The Mathematical Association of Victoria

MATHEMATICAL METHODS

Trial written examination 1
(Facts, skills and applications)

2005

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

PART II
QUESTION AND ANSWER BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer book.
You must complete both parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	Number of questions to be answered	Number of marks
6	6	23

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2005 Mathematical Methods Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

Working Space

Instructions for Part II

Answer **all** questions in the space provided.

A decimal approximation will not be accepted if an **exact** answer is required for a question.

Where an **exact** answer is required for a question, appropriate working must be shown.

In questions where more than one mark is available, appropriate working must be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a.   A random variable X has the probability distribution:

x	1	2	3
$\Pr(X = x)$	$2k$	k	$\frac{1}{12k}$

i.   Show that $k = \frac{1}{6}$.

ii.   Find $E(X)$

b.   For a normal distribution $\mu = 53$ and $\sigma = 1.7$. Find the $\Pr(X > 55 | X > 54)$.

Give your answer correct to four decimal places.

$2 + 1 + 3 = 6$ marks

Question 2

Find the exact coordinates of the points of intersection of $f(x) = 3\cos(2x)$ and $g(x) = \sqrt{3}\sin(2x)$ between $[0, 2\pi]$.

_____

_____

_____

_____

_____

3 marks

Question 3

a. $\frac{6x-4}{2x-3}$ can be written in the form $A + \frac{B}{2x-3}$, where A and B are positive integers. Show that $A = 3$ and $B = 5$.

_____

_____

_____

_____

b. Sketch the graph of $y = f(x) = \frac{6x-4}{2x-3}$ for $\{x: -2 < x < \frac{3}{2}\} \cup \{x: \frac{3}{2} < x \leq 4\}$ on the set of axes below.

Clearly label any intercepts and asymptotes if they exist.
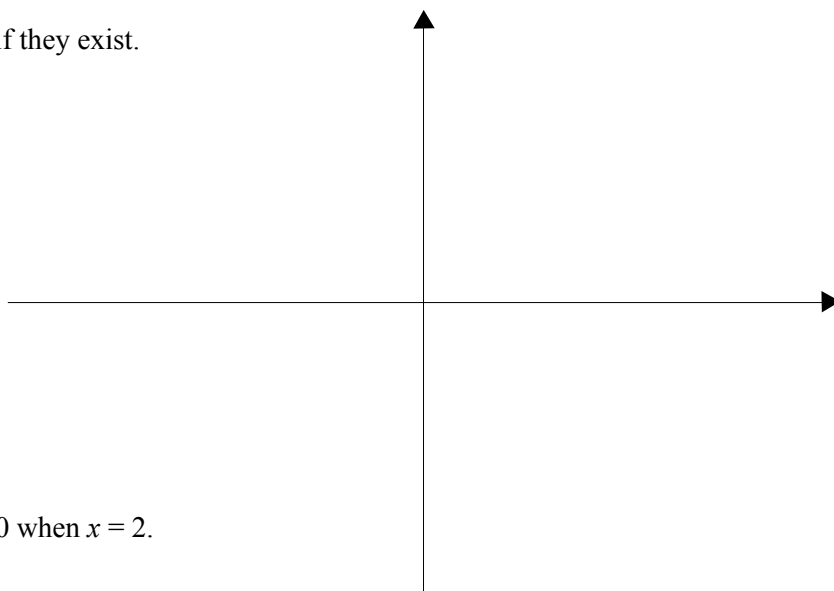
c. Find y in terms of x if $\frac{dy}{dx} = \frac{6x-4}{2x-3}$ and $y = 10$ when $x = 2$.

_____

_____

_____

_____

_____

$2 + 2 + 2 = 6$ marks

Question 4

a.   If $f:(-\infty, 7] \to R$, where $f(x) = -\sqrt{7-x}$, define the function f^{-1}.

_____

_____

_____

_____

_____

b.   Find the exact coordinates of the point where $f = f^{-1}$.

_____

_____

_____

_____

_____

<div align="right">

$2 + 2 = 4$ marks
</div>

Question 5

a.   The curve with equation $y = x^4 + x^3 + cx^2 + x + d$ has a y-intercept at $(0, 2)$ and a stationary point at $x = 1$.
Show that $c = -4$ and $d = 2$.

_____

_____

_____

_____

_____

_____

b.   Hence, find the area bounded by the curves $y = f(x) = x^4 + x^3 + cx^2 + x + d$ and $f'(x)$, correct to two decimal places.

_____

_____

_____

<div align="right">

$2 + 2 = 4$ marks
</div>

END OF PART II QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\dfrac{1}{2}(a+b)h$ volume of a pyramid: $\dfrac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$ volume of a sphere: $\dfrac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$ area of a triangle: $\dfrac{1}{2}bc\sin A$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1}+c, n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$$

$$\int \frac{1}{x}dx = \log_e(x)+c, \text{ for } x>0$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\,\cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)+c$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\,\sin(ax)$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax)+c$$

$$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\,\sec^2(ax)$$

product rule: $\dfrac{d}{dx}(uv)=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$ quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx}=\dfrac{dy}{du}\dfrac{du}{dx}$ approximation: $f(x+h)\approx f(x)+hf'(x)$

Statistics and Probability

$\Pr(A)=1-\Pr(A')$ $\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$

$\Pr(A|B)=\dfrac{\Pr(A\cap B)}{\Pr(B)}$

mean: $\mu=\mathrm{E}(X)$ variance: $\mathrm{var}(X)=\sigma^2=\mathrm{E}((X-\mu)^2)=\mathrm{E}(X^2)-\mu^2$

Discrete distributions			
	$\Pr(X=x)$	mean	variance
general	$p(x)$	$\mu=\sum x\,p(x)$	$\sigma^2=\sum(x-\mu)^2 p(x)$ $=\sum x^2 p(x)-\mu^2$
binomial	$^nC_x\,p^x(1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\dfrac{^DC_x\,^{N-D}C_{n-x}}{^NC_n}$	$n\dfrac{D}{N}$	$n\dfrac{D}{N}\left(1-\dfrac{D}{N}\right)\left(\dfrac{N-n}{N-1}\right)$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu,\sigma^2)$ and $Z=\dfrac{X-\mu}{\sigma}$, then Z is distributed $\mathrm{N}(0,1)$.		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET