\log_2(x-2)} = 3$$
$$\log_2(y) - \frac{\log_2(x-2)^2}{\log_2(x-2)} = 3$$
$$\log_2(y) - \frac{2\log_2(x-2)}{\log_2(x-2)} = 3$$
$$\log_2(y) - 2 = 3$$
$$\log_2(y) = 5$$
$$y = 2^5 = 32$$

Question 18 **B**

f: $[0, 4] \to R$, where $f(x) = e^x + 2e^{-x}$



The minimum occurs at the turning point.

$$f'(x) = e^x - 2e^{-x} = 0$$
$$e^{2x} - 2 = 0$$
$$x = \frac{\log_e 2}{2}$$

Question 19 **E**

$$\frac{d}{dx}(\log_e(\cos(kx))) = \frac{-k\sin(kx)}{\cos(kx)}$$
$$= -k\tan(kx)$$

Question 20 **C**

$$y = 3(x-2)^2, \quad \frac{dy}{dx} = 6(x-2)$$

At $x = 1$, $m_{tangent} = \dfrac{dy}{dx} = 6(1-2) = -6$

$$m_{normal} = \frac{1}{6}$$
$$y - 3 = \frac{1}{6}(x-1)$$
$$6y - 18 = x - 1$$
$$-x + 6y = 17$$

Question 21 **D**

A, **B** and **C** have negative rates of change at $x = -2$.

E doesn't exist at $x = -2$.

$y = e^x - 1$, $\quad \dfrac{dy}{dx} = e^x$

At $x = -2$, $\quad \dfrac{dy}{dx} = e^{-2} > 0$

Question 22 **B**

$$V = \frac{k}{t-6}$$

Average rate of change $= \dfrac{V(10) - V(8)}{10-8}$

$$= \frac{\frac{k}{4} - \frac{k}{2}}{2}$$
$$= \frac{-k}{8}$$

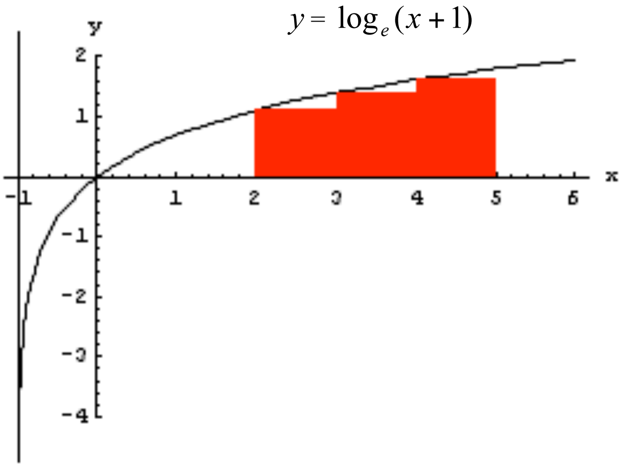
Question 23 **D**

$$f(x+h) \approx f(x) + hf'(x)$$
$$h = -0.2,\ x = 25$$
$$f(25) = \sqrt{25} = 5$$
$$f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{10}$$
$$f(24.8) \approx 5 - 0.2 \times \frac{1}{10}$$

Question 24 **A**

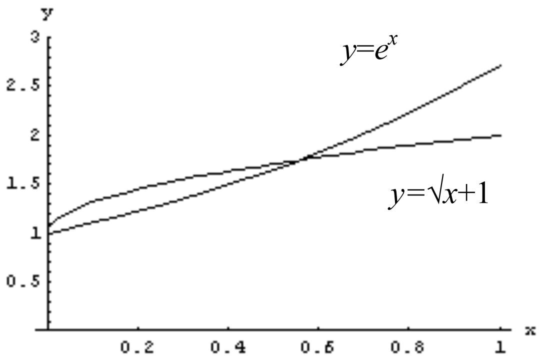
$g(x) = ax^3 + bx^2 + c$ is a cubic function where $a > 0$.
Hence the antiderivative is a quartic function where $a > 0$.

Question 25 **E**



$$\log_e(2+1)\times 1 + \log_e(3+1)\times 1 + \log_e(4+1)\times 1$$
$$= \log_e 3 + \log_e 4 + \log_e 5$$

Question 26 **C**



$e^x = \sqrt{x} + 1$ at $x = 0$ and $x \approx 0.5578$

$$\int_0^{0.5578} (\sqrt{x} + 1 - e^x)\,dx$$

Question 27 **E**

$$\frac{d}{dx}(x\tan(2x)) = \tan(2x) + 2x\sec^2(2x)$$

$$\int(\tan(2x))dx + \int(2x\sec^2(2x))dx = x\tan(2x)$$
$$\int(2x\sec^2(2x))dx = x\tan(2x) - \int(\tan(2x))dx$$
$$\int(x\sec^2(2x))dx = \frac{1}{2}(x\tan(2x) - \int(\tan(2x))dx)$$
$$= \frac{1}{2}(x\tan(2x) + \frac{1}{2}\log_e(\cos(2x)))$$

EXAM 1 PART II

Question 1

a. i $k + 2k + \dfrac{1}{12k} = 1$ **1M**

$$3k + \frac{1}{12k} = 1$$
$$\frac{36k^2 + 1}{12k} = 1$$

$36k^2 - 12k + 1 = 0$ **1M**

$$(6k - 1)^2 = 0$$
$$k = \frac{1}{6}$$

ii $E(X) = \sum_X x.\Pr(X = x)$

$$= 1 \times \frac{2}{6} + 2 \times \frac{1}{6} + 3 \times \frac{6}{12}$$

$= \dfrac{13}{6} = 2\dfrac{1}{6}$ **1A**

b. $\Pr(X > 55 \mid X > 54) = \dfrac{\Pr(X > 55)}{\Pr(X > 54)}$ **1M**

$= \dfrac{0.119703}{0.278187}$ **1M**

$= 0.4303$ **1A**

Question 2

$$3\cos(2x) = \sqrt{3}\sin(2x)$$

$\sqrt{3} = \tan(2x)$ **1A**

$$2x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}$$

$x = \dfrac{\pi}{6}, \dfrac{2\pi}{3}, \dfrac{7\pi}{6}, \dfrac{5\pi}{3}$ **1A**

When $x = \dfrac{\pi}{6}$, $y = 1.5$; when $x = \dfrac{2\pi}{3}$, $y = -1.5$

Intersection points are $(\dfrac{\pi}{6}, 1.5)$, $(\dfrac{2\pi}{3}, -1.5)$, $(\dfrac{7\pi}{6}, 1.5)$, $(\dfrac{5\pi}{3}, -1.5)$ **1A**

Question 3

a. $\frac{6x-4}{2x-3} = A + \frac{B}{2x-3}$

$= \frac{A(2x-3)+B}{2x-3}$ **1M**

$= \frac{2Ax-3A+B}{2x-3}$

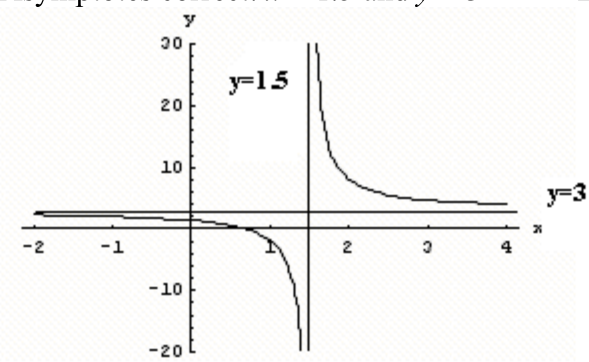
$2A = 6, -3A + B = -4$ **1M**

$A = 3, B = 5$

There are other methods

b. Correct Shape with open circle at $(-2, \frac{16}{7})$ and closed circle at (4, 4). **1A**

Asymptotes correct: $x = 1.5$ and $y = 3$ **1A**



c. $y = \int \frac{6x-4}{2x-3}dx$

$= \int (3 + \frac{5}{2x-3})dx$

$= 3x + \frac{5}{2}\log_e(2x-3) + c$,

where c is a real constant **1M**

At (2, 10), $10 = 6 + c$

$c = 4$

$y = 3x + \frac{5}{2}\log_e(2x-3) + 4$, **1A**

Question 4

a. $f(x) = -\sqrt{7-x}$

Let $y = f(x)$

Inverse: $x = -\sqrt{7-y}$ **1M**

$x^2 = 7 - y$

$y = 7 - x^2, x \le 0$ **1A**

b. $x = 7 - x^2, x \le 0$ **1M**

$x^2 + x - 7 = 0$

$x = \frac{-1-\sqrt{1+28}}{2}$

$= \frac{-1-\sqrt{29}}{2}$.

Coordinates are $(\frac{-1-\sqrt{29}}{2}, \frac{-1-\sqrt{29}}{2})$ **1A**

Question 5

a. Let $y = f(x) = x^4 + x^3 + cx^2 + x + d$

$f(0) = d = 2$ as required **1M**

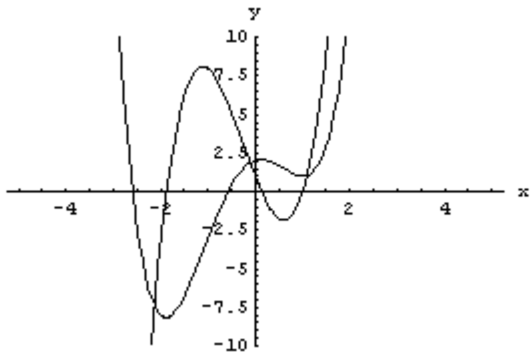
$f'(x) = 4x^3 + 3x^2 + 2cx + 1$

When $x = 1$, $f'(x) = 0$

$4 + 3 + 2c + 1 = 0$ **1M**

$c = -4$ as required

b. There are three regions,



Area =

$\int_{-2.1386}^{-0.1032}(f'(x) - f(x))dx + \int_{-0.1032}^{1.0917}(f(x) - f'(x))dx + \int_{1.0917}^{4.1502}(f'(x) - f(x))dx$

1M

≈ 83.45 units^2 **1A**