# 2004 Mathematical Methods
# Written Examination 1 (Facts, skills and applications)
# Suggested answers and solutions

## Answers – Multiple Choice

| | | | | |
|---|---|---|---|---|
| 1. **B** | 2. **C** | 3. **A** | 4. **E** | 5. **C** |
| 6. **C** | 7. **E** | 8. **D** | 9. **E** | 10. **A** |
| 11. **A** | 12. **B** | 13. **D** | 14. **E** | 15. **C** |
| 16. **A** | 17. **D** | 18. **E** | 19. **C** | 20. **A** |
| 21. **C** | 22. **B** | 23. **C** | 24. **C** | 25. **D** |
| 26. **A** | 27. **E** | | | |

### Question 1

The graph is a negative quartic with solutions at $x = a$ and $x = b$. **[B]**

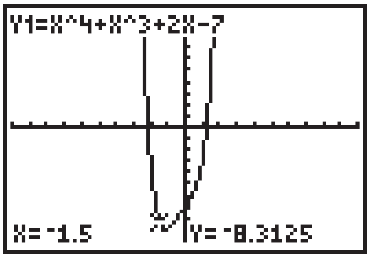
### Question 2

$2x + 3 = 0$

$x = -\frac{3}{2}$

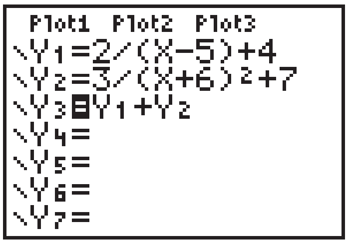
$f(-\frac{3}{2}) = (-\frac{3}{2})4 + (-\frac{3}{2})3 + 2(-\frac{3}{2}) - 7$

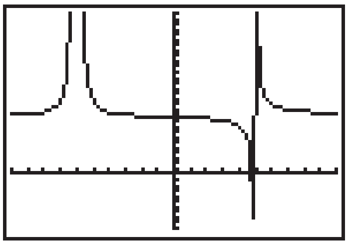
$= -8\frac{5}{16}$




**[C]**

### Question 3



The range is $R$ **[A]**

### Question 4

The asymptotes of $y = \frac{-3}{(x-4)^2} + 2$ are $x = 4$ and $y = 2$.

Hence the equations of the asymptotes of the inverse are $x = 2$ and $y = 4$. **[E]**

### Question 5

$y = -e^x$ reflected in the $y$-axis is $y = -e^{-x}$,

and dilated by a factor of $\frac{1}{2}$ from the $y$-axis is

$y = -e^{-2x}$. **[C]**

## Question 6

A dilation from the $x$-axis is $y = \dfrac{2}{\sin(x)}$.

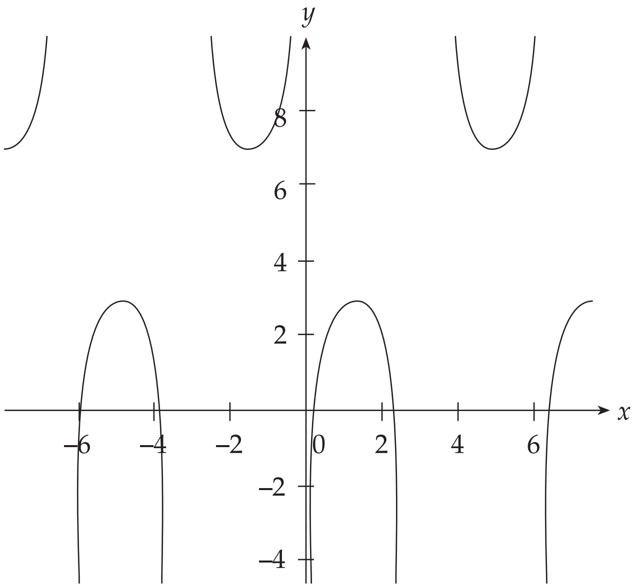
A reflection in both axes is $y = \dfrac{-2}{\sin(-x)} = \dfrac{2}{\sin(x)}$.

A translation 5 units parallel to the $y$-axis is

$y = \dfrac{2}{\sin(x)} + 5$.

A translation 3 units parallel to the $x$-axis is

$y = \dfrac{2}{\sin(x-3)} + 5$.




**[C]**

## Question 7

$f(x) = A - B\sin(Cx + D)$

The amplitude is $|B| = B$

The period is $\dfrac{2\pi}{C}$ **[E]**

## Question 8

Via the chain rule

$$\frac{dy}{dx} = 2 \times 0.5\sin(2x)2\cos(2x)$$

$$= 2\sin(2x)\cos(2x)$$ **[D]**

## Question 9

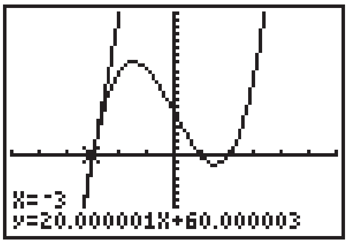
$$\text{Area} = 2\int_0^{\frac{\pi}{4}} \sin(x).dx = 2[-\cos(\tfrac{\pi}{4}) + \cos(0)]$$

$$= 2[-\frac{1}{\sqrt{2}} + 1]$$

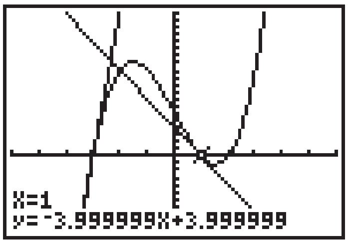
$$= 2 - \sqrt{2}$$ **[E]**

## Question 10

The equation of the tangent at $x = -3$ is
$y = 20x + 60$,

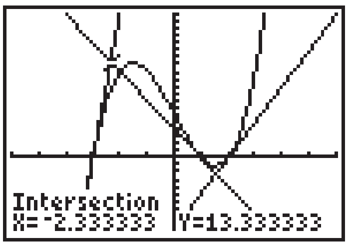



and at $x = 1$, $y = -4x + 4$




$y = -4x + 4$ and $y = 20x + 60$ intersect at

$(-2\frac{1}{3}, 13\frac{1}{3})$.




The gradients are –4 and 20. **[A]**

## Question 11

$$f(x+h) \approx f(x) + h\,f'(x)$$

$$f(x+h) - f(x) \approx h\,f'(x)$$

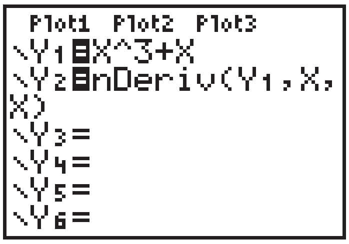
$$= 0.001 \times \frac{-2}{(x+4)^2}$$ **[A]**

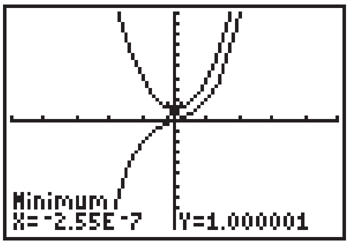
## Question 12

$f(x) = x^3 + x$ has a point of inflection at $x = 0$.

$f'(x) = 3x^2 + 1$

$f'(0) = 1$




**[B]**

## Question 13

$$\frac{d}{dx}\log_e(1-\sin^2(x))$$

$$=\frac{d}{dx}\log_e(\cos^2(x))$$

$$=\frac{-2\cos(x)\sin(x)}{\cos^2(x)}$$

$$=\frac{-2\sin(x)}{\cos(x)}$$

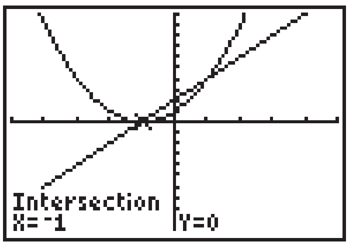
$= -2\tan(x)$ **[D]**

## Question 14

$g'(x) = 2x + 2$

$g'(x)$ intersects $g(x)$ at $x = -1$.

$g'(-1) = 0$ and the gradient of $g'(x) = 2$.




Hence $g'(x)$ is not a tangent to $g(x)$ at $x = -1$. **[E]**

## Question 15

$$\int (2x^2+4)^3\,dx$$

$$= \int (8x^6+48x^4+96x^2+64)\,dx$$

by the binomial expansion or otherwise

$$= \frac{8x^7}{7}+\frac{48x^5}{5}+32x^3+64x+c,$$

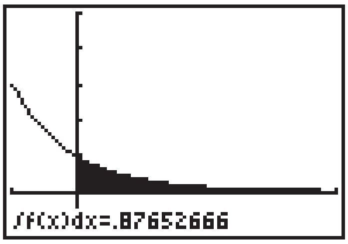
where $c$ is a constant.

Hence $\frac{8x^7}{7}+\frac{48x^5}{5}+32x^3+64x+8$

is an antiderivative **[C]**

## Question 16

$\frac{13}{27}$ is less than the exact answer.




Hence the right rectangle rule must have been used.

Try width 1 unit.

Area $\approx f(1) + f(2) + f(3)$

$$= \frac{1}{3}+\frac{1}{9}+\frac{1}{27}$$

$$= \frac{13}{27}$$

**[A]**

## Question 17

Top curve minus the bottom curve.

$$\int_a^b (g(x)-f(x))dx + \int_b^c ((f(x)-g(x))dx$$

**[D]**

## Question 18

$^9C_5(2x)^4(-1)^5 = -126 \times 16x^4$

$= -2016x^4$

The term is $-2016x^4$. **[E]**

## Question 19

$f(x) = 2\sqrt{1-3x}$

The range is $(0, \infty)$.

Let $y = 2\sqrt{1-3x}$

Then the inverse is

$x = 2\sqrt{1-3y}$

$(\frac{x}{2})^2 = 1 - 3y$

$y = \dfrac{1 - \frac{x^2}{4}}{3}$

$= \dfrac{4 - x^2}{12}$

The domain of the inverse is $(0, \infty)$.

$f^{-1}: (0, \infty) \to R$, where $f^{-1}(x) = -\frac{x^2}{12} + \frac{1}{3}$ **[C]**

## Question 20

$y = \log_e(2x + 1) + 3$

$x$-intercept

Let $y = 0$

$0 = \log_e(2x + 1) + 3$

$e^{-3} = 2x + 1$

$x = \dfrac{e^{-3} - 1}{2}$

$= \dfrac{1 - e^3}{2e^3}$

$y$-intercept

Let $x = 0$

$y = \log_e(1) + 3$

$= 3$ **[A]**

## Question 21

$3e^{2x} - 17e^x + 10 = 0$

Let $a = e^x$

$3a^2 - 17a + 10 = 0$

$(3a - 2)(a - 5) = 0$

$a = \frac{2}{3}$ or 5

$e^x = \frac{2}{3}$ or $e^x = 5$

$x = \log_e(\frac{2}{3}) = \log_e(2) - \log_e(3)$ or $x = \log_e(5)$ **[C]**

## Question 22

$\log_2(2x) - 5\log_2(x - 1) - \log_2(y) = 2$

$\log_2(\dfrac{2x}{y(x-1)^5}) = 2$

$\dfrac{2x}{y(x-1)^5} = 4$

$y = \dfrac{2x}{4(x-1)^5} = \dfrac{x}{2(x-1)^5}$ **[B]**

## Question 23

$\Pr(X > 15) = \text{normalcdf}(15,1E99,10,3)$

$= 0.0478$ **[C]**

## Question 24

Hypergeometric distribution with $N = 16$, $n = 10$, $D = 4$, $x = 0, 1$ **[C]**

## Question 25

$\text{Var}(X) = E(X^2) - [E(X)]^2$

$= 4k - 4k^2$

$\text{Sd}(X) = \sqrt{\text{Var}(X)} = 2\sqrt{k(1-k)}$ **[D]**

## Question 26

Recognise the symmetry; 0.7 + 0.15 = 0.85

InvNorm (0.85,1,1.5) = 2.555

$M + k = 2.555$, $k = 1.555$ **[A]**

### Question 27

For binomial
$E(X) = np$, $\mathrm{Var}(X) = np(1-p)$

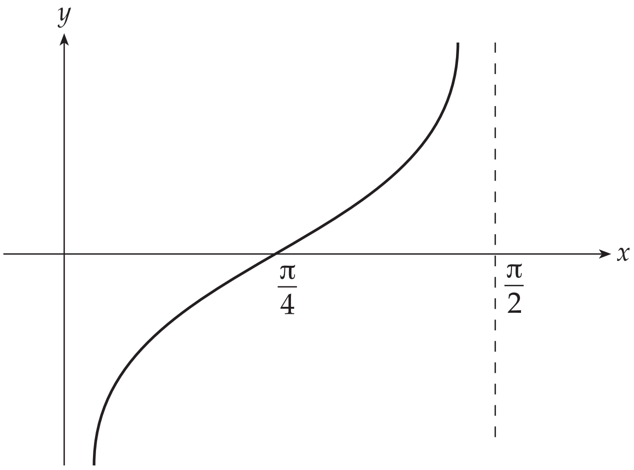
For hypergeometric $E(X) = \dfrac{nD}{N}$,

$\mathrm{Var}(X) = \dfrac{nD(N-D)(N-n)}{N^2(N-1)}$

$< np(1-p)$ **[E]**

## Solutions Short Answer 2004

### Question 1



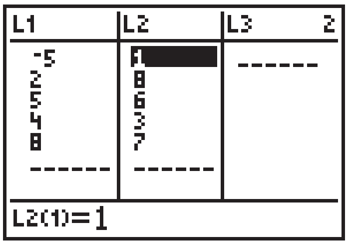

Asymptotes and intercept shown **[1A]**

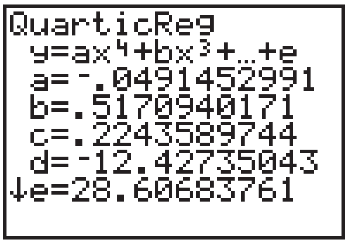
Correct shape **[1A]**

$n = 2, k = \dfrac{\pi}{2}$ **[2A]**

### Question 2

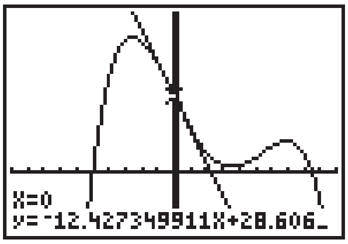
**a**




$y = -0.049x^4 + 0.517x^3 + 0.224x^2$
$- 12.427x + 28.607$ **[1A]**

**b**




The gradient of the tangent is $\approx -12.42735$.

The gradient of the normal is $\approx \dfrac{1}{12.42735} \approx 0.08047$

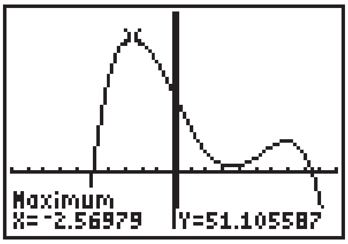
The $y$-intercept is $\approx 28.607$.

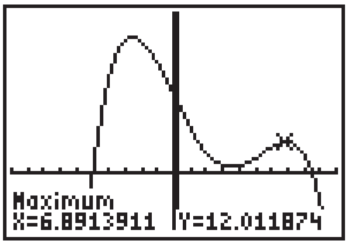
The equation of the normal is

$y = 0.08x + 28.61$

correct to two decimal places. **[2A]**

**c**



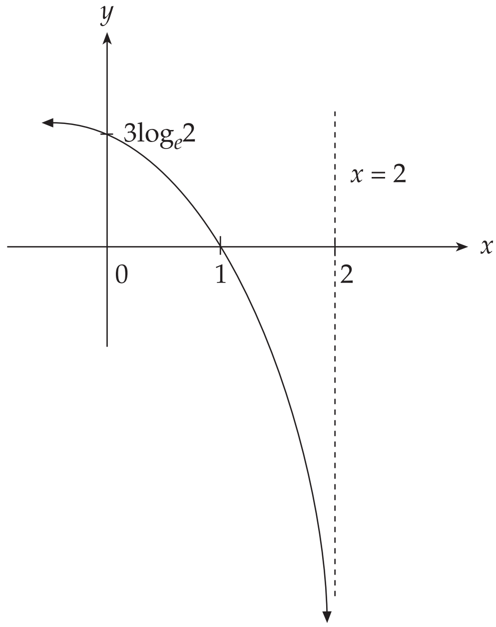




Average rate of change $\approx \dfrac{12.01187 - 51.10559}{6.89139 - -2.56979}$ **[1M]**

$\approx -4.13$ **[1A]**

## Question 3

**a**




Asymptotes and cuts on axes **[3A]**

**b** $f'(x) = \dfrac{-3}{2-x} = \dfrac{3}{x-2}$ **[1A]**

## Question 4

**a** $x = 2\sqrt{3x+2}$

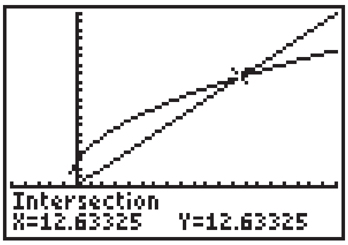
$x^2 = 4(3x + 2)$

$x^2 - 12x - 8 = 0$ **[1M]**

$x = \dfrac{12 \pm \sqrt{144 - (4 \times 1 \times -8)}}{2}$

$= \dfrac{12 \pm \sqrt{176}}{2}$

$= 6 + 2\sqrt{11}$ as $6 - 2\sqrt{11}$ is outside the domain. **[1A]**



**b** Let $y = 2\sqrt{3x+2}$

Inverse $x = 2\sqrt{3y+2}$ **[1M]**

$$\frac{x^2}{4} = 3y + 2$$

$$y = \frac{x^2}{12} - \frac{2}{3}$$

$f^{-1}: [0, \infty) \to R$, where $f^{-1}(x) = \frac{x^2}{12} - \frac{2}{3}$ **[1A]**

**c** $\int_0^{6+2\sqrt{11}} \left( x - \left( \frac{x^2}{12} - \frac{2}{3} \right) \right) dx$ **[1M]**

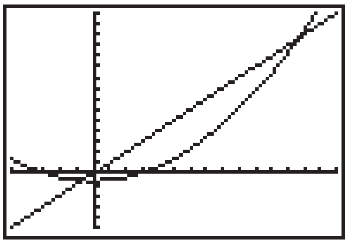
$= \left[ \frac{x^2}{2} - \frac{x^3}{36} + \frac{2}{3}x \right]_0^{6+2\sqrt{11}}$ **[1M]**

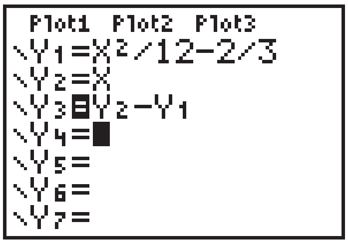
$= \frac{(6+2\sqrt{11})^2}{2} - \frac{(6+2\sqrt{11})^3}{36} + \frac{2(6+2\sqrt{11})}{3}$

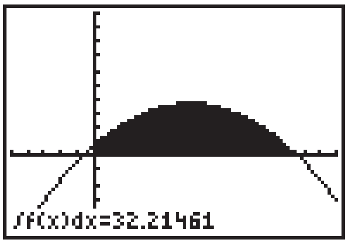
$= 16 + \frac{44\sqrt{11}}{9}$

$\approx 32.2146$ **[1A]**

Calculator check






## Question 5

**a** $\Pr(B \geq 1) = 1 - \Pr(B = 0)$

$= 1 - 0.49^3$

$= 0.8824$ **[1A]**

**b** $\Pr(B = 3 \mid B \geq 1) = \frac{\Pr(B = 3)}{\Pr(B \geq 1)}$ **[1M]**

$= \frac{0.51^3}{0.8824}$

$= 0.1503$ **[1A]**