Year 2004

VCE

Mathematical Methods

Trial Examination 2

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STUDENT NUMBER									Letter
Figures									
Words									

VICTORIAN CERTIFICATE OF EDUCATION 2004

MATHEMATICAL METHODS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Number of questions	*Number of questions to be answered*	*Number of marks*
4	4	55

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or whiteout liquid/tape.

Materials supplied

- Question and answer book of 10 pages with a detachable sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x} \qquad \int \frac{1}{x}dx = \log_e x + c, \text{for } x > 0$$

$$\frac{d}{dx}(\sin ax) = a\cos ax \qquad \int \sin ax\, dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax \qquad \int \cos ax\, dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}(\tan ax) = \frac{a}{\cos^2 ax} = a\sec^2 ax$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$ $= \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\ {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $\mathrm{N}(0, 1)$.		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET

Question 1

A certain liquid at temperature T^0C, cools down according to the model $T = 50e^{-kt} + 20$ where t is the time in minutes from the moment the heating device is switched off.

a. What is the initial temperature of the liquid when the heating is first switched off?

_____

_____

_____

1 mark

b. If the liquid cools to 45^0C after three minutes, find the value of k to three decimal places.

_____

_____

_____

_____

_____

2 marks

c. How long will it take for the liquid to cool to half its temperature when the heating device was first switched off? Give your answer to one decimal place.

_____

_____

_____

_____

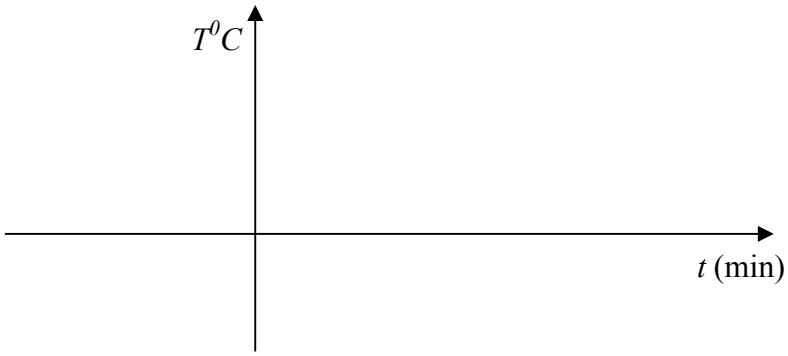
_____

_____

1 mark

Question 1 (continued)

d. Sketch the graph of $T = 50e^{-kt} + 20$ on the axes below, showing any intercepts with axes and giving the equation of any asymptotes.



3 marks

e. Find the rate at which the temperature is decreasing when $T = 35^0C$ Give your answer to two decimal places.

_____

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2 marks

Question 1 (continued)

f. Another liquid heats up according to the model $T = a(3t - b)^2$ where a and b are positive constants and where T is measured in ^0C and t is measured in minutes. When this graph of temperature versus time is sketched, it is found that the graph passes through the point (4,5). It is also found that the tangent to the point (3,0) on the curve is parallel to the time axis.

i Find the value of b

ii Find the value of a

iii Find the value of t to one decimal place when the liquid from part (a) and this liquid have the same temperature.

1 + 1+ 1= 3 marks

(Total = 12 marks)

Question 2.

The time for the 109 tram to go from Box Hill to Port Melbourne is normally distributed with a mean of 65 minutes and a standard deviation of 4 minutes. A tram is considered too slow when it takes more than 75 minutes for the journey.

a. What is the probability, correct to four decimal places, that a tram is too slow?

2 marks

b. If 1% of trams are too fast for the journey, what is the minimum time for a journey not to be considered too fast? Give your answer to one decimal place.

2 marks

c. Trams that are neither too fast nor too slow on this line are said to have acceptable times. What is the probability that a tram on this line has an acceptable time? Give your answer to four decimal places.

1 mark

Question 2 (continued)

d. When trams do not have acceptable times, the tram ways board must pay a fine of $10. When trams do have acceptable times, their profit is $4.

i Complete the table below where $\$Y$ is the profit per trip.

	Acceptable	Not acceptable
Y		
$\Pr(Y=y)$		

4 marks

ii What is the expected daily profit if there are 80 trips each day on the 109 line?

Give your answer to the nearest dollar.

_____

_____

_____

_____

_____

_____

2 marks

e. It is known that the 10.00AM tram from Box Hill to Port Melbourne always takes longer than 65 minutes. What is the probability that its time is acceptable? Give your answer to four decimal places.

_____

_____

_____

_____

_____

_____

2 marks

Question 2 (continued)

f. If Delta catches 20 trams from Box Hill to Port Melbourne, what is the probability that at least one of these trams will be too slow? Give your answer to three decimal places.

2 marks

g. Delta knows that if she catches as many 109 trams from Box Hill to Port Melbourne as she caught last year, the probability of at least one tram being too slow is 0.285. How many of these trams did she take last year?

2 marks

(Total = 17 marks)

Question 3

a. Differentiate $\frac{1}{2}(\sin(x)-\cos(x))e^{-x}$

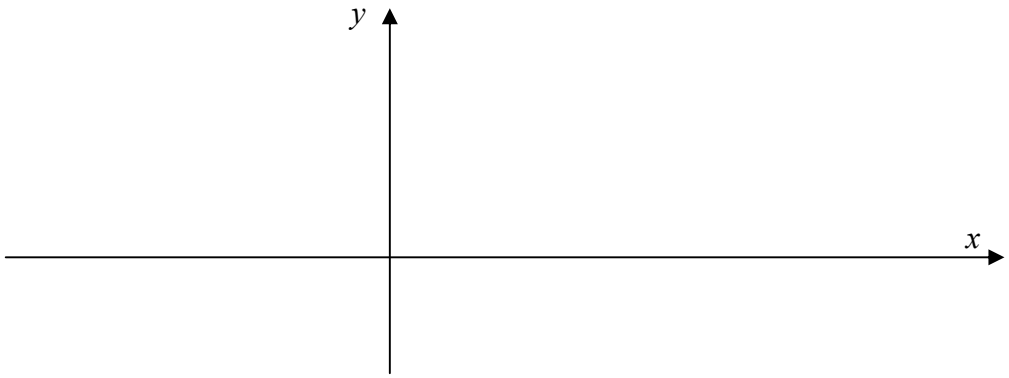
2 marks

Question 3 (continued)

b.

On the axes below, sketch the graph of $f(x) = 3e^{-x}\cos(x)$ $\quad -\pi \le x \le \frac{\pi}{2}$

Give exact values for X and Y intercepts, any turning points and the end points.



8 marks

Question 3 (continued)

c.

Using the information obtained in part(a), find the area between the X axis and the graph of $f(x) = 3e^{-x}\cos(x)$ from $x = -\frac{\pi}{4}$ to $x = \frac{\pi}{2}$. Give an exact answer.

3 marks

(Total = 13 marks)

Question 4

The drama department of a university sells tickets to the annual revue. They estimate that 20,000 tickets would be bought if the price of each ticket is \$5.00, and that only 10,000 tickets would be bought if they cost \$10.00 each. The cost, C, of organizing the costumes for the revue is \$1,500 and 20 cents per ticket for printing. They only print the number of tickets they can sell.

a The number of tickets sold is modelled by the equation $n = a + bs$ where s is the selling price per ticket and a and b are constants.
Find a and b

2 marks

Question 4 (continued)

b. If the selling price per ticket is $\$s$ then show that $C = 7500 - 400s$

2 marks

c.

Show that in terms of the selling price, s, the profit for the revue is $P = 30,400s - 2000s^2 - 7500$

2 marks

d. What selling price per ticket, s, will give a maximum profit?

2 marks

e. What is the maximum profit?

1 mark

Question 4 (continued)

f. How many tickets would have to be sold to achieve this profit?

_____

_____

_____

1 mark

g. What is the minimum price for which a ticket can be sold if the department does not want to make a loss?

_____

_____

_____

_____

_____

1 mark

h. If only 12,000 people purchase tickets, what is the maximum profit?

_____

_____

_____

_____

_____

2 marks

(Total = 13 marks)

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End of 2004 Mathematical Methods Trial Examination 2
Question and Answer Book

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