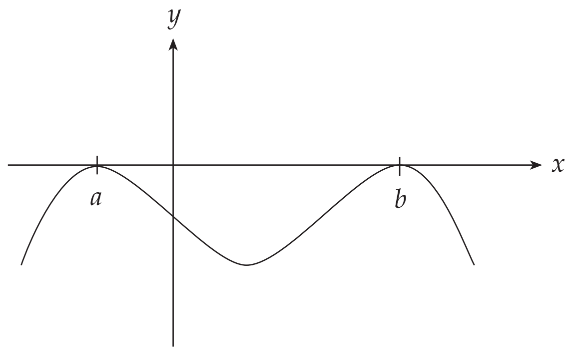
Multiple Choice Questions

Question 1



The equation of the above graph could be

A $y = (x - a)^2(x - b)^2$

B $y = -(x - a)^2(x - b)^2$

C $y = -(x + a)^2(x - b)^2$

D $y = (x + a)^2(x - b)^2$

E $y = -(x + a)^2(x + b)^2$

Question 2

If $x^4 + x^3 + 2x - 7$ is divided by $2x + 3$ then the remainder is

A 41

B $4\frac{7}{16}$

C $-8\frac{5}{16}$

D $8\frac{5}{16}$

E 0

Question 3

If $p(x) = \dfrac{2}{x-5} + 4$ and $q(x) = \dfrac{3}{(x+6)^2} + 7$ then the largest possible range of $p(x) + q(x)$ is

A R

B $R \setminus \{-6, 5\}$

C $(11, \infty)$

D $R \setminus \{4, 7\}$

E $R \setminus \{11\}$

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Question 4

The equations of the asymptotes of the graph of the inverse function of $y = \frac{-3}{(x-4)^2} + 2$ are

A $x = 4$, $y = 2$

B $x = -4$, $y = 2$

C $x = 2$, $y = -4$

D $x = -2$, $y = 4$

E $x = 2$, $y = 4$

Question 5

If $y = -e^x$ is reflected in the y-axis and dilated by a factor of $\frac{1}{2}$ from the y-axis then the equation of the new graph is

A $y = e^{2x}$

B $y = -e^{-\frac{1}{2}x}$

C $y = -e^{-2x}$

D $y = e^{\frac{1}{2}x}$

E $y = \frac{-1}{2}e^x$

Question 6

The function $f(x) = \frac{1}{\sin x}$ is transformed by:

- a dilation of a factor of 2 from the x-axis
- a reflection in both the x and y axes
- a translation of 5 units parallel to the y-axis in the positive direction, and then
- a translation of 3 units parallel to the x-axis in the positive direction.

The rule for the new function is

A $g(x) = \frac{-2}{\sin(-x-3)} + 5$

B $g(x) = \frac{2}{-\sin(x+3)} + 5$

C $g(x) = \frac{2}{\sin(x-3)} + 5$

D $g(x) = \frac{-2}{\sin(3+x)} + 5$

E $g(x) = \frac{-2}{\sin(3-x)} - 5$

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Question 7

The function $f(x) = A - B\sin(Cx + D)$, where A, B and C are positive real constants, has an amplitude and period respectively of

A $A - B, \quad C$

B $-B, \quad \frac{2\pi}{C}$

C $A - B, \quad \frac{2\pi}{C}$

D $A, \quad \frac{2\pi}{C}$

E $B, \quad \frac{2\pi}{C}$

Question 8

If $y = 0.5\sin^2(2x)$ then $\frac{dy}{dx}$ is equal to

A $\sin(2x)$

B $\sin(x)\cos(x)$

C $2\sin(x)\cos(x)$

D $2\sin(2x)\cos(2x)$

E $4\sin(2x)\cos(2x)$

Question 9

The total area of the regions enclosed by $f(x) = \sin(x)$, the x-axis, and the lines $x = \pm\frac{\pi}{4}$ is equal to

A $\frac{\pi}{2}$

B $\sqrt{2}$

C $2\sqrt{2}$

D 0

E $2 - \sqrt{2}$

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Question 10

If the intersection of the tangents to $f(x) = (x-1)(x-2)(x+3)$ at two of the x-intercepts is $(-2\frac{1}{3}, 13\frac{1}{3})$, then the gradients of the tangents are

A –4 and 20

B 5 and 20

C –4 and 5

D –10 and 4

E 4 and 60

Question 11

Using the linear approximation formula $f(x+h) \approx f(x) + h\,f'(x)$ where $f(x) = \frac{2}{x+4}$, the approximate change in $f(x)$ when x increases by 0.001 is

A $0.001 \times \frac{-2}{(x+4)^2}$

B $0.001 \times \frac{2}{(x+4)^2}$

C $\frac{2}{x+4} + 0.001 \times \frac{-2}{(x+4)^2}$

D $\frac{2}{x+4} + 0.001 \times \frac{2}{(x+4)^2}$

E $0.001 \times 2\log_e(x+4)$

Question 12

The instantaneous rate of change of $f(x) = x^3 + x$ at the point where $f'(x)$ is a minimum is

A 0

B 1

C $\sqrt{\frac{1}{3}}$

D $-\sqrt{\frac{1}{3}}$

E –1

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Question 13

$\frac{d}{dx}\log_e(1-\sin^2(x))$ equals

A $\frac{1}{\cos(x)}$

B $2\tan(x)$

C $\frac{1}{\cos^2(x)}$

D $-2\tan(x)$

E $\cos^2(x)$

Question 14

If $g(x) = x^2 + 2x + 1$ and $f(x) = 2e^x$ then which one of the following statements is **false**?

A $y = g'(x)$ is a tangent to the graph of $y = f(x)$

B $y = g'(x)$ is a tangent to the graph of $y = f'(x)$

C $y = f(x) = f'(x)$

D $y = g'(x)$ is a tangent to the graph of y = an antiderivative of $f(x)$

E $y = g'(x)$ is a tangent to the graph of $y = g(x)$ at $x = -1$

Question 15

An antiderivative of $(2x^2+4)^3$ is

A $\frac{(2x^2+4)^4}{8}$

B $\frac{(2x^2+4)^4}{16x}$

C $\frac{8x^7}{7}+\frac{48x^5}{5}+32x^3+64x+8$

D $8x^7 + 48x^5 + 96x^3 + 64x$

E $12x(2x^2 + 4)^2$

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Question 16

A student found an approximation to the area bounded by the graph of $f(x) = 3^{-x}$, the x-axis and the lines $x = 0$ and $x = 3$ to be $\frac{13}{27}$ units squared. Which one of the following methods did the student use to calculate the area?

A Right rectangle rule with strips of width 1 unit.

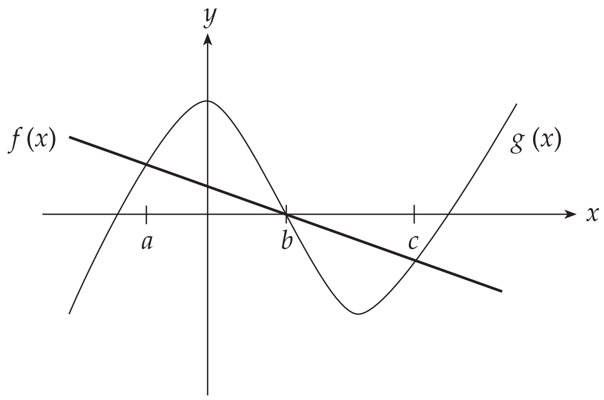
B Left rectangle rule with strips of width 1 unit.

C $\int_0^3 f(x)dx$

D Right rectangle rule with strips of width 0.5 units.

E Left rectangle rule with strips of width 0.5 units.

Question 17



If $f(x)$ and $g(x)$ intersect at $x = a$, $x = b$ and $x = c$ then the area bounded by $f(x)$ and $g(x)$ can be found by evaluating

A $\int_b^a (g(x) - f(x))dx + \int_b^c ((f(x) - g(x))dx$

B $\int_a^b (f(x) - g(x))dx + \int_b^c ((g(x) - f(x))dx$

C $\int_a^c (g(x) - f(x))dx$

D $\int_a^b (g(x) - f(x))dx + \int_b^c ((f(x) - g(x))dx$

E $\int_a^c (f(x) - g(x))dx$

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Question 18

In the expansion of $(2x-1)^9$ one of the terms is

A $-16x^4$

B -2016

C $-252x^4$

D $-2016x^5$

E $-2016x^4$

Question 19

The rule for the inverse of $f: (-\infty, \frac{1}{3}) \rightarrow R$, where $f(x) = 2\sqrt{1-3x}$ is

A $f^{-1}: (-\infty, \frac{1}{3}) \rightarrow R$, where $f^{-1}(x) = -\frac{x^2}{12} + \frac{1}{3}$

B $f^{-1}: [0, \infty) \rightarrow R$, where $f^{-1}(x) = -\frac{x^2}{12} + \frac{1}{3}$

C $f^{-1}: (0, \infty) \rightarrow R$, where $f^{-1}(x) = -\frac{x^2}{12} + \frac{1}{3}$

D $f^{-1}: (-\infty, 0) \rightarrow R$, where $f^{-1}(x) = -\frac{x^2}{12} + \frac{1}{3}$

E $f^{-1}: (0, \infty) \rightarrow R$, where $f^{-1}(x) = -\frac{x^2}{6} + \frac{1}{3}$

Question 20

The x and y intercepts of $y = \log_e(2x + 1) + 3$ are respectively

A $\frac{1-e^3}{2e^3}$ and 3

B 3 and $\frac{1-e^3}{2e^3}$

C $-\frac{1}{2}$ and 3

D 3 and $\frac{e^3-1}{2}$

E 3 and $e^{-3} + \frac{1}{2}$

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Question 21

If $3e^{2x} - 17e^{x} + 10 = 0$ then the solutions for x are

A $\frac{2}{3}$ and 5

B $\log_{10}\frac{2}{3}$ and $\log_{10}5$

C $\log_e 2 - \log_e 3$ and $\log_e 5$

D $\log_e 2 + \log_e 3$ and $\log_e 5$

E $\frac{\log_e 2}{\log_e 3}$ and $\log_e 5$

Question 22

If $\log_2(2x) - 5\log_2(x-1) - \log_2(y) = 2$ then y equals

A $\frac{x}{(x-1)^5}$

B $\frac{x}{2(x-1)^5}$

C $\frac{8x}{(x-1)^5}$

D $\frac{2(x-1)^5}{x}$

E $\frac{(x-1)^5}{x}$

Question 23

The number of currants in small buns is normally distributed with a mean of 10 and a variance of 9. The probability of a randomly chosen bun having more than 15 currants is closest to

A 0.9522

B 0.2892

C 0.0478

D 0.7107

E 0.0332

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Question 24

A punnet contains 16 strawberries, of which 12 are perfect and the rest are damaged. Ten strawberries are taken from the punnet at random to decorate a dessert. The probability that no more than one of the strawberries selected is damaged is

A $^{16}C_{10} \times (0.75)^9 \times (0.25)^1$

B $^{16}C_{11} \times {}^4C_1 + {}^{16}C_{10} \times {}^4C_2 \div {}^{16}C_{10}$

C $({}^{12}C_9 \times {}^4C_1 + {}^{12}C_{10} \times {}^4C_0) \div {}^{16}C_{10}$

D $1 - ({}^{12}C_1 \times {}^4C_1 + {}^{12}C_0 \times {}^4C_2) \div {}^{16}C_{10}$

E $1 - ({}^{16}C_{10} \times (0.75)^9 \times (0.25)^1)$

Question 25

The random variable X has the following probability distribution, where $0 < k < 1$.

x	0	2
$\Pr(X = x)$	k	$1-k$

The standard deviation of X is

A $2 - 2k$

B $2\sqrt{k(k-1)}$

C $4k(1-k)$

D $2\sqrt{k(1-k)}$

E $2\sqrt{k(1+k)}$

Question 26

If X is a normally distributed random variable with a mean $\mu = 1$, and standard deviation $\sigma = 1.5$ and $\Pr(\mu - k < X < \mu + k) = 0.7$, then k is closest to

A 1.555

B 1.037

C 0.787

D 0.524

E 2.332

Question 27

A box contains a number of Chocolates and Toffees and a random selection of the sweets is made. If A is the number of chocolates in the sample when a sample is taken without replacement, and B is the number of chocolates in the sample when a sample is selected with replacement then

A $E(A) = E(B)$ and $\text{Var}(A) = \text{Var}(B)$

B $E(A) > E(B)$ and $\text{Var}(A) > \text{Var}(B)$

C $E(A) < E(B)$ and $\text{Var}(A) < \text{Var}(B)$

D $E(A) = E(B)$ and $\text{Var}(A) > \text{Var}(B)$

E $E(A) = E(B)$ and $\text{Var}(A) < \text{Var}(B)$

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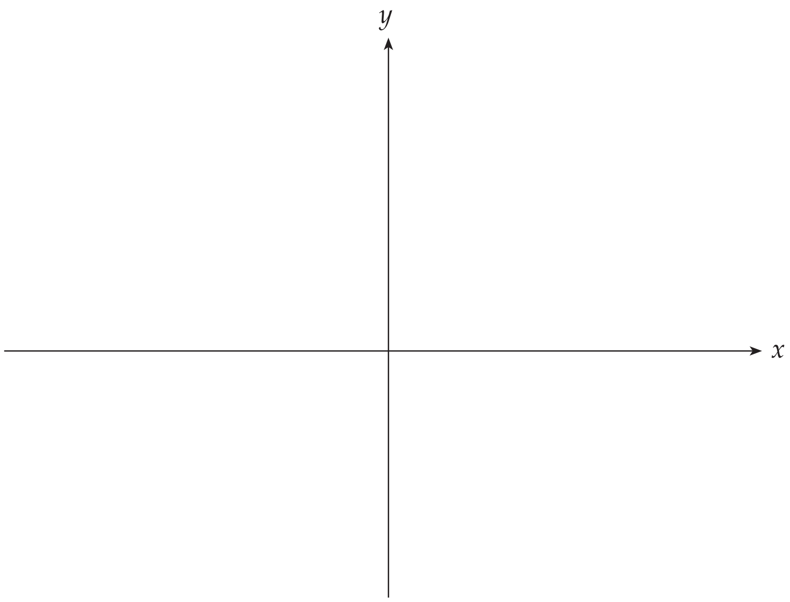
Short Answer (23 marks)

Question 1

An increasing function $f : (0, \frac{\pi}{2}) \rightarrow R$, where $f(x) = \tan(nx + k)$, with $k > 0$, has one x intercept at the point $(\frac{\pi}{4}, 0)$ and asymptotes at $x = 0$ and $x = \frac{\pi}{2}$.

a Sketch the function on the axes provided.



b Find the values of

i n

ii k.

2 + 2 = 4 marks

Question 2

The points (–5, 1), (2, 8), (5, 6), (4, 3) and (8, 7) belong to a quartic curve.

a Write down the equation of the quartic, giving the coefficients correct to three decimal places.

b Find the equation of the normal to the curve where it cuts the *y*-axis. Give the gradient and the *y*-intercept correct to two decimal places.

c Find the average rate of change, correct to two decimal places, between the two local maximums of the quartic.

1 + 2 + 2 = 5 marks

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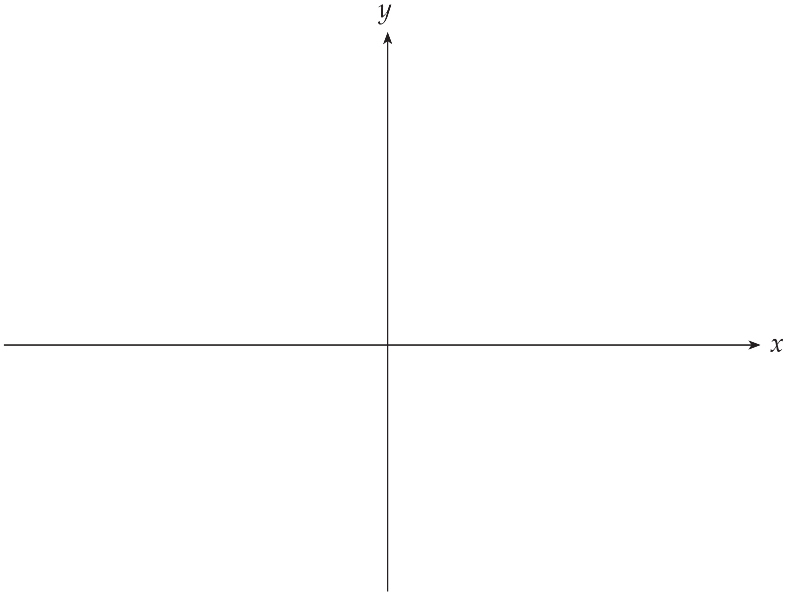
Question 3

a Sketch the graph of $y = f(x) = 3\log_e(2 - x)$, clearly labelling any cuts on the axes and asymptotes.



b Find $f'(x)$.

3 + 1 = 4 marks

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Question 4

Let $f(x) = 2\sqrt{3x+2}$ and $g(x) = x$, with both functions having their largest possible domains.

a Solve $x = 2\sqrt{3x+2}$ algebraically. Give an exact answer.

b Find $f^{-1}(x)$. State the domain.

c Find correct to four decimal places the area enclosed by the graphs of $y = f^{-1}(x)$, $y = g(x)$ and the y-axis.

2 + 2 + 3 = 7 marks

Question 5

The probability of a baby being a girl is 0.49. Find, correct to 4 decimal places,

a the probability of a family with three children including at least 1 boy;

b the probability of a family with 3 children being all boys given that at least 1 child is a boy.

1 + 2 = 3 marks

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