THE HEFFERNAN GROUP

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MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 2

(ANALYSIS TASK)

2004

Reading Time: 15 minutes
Writing time: 90 minutes

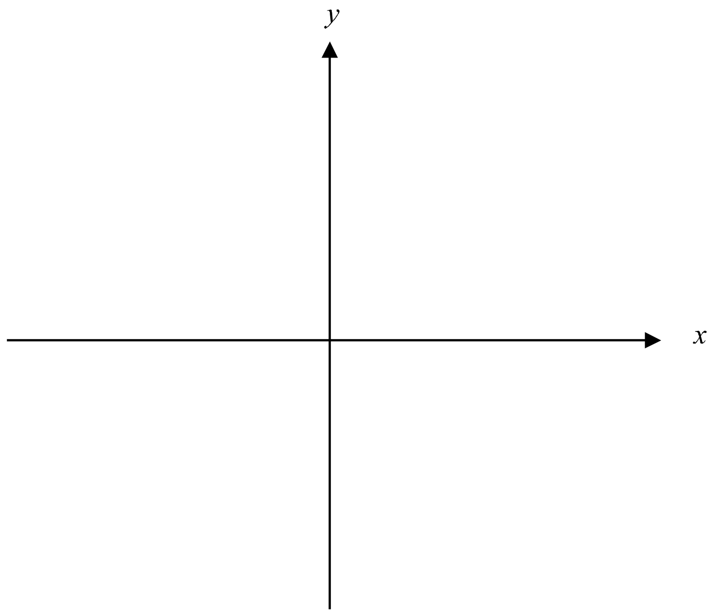
Instructions to students

This exam consists of 4 questions.
All questions should be answered.
There is a total of 55 marks available.
The marks allocated to each of the four questions are indicated throughout.
Students may bring up to two A4 pages of pre-written notes into the exam.
Formula sheets and a table of the Normal distribution - cdf can be found on pages 14-16 of this exam.
Diagrams in this exam are not to scale except where otherwise stated.

Question 1

a. **i.** Sketch the graph of $y = \frac{-2}{1-x} + 4$ on the set of axes below.

Show any asymptotes or intercepts clearly on your graph.



2 marks

ii. Write down the domain and range of the function.

1 mark

b. **i.** Find $\frac{dy}{dx}$.

1 mark

ii. Using part **a. i.**, explain why there is no solution to the equation $\frac{dy}{dx}=0$.

1 mark

c. Let $f: R\setminus\{1\}\to R$ where $f(x)=\frac{-2}{1-x}+4$.

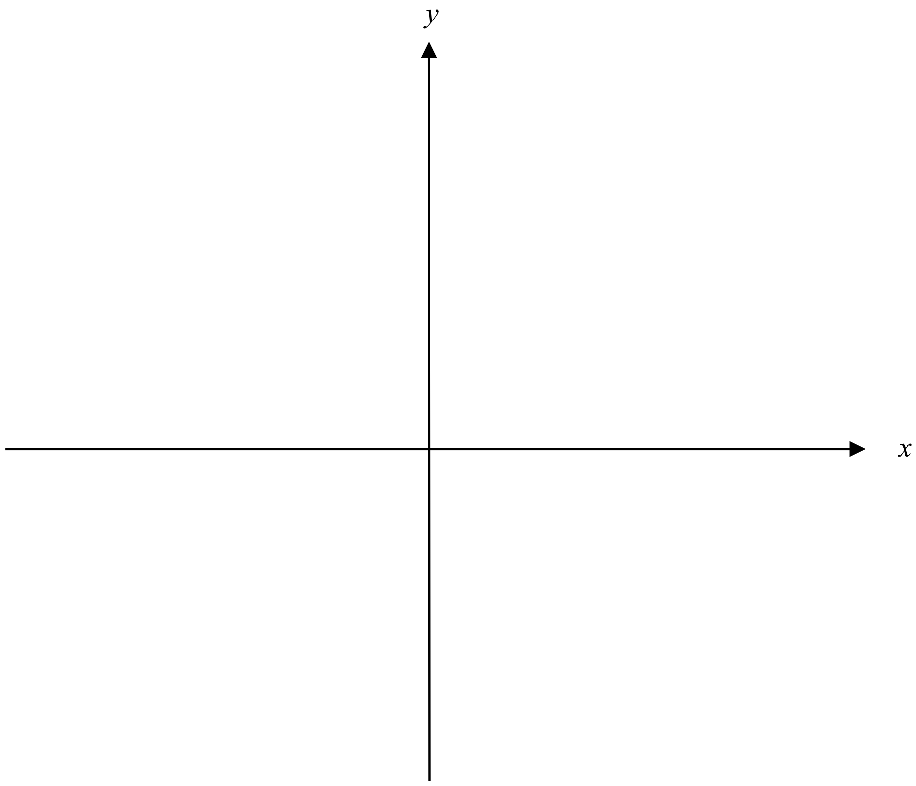
i. Explain why f^{-1} exists.

1 mark

ii. Show that $f^{-1}(x)=\frac{2}{x-4}+1$.

1 mark

iii. On the set of axes below, sketch the graph of $y = f^{-1}(x)$.
Show any asymptotes, or intercepts clearly on your graph.



2 marks

d. Show that $\int_0^{\frac{1}{2}} f(x)\,dx = \int_0^{2} f^{-1}(x)\,dx$.

5 marks

e. Find correct to 2 decimal places the value(s) of x for which $f(x) = f^{-1}(x)$.

2 marks

Total 16 marks

Question 2

At a blood donation facility, donors have their haemoglobin levels checked prior to donating blood. The haemoglobin levels for women over time at this facility are found to be normally distributed with a mean of 140 g/L and a standard deviation of 19.5 g/L.
Approximately 10% of female donors who attend the facility have a haemoglobin level that is considered to be high.

a. What is the probability that a randomly selected female donor at the facility will have a haemoglobin level that is considered high?

1 mark

b. What is the probability that if ten female donors at the facility are randomly selected, at least two will have a haemoglobin level that is considered high?

2 marks

c. What is the minimum haemoglobin level that is considered high? Express your answer to the nearest whole number.

2 marks

At the other end of the scale, female donors who have a haemoglobin level that is below 120 g/L are not allowed to donate blood.

d. What percentage of female donors at the facility are not allowed to donate blood because their haemoglobin level is too low? Express your answer to the nearest whole percent.

2 marks

e. If the blood donation facility has 300 female donors attend in a day, use your answer to part **d.** to find the expected number of those donors who don't suffer from a haemoglobin level that is either considered high or is too low to donate.

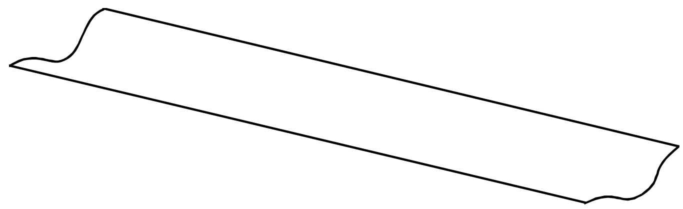
2 marks

f. A group of 12 female friends attend the blood donation facility together and then all go out afterwards for lunch.
On a particular day, two of them are found to have a haemoglobin level that is either considered high or too low.
What is the probability that if three of the women are randomly selected at lunch, all three had haemoglobin levels that were considered neither high nor too low?

3 marks
Total 12 marks

Question 3

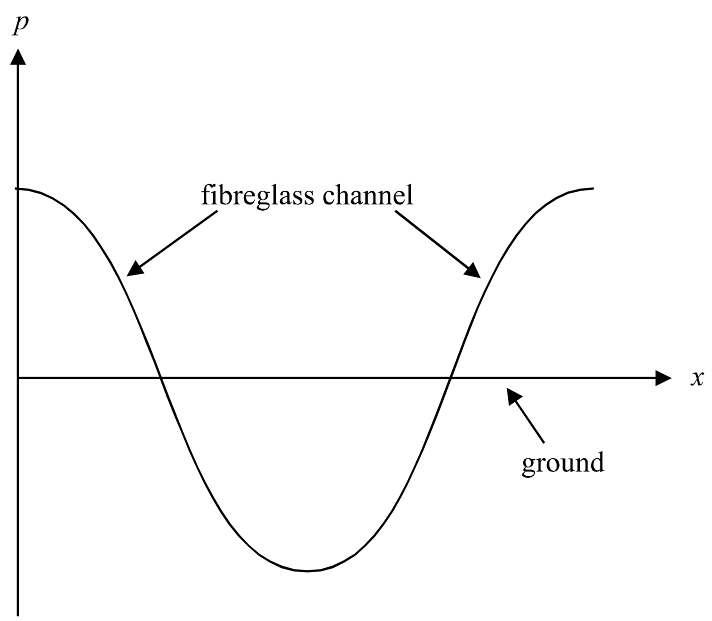
On a farm, irrigation channels are used to water crops. A prefabricated fibreglass channel is set into a dirt channel. These fibreglass channels are used to avoid water soaking into the soil before reaching its intended destination. A length of the fibreglass channel is shown below.



The bottom half of the fibreglass channel is set into the soil and the top half sits above the soil.
When correctly installed, the vertical position, p, in metres of the fibreglass channel is given by

$$p(x) = \cos(\pi x),\ x \in [0,2]$$

where $p = 0$ represents ground level and x is the distance in metres from the side of the channel.
A cross-sectional view of a fibreglass channel has been placed on a set of axes and is shown on the graph below.



The level of irrigation water in the channel is normally the same as ground level.

a. What is the normal height of water in the fibreglass channel measured from the lowest point in that channel?

1 mark

b. **i.** Find the area of the graph enclosed by the function and the x-axis. Express your answer correct to 4 decimal places.

2 marks

ii. Hence find the volume of water to the nearest litre, in a 10 metre length of fiberglass channel given that $1\text{mL} = 1\text{cm}^3$.

1 mark

c. **i.** Find $\dfrac{dp}{dx}$.

1 mark

ii. Hence find the maximum and minimum gradient of the sides of the channel.

2 marks

iii. How far from the edge of the fibreglass channel do the maximum and minimum gradients occur?

2 marks

The fibreglass channels are designed to withstand the force of water that rises to $\frac{\sqrt{3}}{2}$ metres **above** ground level.

d. Find the cross-sectional area of water in one of the fibreglass channels if it reaches this height.
Express your answer as an exact value.

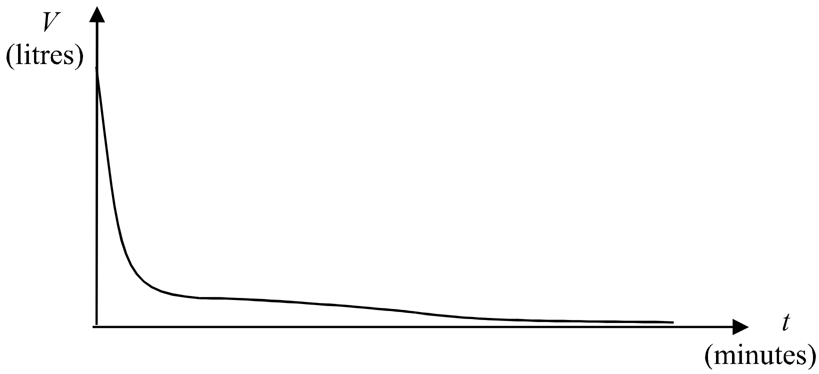
4 marks
Total 13 marks

Question 4

A piece of machinery has 10 litres of a coolant fluid contained in its system, that assists in the cooling of the motor.
One of the hoses that carries this fluid bursts and the fluid escapes.
The volume V, in litres, of the coolant fluid in the system at time t minutes after the hose bursts is modelled by

$$V(t) = ke^{-t} + ke^{-t}(t-1)^2, \quad t \geq 0$$

The graph of this equation is shown in the diagram below.



a. Show that $k = 5$.

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1 mark

b. What value does $V(t)$ approach as time increases?

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1 mark

c. When the volume of the fluid remaining in the system drops below 5 litres an alarm goes off. Use your graphics calculator to find out how long after the hose bursts, does the alarm go off.
Express your answer, in minutes, correct to 2 decimal places.

2 marks

d. By considering the appropriate average rates of change, explain whether the rate of change of V with respect to t is greater in the first minute or in the second minute after the hose bursts.

2 marks

e. **i.** Use calculus to find the value of t for any stationary points on the graph of $y = V(t)$.

3 marks

ii. State the nature of this stationary point. Give a reason for your answer.

2 marks

iii. The model includes an instant when an employee momentarily managed to stop the fluid from leaking from the hose. Find the exact volume of liquid in the system at that instant.

1 mark

f. Use the derivative function $V'(t)$ to explain why the gradient of the function $y = V(t)$ can never be positive.

2 marks
Total 14 marks

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

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Mathematical Methods Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x}\,dx = \log_e(x) + c, \text{ for } x > 0$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

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Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A / B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$ variance: $\mathrm{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

Discrete distributions

	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2\, p(x)$ $= \Sigma\, x^2\, p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\dfrac{{}^DC_x\;{}^{N-D}C_{n-x}}{{}^NC_n}$	$n\dfrac{D}{N}$	$n\dfrac{D}{N}\left(1-\dfrac{D}{N}\right)\left(\dfrac{N-n}{N-1}\right)$

Continuous distributions

normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \dfrac{X-\mu}{\sigma}$ then Z is distributed $\mathrm{N}(0, 1)$.

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