VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2003 Mathematical Methods GA 3: Written examination 2

GENERAL COMMENTS

There were 17 594 students who presented for this examination and the entire range of marks from 0 to 55 was scored. The paper was accessible to students, although both the number of scores at the very top of the range and at the very bottom of the range were significantly less than in the last few years.

There was evidence of ineffective calculator use, leading to answers incorrect in the last decimal place. The use of the TRACE button is invariably inaccurate and inefficient and students need to learn to use the calculator more efficiently.

This year, students were required to show working for any question worth more than 1 mark. Suitable working in the early probability questions, where many answers are obtained from a calculator without any need to write down anything else, could be:

- the instruction for evaluation entered into the calculator, e.g. **normcdf (145,9999,140,1.2)** for Question 1a
- a normal distribution graph with the mean marked and a relevant area shaded
- an appropriate value of z
- identification of the distribution, relevant parameters and values of the random variable for binomial or hypergeometric distributions.

In Questions 2b, 3ci and 3ciii, students were required to *show* a given result. These questions were generally not well done – students need to practise this kind of question. \
Suitable responses were:

Question 2b

Gradient $= \dfrac{dy}{dx} = \sin\left(\dfrac{x}{2}\right)$, but $\sin\left(\dfrac{x}{2}\right) \le 1$ for all x, so the gradient of the ramp is always less than or equal to 1.

Question 3ci

$f(1) = e^{-2}$ and $f'(1) = e^{-2}$

So the equation to the tangent is

$y - e^{-2} = e^{-2}(x-1) \Rightarrow y - e^{-2} = e^{-2}x - e^{-2} \Rightarrow y = e^{-2}x$

ciii

Equation of the tangent is of the form $y = mx + c$;

when $x = a$, $m = e^{-2a}(3a^2 - 2a^3)$ and $y = a^3e^{-2a}$

then $c = a^3e^{-2a} - e^{-2a}(3a^2 - 2a^3)a$

$= e^{-2a}(a^3 - 3a^3 + 2a^4)$

$= a^3e^{-2a}(2a - 2)$

This gives c = 0, and tangents passing through the origin, only when $a = 0$ or $a = 1$. So the two tangents in i and ii are the only that pass through the origin.

Students lost marks they might have otherwise gained because they:

- did not show working for questions worth more than 1 mark
- did not answer the specific question asked
- gave decimal answers when an exact answer was required or gave the incorrect number of decimal places
- misread the question in other ways
- did not pay enough attention to detail in sketching graphs
- were not sufficiently careful, particularly with algebra and differentiation.

SPECIFIC INFORMATION

Question 1

a

Marks	0	1	2	Average
%	13	15	72	**1.59**

Answer: 0.106

b

Marks	0	1	2	Average
%	70	9	21	**0.51**

Answer: 1.7

c

Marks	0	1	2	Average
%	31	9	60	**1.29**

Answer: 0.292

d

Marks	0	1	2	Average
%	18	29	53	**1.35**

Answer: 0.672

ei

Marks	0	1	Average
%	10	90	**0.90**

Answer: 0.68

eii

Marks	0	1	Average
%	42	58	**0.58**

Answer: $E(Y) = 0.85x - 4.76$

eiii

Marks	0	1	Average
%	52	48	**0.48**

Answer: 5.60

eiv

Marks	0	1	Average
%	87	13	**0.13**

Answer: 0.80

Most students identified the correct distribution in parts a to d, and there many used calculator programs to handle the calculations. Errors frequently arose from arithmetic miscalculations, giving answers to the wrong number of decimal places and from misinterpretation of 'at least 2' in part d. A common mistake in part b was to ignore the two-tailed nature of the distribution.

Part e was generally well answered, although some students did not recognise the conditional probability in part eiv. This can be answered without explicit reference to conditional probability by arguing from first principles that the probability of a rod being ready to be sold is 0.85 and the probability it is a good one is 0.68, so the proportion of good ones ready for sale is $\frac{0.68}{0.85} = 0.80$.

Question 2

a

Marks	0	1	Average
%	18	82	**0.82**

Answer: 2.83

b

Marks	0	1	2	Average
%	25	48	27	**1.01**

Answer: Gradient = $\sin(\frac{x}{2}) \leq 1$

c

Marks	0	1	2	3	Average
%	22	22	10	46	**1.78**

Answer: 14.18

di

Marks	0	1	2	Average
%	28	16	56	**1.27**

Answer: $\frac{2\pi}{3}$

dii

Marks	0	1	2	Average
%	43	18	39	**0.95**

Answer: $-\frac{2}{\sqrt{3}}$

diii

Marks	0	1	2	3	Average
%	59	20	7	14	**0.76**

Answer: $\frac{\sqrt{7}}{2}$

Most students handled the early parts of this question reasonably well. Explanations given in part b were of variable quality and it is clear that 'show' questions require more attention during the year. A number of students believed they could show the gradient is always less than or equal to 1 by finding its value at a few points (usually including the end-points). This kind of fallacious argument must be discussed in the classroom. Students should be aware that while a single counter-example suffices to disprove a mathematical statement, a general argument is typically required to establish one. It was disappointing when students tried to apply $\frac{\text{rise}}{\text{run}}$ to find the gradient.

In part c, a number of students ignored the instruction to use calculus (requiring that an appropriate antiderivative be written down), simply finding an answer via their calculator, gaining a maximum of 1 mark out of 3. Some incorrect answers were obtained by finding half the area required, rounding to two decimal places and doubling; students must remember to round at the very last step.

Part d caused problems for many. Part di required the solution of a trigonometric equation and while $x = 2\cos^{-1}(0.5)$ is a correct answer, a simplified answer would lead to easier working in part ii. If students do not remember the trigonometric ratios for the multiples of $\frac{\pi}{6}$, they should include them on their four pages of notes. In part dii, the instruction to find an exact answer was often ignored and a number of students went needlessly on to find the equation of the normal. Part iii was not done well, yet there are a number of correct methods, probably the easiest using similar triangles:

Let the foot of the perpendicular from A to the x-axis be C. Since the gradient of the normal (from part ii) is $-\frac{2}{\sqrt{3}}$

Then $\frac{AC}{BC} = \frac{2}{\sqrt{3}}$. But $AC = 1$, so $BC = \frac{\sqrt{3}}{2}$ and $AB = \sqrt{AC^2 + BC^2} = \sqrt{1 + \frac{3}{4}} = \frac{\sqrt{7}}{2}$

Question 3

a

Marks	0	1	Average
%	41	59	**0.59**

Answer: $a = -2$, $b = 3$

b

Marks	0	1	2	Average
%	32	39	29	**0.96**

Answer: Stationary point of inflection at (0, 0)

Local maximum at $(1.5, \frac{27e^{-3}}{8})$

ci

Marks	0	1	2	3	Average
%	38	18	6	38	**1.43**

Answer: See above

cii

Marks	0	1	Average
%	52	48	**0.48**

Answer: $y = 0$

ciii

Marks	0	1	2	3	Average
%	90	5	2	3	**0.18**

Answer: See general comments

di

Marks	0	1	2	3	Average
%	64	14	4	18	**0.76**

Answer: $p = 6$, $q = 6$, k = –8

dii

Marks	0	1	2	3	Average
%	73	15	6	6	**0.46**

Answer: $\frac{23e^{-2}}{8} - \frac{3}{8}$

Some students did not use the product rule or failed to factorise the expression correctly in part a. Others just gave the wrong values for a and b from correct working. Many students lost marks in Question 3b because they did not answer the question – they did not give *coordinates* of the stationary points or they did not *state their nature*.

Part 3c required showing two things and was not handled well by many students. It is essential that students practise questions of this type and produce arguments which are clear and follow logically from one line of working to the next. In part ii, answers of $y = x$ or $y = e^{-2}x$ were very common. In part iii, many students started out well by attempting to write an equation for the tangent in terms of x. In order to do this correctly, it is necessary to let the x-coordinate of the point of tangency be a, or some other parameter (but not x). Not doing this leads to erroneous arguments such as 'the equation to the tangent is $y = e^{-2x}(3x^2 - 2x^3)x + c$' where this equation is certainly not that of a straight line. Some students showed that the two tangents were in fact correct and thought this a sufficient answer.

Part 3d required application of the Fundamental Theorem of Calculus. In part 3di, some students did not attempt to differentiate the given expression; others who did, did not apply the product rule correctly, or did not know to equate co-efficients. In part ii, careless working was responsible for many lost marks.

Question 4

a

Marks	0	1	2	3	Average
%	14	31	17	38	**1.78**

Answer: 6.02 hours

b

Marks	0	1	2	3	4	Average
%	39	21	4	7	29	**1.67**

Answer: $\sqrt{5}, \frac{3\sqrt{5}}{10}$

c

Marks	0	1	Average
%	74	26	**0.26**

Answer: 1.22

di

Marks	0	1	Average
%	60	40	**0.40**

Answer: $\sqrt{5}$

dii

Marks	0	1	2	Average
%	76	20	4	**0.27**

Answer: Graph of inverse

diii

Marks	0	1	2	3	Average
%	32	62	3	3	**0.76**

Answer: $x = \frac{3 + \sqrt{(9 - 20t^2)}}{2t}$

Most students handled the early parts of the question well, although lack of care with differentiation was all too evident in part b, whereas other students found non-exact values via a calculator and did not gain marks accordingly. Part d was challenging for many students with only a few using the quadratic formula in part iii to find $x = \frac{3 \pm \sqrt{9 - 20t^2}}{2t}$, let alone selecting one of the two solutions, and many writing $x = 0$ for the asymptote in part ii.