

**VCE**  
**2005 Mathematical Methods**  
**Trial Examination 2**

# **Suggested Solutions**

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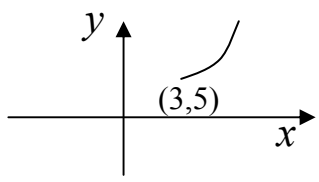
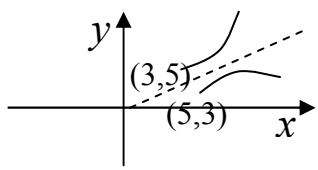


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Question 1

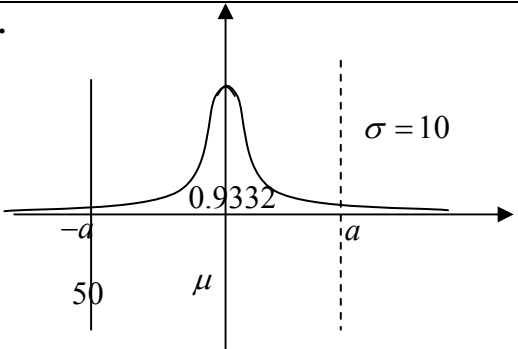
<p><b>a.</b></p> $f(x) = 2 \left[ x^2 - 6x + \frac{23}{2} \right]$ $f(x) = 2 \left[ x^2 - 6x + 9 + \frac{23}{2} - 9 \right] \quad (1 \text{ mark})$ $f(x) = 2 \left[ (x-3)^2 + \frac{5}{2} \right]$ $f(x) = 2(x-3)^2 + 5$ $A = 2, B = 3, C = 5 \quad (1 \text{ mark})$	<p><b>b.</b></p>  <p><math>f(x)</math> must have one - one correspondence for <math>f^{-1}(x)</math> to exist.</p> $a = 3 \quad (1 \text{ mark})$
<p><b>c.</b></p> $x = 2(y-3)^2 + 5$ $x - 5 = 2(y-3)^2$ $\frac{x-5}{2} = (y-3)^2$ $y - 3 = \pm \sqrt{\frac{x-5}{2}}$ $y = 3 \pm \sqrt{\frac{x-5}{2}} \quad (1 \text{ mark})$ <p>But <math>y \geq 3</math></p> $\therefore y = 3 + \sqrt{\frac{x-5}{2}}$ $\therefore f^{-1}(x) = 3 + \sqrt{\frac{x-5}{2}} \quad x \geq 5 \quad (1 \text{ mark})$ <p>Domain <math>[5, \infty)</math> <span style="float: right;">(1 mark)</span></p> <p>Range <math>[3, \infty)</math> <span style="float: right;">(1 mark)</span></p>	<p><b>d.</b></p> <p>1 mark for each shape with its end point.</p> 
	<p><b>e.(i)</b></p> $f'(x) = 4x - 12$ <p>When <math>x = 4, f'(x) = 4</math> <span style="float: right;">(1 mark)</span></p>

<p><b>e.(ii)</b></p> $f^{-1}(x) = 4 = 3 + \sqrt{\frac{x-5}{2}}$ $\sqrt{\frac{x-5}{2}} = 1$ $\frac{x-5}{2} = 1$ $x-5 = 2$ $x = 7 \quad (1 \text{ mark})$ $f^{-1}(x) = \frac{1}{2} \left( \frac{x-5}{2} \right)^{-\frac{1}{2}} \times \frac{1}{2} = \frac{1}{4} \left( \frac{x-5}{2} \right)^{-\frac{1}{2}} \quad (1 \text{ mark})$ <p>When <math>x = 7</math>,</p> $f^{-1}(x) = \frac{1}{4} \quad (1 \text{ mark})$	<p><b>f.</b></p> <p>Let point of intersection be <math>(a, b)</math></p> <p>For <math>f^{-1}(x)</math> when <math>x = 7, y = 4</math></p> <p><math>\therefore</math> for <math>f(x)</math> when <math>x = 4, y = 7</math></p> <p>Gradient of <math>f(x) = 4</math></p> $\therefore \frac{7-b}{4-a} = 4$ $\therefore 7-b = 16-4a$ $4a-b = 9 \quad (1) \quad (1 \text{ mark})$ <p>But gradient of <math>f^{-1}(x) = \frac{1}{4}</math></p> $\therefore \frac{4-b}{7-a} = \frac{1}{4}$ $\therefore 7-a = 16-4b$ $-a+4b = 9$ $-4a+16b = 36 \quad (2)$ $(1) + (2)$ $15b = 45$ $b = 3$ <p>Sub <math>b = 3</math> in (1)</p> $a = 3$ <p><math>(3, 3) \quad (1 \text{ mark})</math></p>
<p><b>g.</b></p> <p>All the points will be of the form <math>(a, a)</math>, so the equation of the line will be <math>y = x</math> (1 mark)</p>	

Question 2

<p><b>a.</b> <math>\frac{35}{100} \times 18,000 = 6,300</math>  (1 mark)</p>	<p><b>b.</b> <math>4k^2 + k + 6k^2 + 4k - 14k^2 = 1</math> <math>-4k^2 + 5k - 1 = 0</math> <math>4k^2 - 5k + 1 = 0</math> <math>(4k - 1)(k - 1) = 0</math> (1 mark) <math>k = \frac{1}{4}</math> or 1 But <math>0 &lt; k &lt; 1</math> <math>\therefore k = \frac{1}{4}</math> (1 mark)</p>
<p><b>c.</b> <math>\Pr = 4k^2 + k + 6k^2 = 10k^2 + k = \frac{7}{8}</math> (1 mark)</p>	<p><b>d.(i)</b> <math>\Pr(X = 1) = \binom{10}{1} \left(\frac{3}{10}\right)^1 \left(\frac{7}{10}\right)^9 = 0.121</math>  (1 mark)</p> <p><b>d.(ii).</b> <math>\Pr(X \geq 2) = 1 - [\Pr(X = 0) + \Pr(X = 1)]</math> (1 mark) <math>= 1 - \left[ \binom{10}{0} \left(\frac{3}{10}\right)^0 \left(\frac{7}{10}\right)^{10} + \binom{10}{1} \left(\frac{3}{10}\right)^1 \left(\frac{7}{10}\right)^9 \right]</math> <math>= 0.851</math> to 3 dec. places. (1 mark)</p>

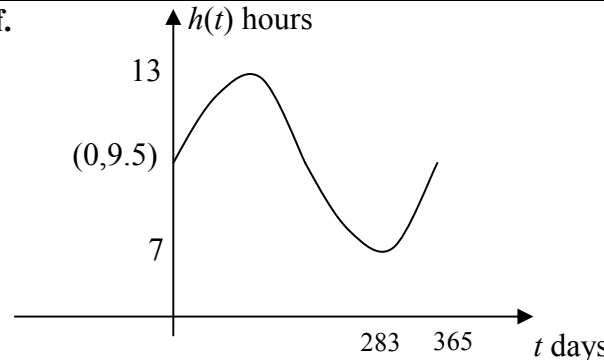
Question 2(continued)

<p><b>e(i).</b></p>  <p><math>\Pr(X &gt; 50) = 0.9332</math>  <math>\Pr(Z &gt; -a) = 0.9332</math>  <math>\Pr(Z &lt; a) = 0.9332</math>  <math>a = 1.5</math>  <math>-a = -1.5</math> (1 mark)  <math>Z = \frac{x - \mu}{\sigma}</math>  <math>-1.5 = \frac{50 - \mu}{10}</math>  <math>\mu = 65</math> (1 mark)</p>	<p><b>e(ii).</b></p> $\Pr(X > 82 / X > 50)$ $= \frac{\Pr(X > 82 \cap X > 50)}{\Pr(X > 50)}$ $= \frac{\Pr(X > 82)}{0.9332}$ (1 mark) $Z = \frac{x - \mu}{\sigma} = \frac{82 - 65}{10} = 1.7$ $\Pr(X > 82 / X > 50) = \frac{\Pr(Z > 1.7)}{0.9332}$ $= \frac{1 - \Pr(Z < 1.7)}{0.9332} = \frac{1 - 0.9554}{0.9332} = 0.05$ (1 mark)
<p><b>f.</b></p> $\Pr(X = 4) = \frac{\binom{20}{7} \binom{10}{4}}{\binom{30}{11}}$ (1 mark) $= 0.298$ (1 mark)	

Question 3

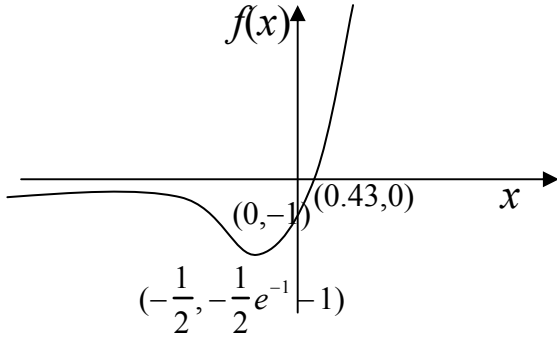
<p><b>a.</b> Maximum <math>h = 10 + 3 = 13</math> hours (1 mark)</p>	<p><b>b.</b> Minimum <math>h = 10 - 3 = 7</math> hours (1 mark)</p>
<p><b>c.</b> Period = <math>\frac{2\pi}{n}</math> where <math>n = \frac{2\pi}{365}</math> Period = <math>2\pi \div \frac{2\pi}{365} = 365</math> days (1 mark)</p>	<p><b>d.</b> Minimum occurs when <math>\cos \frac{2\pi(t-100.5)}{365} = -1</math> (1 mark) <math>\frac{2\pi(t-100.5)}{365} = -\pi, \pi, 3\pi, 5\pi, \dots</math> (1 mark) <math>\frac{2(t-100.5)}{365} = -1, 1, 3, 5, \dots</math> <math>2(t-100.5) = -365, 365, \dots</math> <math>t - 100.5 = -182.5, 182.5, \dots</math> <math>t = 283 \quad t &gt; 0</math> Minimum daylight hours occur on the 283rd day of the year. (1 mark)</p>
<p><b>e.</b> <math>10 + 3 \cos \frac{2\pi(t-100.5)}{365} = 12</math> <math>\cos \frac{2\pi(t-100.5)}{365} = \frac{2}{3}</math> <math>\frac{2\pi(t-100.5)}{365} = -0.8411, 0.8411, 5.442</math> (1 mark) <math>2\pi(t-100.5) = -306.99, 307.0015, 1986.37</math> <math>t = 51.64, 149.36</math> <math>t = 52, 149</math> (1 mark) 52nd day of the year is February 21 149th day of the year is May 29 (1 mark)</p>	

Question 3 (continued)

<p><b>f.</b></p>  <p>When <math>t = 0, h = 10 + 3\cos\left[\frac{2\pi(-100.5)}{365}\right] = 9.5</math> (1 mark)</p>	<p><b>g.</b> There are 12 hours of sunlight on day 52 and day 149. Number of days from day 52 to day 149 = 97 More than 12 hours of sunlight = 96 days because day 52 and day 149 should not be included.</p> <p>(1 mark)</p>
<p><b>h.</b></p> $\frac{dh}{dt} = -3 \sin\left[\frac{2\pi(t-100.5)}{365}\right] \times \frac{2\pi}{365} \quad (1 \text{ mark})$ <p>When <math>t = 30</math>,</p> $\frac{dh}{dt} = -3 \sin\left[\frac{2\pi(-70.5)}{365}\right] \times \frac{2\pi}{365}$ $\frac{dh}{dt} = 0.048 \text{ hours / day} = 2.89 \text{ min / day}$ $\frac{dh}{dt} = 3 \text{ min / day} \quad (1 \text{ mark})$	



Question 4

<p><b>a.</b></p> $f(0) = 0 \times 1 - 1 = -1 \quad (1 \text{ mark})$	<p><b>b.</b></p> $f'(x) = xke^{kx} + e^{kx} \quad (1 \text{ mark})$
<p><b>c.</b></p> <p>Turning Point occurs when <math>f'(x) = 0</math></p> $e^{kx}(kx + 1) = 0 \quad (1 \text{ mark})$ $e^{kx} \neq 0$ $\therefore kx + 1 = 0$ $kx = -1$ $x = -\frac{1}{k} = -\frac{1}{2}$ $\therefore k = 2 \quad (1 \text{ mark})$	<p><b>d.</b></p>  <p>Asymptote: <math>y = 0</math></p> <p>When <math>x = -\frac{1}{2}, y = -\frac{1}{2}e^{-1} - 1</math></p> <p>When <math>x = 0, y = -1</math></p> <p>1 mark for shape and <math>x, y</math> intercepts 1 mark for equation of asymptote 1 mark for exact value of minimum turning point.</p>

Question 4 (continued)

<p><b>e.</b></p> $\int xe^{5x}$ $f(x) = xe^{kx} - 1$ $f'(x) = xke^{kx} + e^{kx}$ $\int (xke^{kx} + e^{kx})dx = xe^{kx} - 1 \quad (1 \text{ mark})$ <p>If <math>k = 5</math></p> $\int 5xe^{5x} dx + \int e^{5x} dx = xe^{5x} - 1$ $5 \int xe^{5x} dx + \frac{1}{5}e^{5x} = xe^{5x} - 1$ $5 \int xe^{5x} dx = xe^{5x} - 1 - \frac{1}{5}e^{5x}$ $\int xe^{5x} dx = \frac{1}{5}(xe^{5x} - 1 - \frac{1}{5}e^{5x}) + c \quad (1 \text{ mark})$ <p>where <math>c</math> is a constant.</p> <p>So <math>\int xe^{5x} dx = \frac{1}{5}\left(xe^{5x} - \frac{1}{5}e^{5x}\right) + c_1</math></p> <p>where <math>c_1</math> is a constant.</p> $\int xe^{5x} dx = \frac{1}{25}(5x - 1)e^{5x} + c_1$	<p><b>f.</b></p> $A = \left  \int_0^{0.43} f(x)dx \right  + \int_{0.43}^1 f(x)dx$ $\int f(x)dx = \int (xe^{2x} - 1)dx$ $A = \left  \frac{1}{4}(2x - 1)e^{2x} - x \right _0^{0.43} + \frac{1}{4}(2x - 1)e^{2x} - x \Big _{0.43}^0$ <p style="text-align: right;">(1 mark)</p> $A =  -0.2627  + 1.36 \quad (1 \text{ mark})$ $A = 0.2627 + 1.36$ $A = 1.62 \text{ to 2 decimal places.}$ <p style="text-align: right;">(1 mark)</p>
<p><b>g.</b></p> <p><math>g(x)</math> and <math>g^{-1}(x)</math> intersect on the line <math>y = x</math></p> $\therefore x = xe^{5x}$ $x - xe^{5x} = 0 \quad (1 \text{ mark})$ $x(1 - e^{5x}) = 0$ $\Rightarrow x = 0 \text{ or } e^{5x} = 1$ $\Rightarrow x = 0 \text{ or } 5x = 0$ $\Rightarrow x = 0$ <p>When <math>x = 0, y = 0</math></p> $\therefore \text{ point is } (0, 0) \quad (1 \text{ mark})$	

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