The Mathematical Association of Victoria

MATHEMATICAL METHODS

Trial written examination 1

(Facts, skills and applications)

2005

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ____

PART I
MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
27	27	27

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2005 Mathematical Methods Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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Multiple-Choice Answer Sheet

Student's Name ____________________

Circle the letter that corresponds to each answer.

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**
11.	**A**	**B**	**C**	**D**	**E**
12.	**A**	**B**	**C**	**D**	**E**
13.	**A**	**B**	**C**	**D**	**E**
14.	**A**	**B**	**C**	**D**	**E**
15.	**A**	**B**	**C**	**D**	**E**
16.	**A**	**B**	**C**	**D**	**E**
17.	**A**	**B**	**C**	**D**	**E**
18.	**A**	**B**	**C**	**D**	**E**
19.	**A**	**B**	**C**	**D**	**E**
20.	**A**	**B**	**C**	**D**	**E**
21.	**A**	**B**	**C**	**D**	**E**
22.	**A**	**B**	**C**	**D**	**E**
23.	**A**	**B**	**C**	**D**	**E**
24.	**A**	**B**	**C**	**D**	**E**
25.	**A**	**B**	**C**	**D**	**E**
26.	**A**	**B**	**C**	**D**	**E**
27.	**A**	**B**	**C**	**D**	**E**

PART 1

Instructions for Part 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

Two events X and Y are independent. If the $\Pr(X) = 0.7$ and the $\Pr(Y) = 0.5$, then the $\Pr(X \cup Y)$ is

A. 0.20
B. 0.35
C. 0.5
D. 0.7
E. 0.85

Question 2

Consider a binomial experiment with 12 trials and a probability of success in any one trial of $p = 0.35$. The mean and standard deviation, correct to two decimal places are

A. mean = 4.20, standard deviation = 2.05
B. mean = 2.05, standard deviation = 4.20
C. mean = 4.20, standard deviation = 2.73
D. mean = 4.20, standard deviation = 1.65
E. mean = 1.65, standard deviation = 4.20

Question 3

A light bulb has a probability of failure of 0.01. In a room with 8 such light bulbs, the probability that more than one fails is given by

A. $1 - {}^8C_1\,(0.01)\,(0.99)^7$
B. $1 - {}^8C_7\,(0.01)^7\,(0.99)$
C. $1 - (0.99)^8$
D. $1 - [(0.99)^8 + (0.01)\,(0.99)^7]$
E. $1 - [(0.99)^8 + {}^8C_7\,(0.01)\,(0.99)^7]$

Question 4

In a packet there are 5 Milky Bars, 7 Mars Bars and 3 Cherry Ripe Bars. Three bars are chosen at random. The probability that exactly one of them is a Cherry Ripe bar is given by

A. $\frac{1}{5}$

B. $\frac{1}{5} \times \frac{2}{14} \times \frac{1}{13}$

C. $\frac{1}{5} \times \frac{2}{14} \times \frac{1}{13} \times 3$

D. $\frac{1}{5} \times \frac{12}{14} \times \frac{11}{13} \times 3$

E. $\frac{1}{5} \times \frac{12}{14} \times \frac{11}{13}$

Question 5

A normal distribution has $\mu = 11.2$ and $\sigma = 6.5$. If the random variable, X, has a value of 6, the value of the standardised normal variable, Z, correct to one decimal place, is

A. –5.2

B. 0.8

C. 1.3

D. 0.5

E. –0.8

Question 6

The weight of packets of spices is normally distributed with a mean of 171 gram. If 97% of the packets have a weight of less than 181 gram, then the standard deviation, correct to one decimal place, is

A. 3.3 gram

B. 5.3 gram

C. 6.1 gram

D. 10.0 gram

E. 12.1 gram

Question 7

If $\cos(\theta) = \frac{1}{4}$, and $\frac{\pi}{2} < \theta < \pi$, the exact value for $\tan(\theta)$ is

A. $\frac{\sqrt{15}}{15}$

B. $\frac{\sqrt{15}}{4}$

C. $\sqrt{15}$

D. $-\sqrt{15}$

E. $-\frac{\sqrt{15}}{15}$

Question 8

The height of the tide (h metres) is given by $h = 3\sin\left(\frac{\pi}{12}t\right) + 4$ where t is the time after midnight.

The height of the tide at 6 pm will be

A. 1 metre

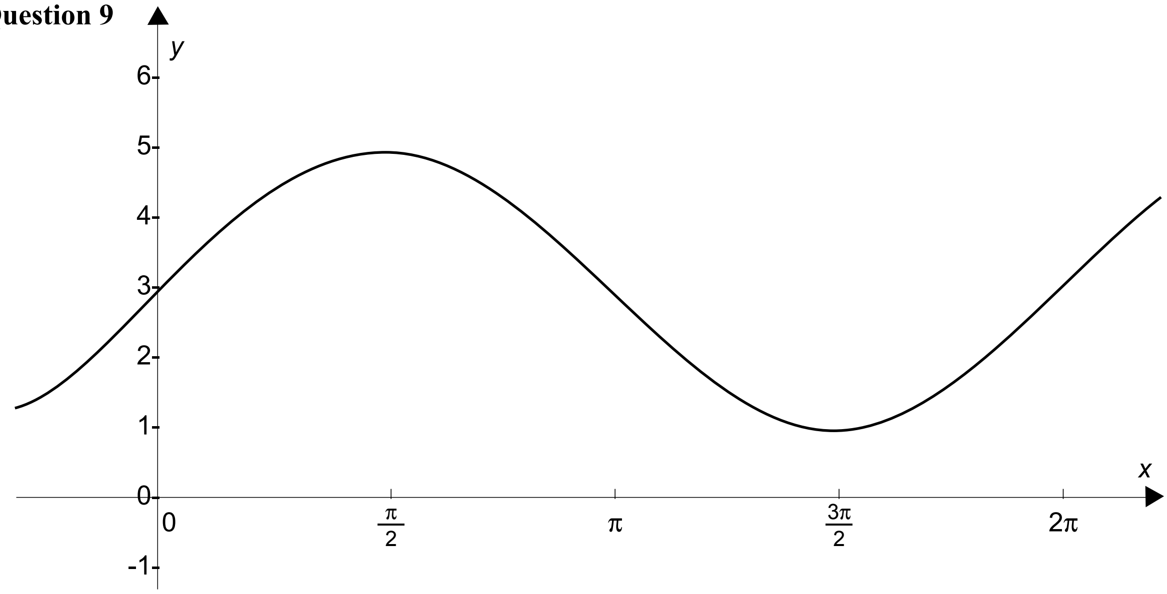
B. 2 metres

C. 3 metres

D. 4 metres

E. 5 metres

Question 9



The rule for the graph shown above could be

A. $y = 2\sin(x) + 1$

B. $y = 2\ \sin(x - \frac{\pi}{4}) + 3$

C. $y = 2\cos\ (x + \frac{\pi}{4}) + 3$

D. $y = 2\cos(x - \frac{\pi}{2}) + 3$

E. $y = 2\cos\ (x + \frac{\pi}{2}) + 3$

Question 10

When $y = \sin(\theta)$ is dilated by a factor of 2 from the x-axis, then **reflected in the y-axis**, and translated 3 units to the right, the new rule is

A. $y = -2\ \sin(\theta + 3)$

B. $y = -\sin(2\theta + 3)$

C. $y = \sin\ (3 - 2\theta)$

D. $y = 2\ \sin(3 - \theta)$

E. $y = \sin(\theta - 3) + 2$

Question 11

Given that $a > 0$ and $b > 0$, the maximum and minimum values for $y = a - b\ \sin(x - \pi)$ are respectively

A. a, $-b$

B. $a - b$, $a + b$

C. $a + b$, $a - b$

D. $a - b$, $b - a$

E. a, $a + b$

Question 12

The graph of $y = f(x)$ is a third degree polynomial with a stationary point of inflection at $(2, b)$ where b is a negative real constant.

A possible equation for the graph is

A. $y = (x + 2)^3 + b$

B. $y = -(x - 2)^3 - b$

C. $y = -(x - 2)^3 + b$

D. $y = (x - 2)^3 - b$

E. $y = (x + 2)^3 - b$

Question 13

$ax^4 + 4x^3 + 2x^2 = 0$, where a is a real constant, has exactly two distinct real solutions.

Hence the graph of $y = ax^4 + 4x^3 + 2x^2$ has

A. local maximum turning points at $x = 0$ and $x = -1$.

B. local maximum turning points at $x = 0$ and $x = 1$.

C. local minimum turning points at $x = 0$ and $x = 1$.

D. local minimum turning points at $x = 0$ and $x = -1$.

E. local maximum turning points at $x = 0$ and $x = 2$.

Question 14

If $f(x) = \frac{2}{\sqrt{x-1}}$ and $g(x) = \frac{-3}{(x+7)^2}$ then the implied domain of $f + g$ is

A. $(1, \infty)$

B. $R \backslash \{-7\}$

C. R

D. $[1, \infty)$

E. $R \backslash \{-7, 1\}$

Question 15

The graph of the curve with rule $y = f(x)$, where f is a one-to-one function, has exactly one asymptote whose equation is $y = 1$.

A possible equation for $y = f^{-1}$ could be

A. $y = \log_e (x - 1)$

B. $y = \log_e (x)$

C. $y = -e^x + 1$

D. $y = e^x + 1$

E. $y = 2^x + 1$

Question 16

If $0.3^x > 0.09$ then

A. $x > \frac{\log_{10}(0.09)}{\log_{10}(0.3)}$

B. $x < 2$

C. $x < \log_{10}(0.03)$

D. $x > \log_{10}(0.03)$

E. $x < \log_{10}(0.09) - \log_{10}(0.03)$

Question 17

If $\log_2(y) - \frac{\log_2(x^2 - 4x + 4)}{\log_2(x-2)} = 3$, then y equals

A. 5

B. 32

C. $3(x-2)$

D. $8(x-2)$

E. $8(x-2)^3$

Question 18

The minimum value of $f: [0, 4] \to R$, where $f(x) = e^x + 2e^{-x}$ occurs at x equals

A. 0

B. $\frac{1}{2}\log_e(2)$

C. 3

D. 4

E. $2\sqrt{2}$

Question 19

$\frac{d}{dx}\left(\log_e(\cos(kx)\right)$, where k is a real constant, equals

A. $\sec(kx)$

B. $k\tan(kx)$

C. $-\tan(kx)$

D. $\tan(kx)$

E. $-k\tan(kx)$

Question 20

The equation of the normal to the curve $y = 3(x-2)^2$ at the point (1, 3) is

A. $6x + y = 9$

B. $x + 6y = 19$

C. $-x + 6y = 17$

D. $x + 3y = 10$

E. $-6x + y = -3$

Question 21

Which one of the following functions has a positive rate of change of y with respect to x at $x = -2$?

A. $y = x^2$

B. $y = -x^3$

C. $y = x^4$

D. $y = e^x - 1$

E. $y = \sqrt{x-3}$

Question 22

The volume of water, V mL, in a container is described by the rule

$V = \frac{k}{t-6}$ for $8 \le t \le 10$, where t is the time in seconds from the start of the flow.

The **average** rate of change of the volume of water, in mL/s, from $t = 8$ s to $t = 10$ s is

A. $\frac{-k}{4}$

B. $\frac{-k}{8}$

C. $\frac{k \log_e 2}{2}$

D. $k \log_e 2$

E. $\frac{k}{8}$

Question 23

Using the approximation formula $f(x + h) \approx f(x) + h f'(x)$ where $f(x) = \sqrt{x}$,
then the approximate value of $\sqrt{24.8}$ can be found by evaluating

A. $5 + 0.2 \times \frac{1}{10}$

B. $\frac{1}{10} + 0.2 \times 5$

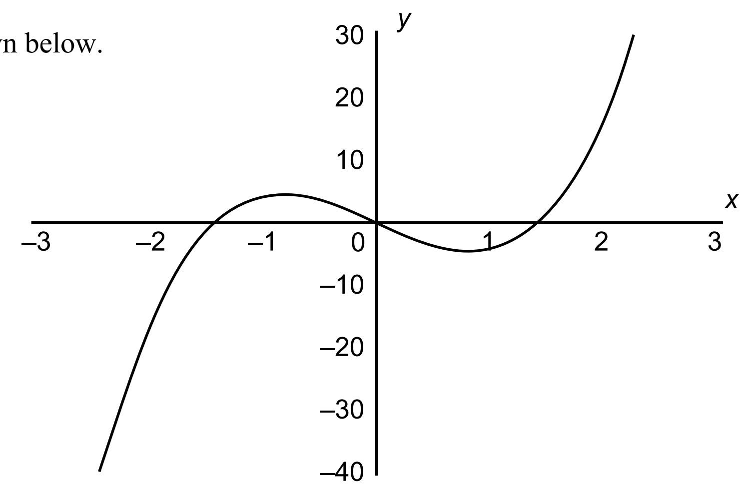
C. $\frac{1}{10} - 0.2 \times 5$

D. $5 - 0.2 \times \frac{1}{10}$

E. $5 - 0.02 \times \frac{1}{10}$

Question 24

The graph of the function g with rule $y = g(x)$ is shown below.



If $g(x) = f'(x)$ then a possible equation for $y = f(x)$ is

A. $y = x^2(x-2)(x+2)$

B. $y = 4x(x^2 - 2)$

C. $y = 12x^2 - 8$

D. $y = -12x^2 + 8$

E. $y = -x^2(x-2)(x+2)$

Question 25

Using the **left rectangle approximation** with rectangles of width 1 unit, the area of the region bounded by the x-axis, the lines $x = 2$ and $x = 5$ and the curve with equation $y = \log_e(x + 1)$ is approximated by

A. $\log_e 3 + \log_e 4 + \log_e 5 + \log_e 6$

B. $\log_e 2 + \log_e 3 + \log_e 4$

C. $2\log_e 2 + \log_e 3 + \log_e 5$

D. $\log_e 4 + \log_e 5 + \log_e 6$

E. $\log_e 3 + \log_e 4 + \log_e 5$

Question 26

The approximate area of the region enclosed by the curves with equations $y = e^x$ and $y = \sqrt{x} + 1$ is given by

A. $\int_0^{0.5578} (e^x - \sqrt{x} + 1)dx$

B. $\int_0^{0.5578} (e^x - \sqrt{x} - 1)dx$

C. $\int_0^{0.5578} (\sqrt{x} + 1 - e^x)dx$

D. $\int_0^1 (e^x - \sqrt{x} + 1)dx$

E. $\int_0^1 (e^x - \sqrt{x} - 1)dx$

Question 27

If $\frac{d}{dx}(x \tan(2x)) = \tan(2x) + 2x \sec^2(2x)$ then an antiderivative of $x \sec^2(2x)$ is:

A. $\frac{1}{2}(x \tan(2x) + \log_e(\cos(2x)))dx$

B. $x \tan(2x) - 2\sec^2(2x)$

C. $\frac{1}{2} x \tan(2x) - 2\sec^2(2x)$

D. $\frac{1}{2}(x\tan(2x) - \frac{1}{2}\log_e(\cos(2x)))dx$

E. $\frac{1}{2}(x \tan(2x) + \frac{1}{2}\log_e(\cos(2x)))dx$

END OF PART 1 MULTIPLE-CHOICE QUESTION BOOK

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$

$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$

$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$

$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\int \frac{1}{x} dx = \log_e(x) + c, \textit{for } x > 0$

$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$

$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \sum x\,p(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$ $= \sum x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x(1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\ {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\left(\frac{N-n}{N-1}\right)$
Continuous distributions			
normal	If X is distributed $N(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed N(0, 1).		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET