VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2003

MATHEMATICAL METHODS

Written examination 1

(Facts, skills and applications)

Friday 7 November 2003

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

PART I
MULTIPLE-CHOICE QUESTION BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
27	27	27

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 13 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
- You may retain this question book.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

This page is blank

Instructions for Part I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

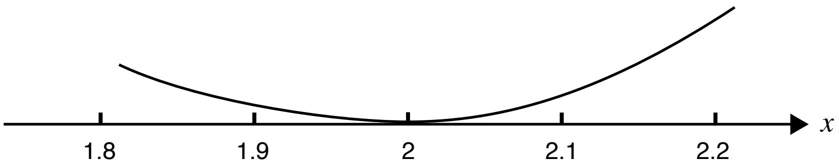
A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

A polynomial function p has degree three. A part of its graph, near the point on the graph with coordinates (2, 0), is shown below.



Which one of the following could be the rule for the third degree polynomial p?

A. $p(x) = x(x + 2)^2$

B. $p(x) = (x - 2)^3$

C. $p(x) = x^2(x - 2)$

D. $p(x) = (x - 1)(x - 2)^2$

E. $p(x) = -x(x - 2)^2$

Question 2

Which one of the following is **not** true of the graph of the function $f: R^+ \to R, f(x) = \log_2(x)$?

A. It has a vertical asymptote with equation $x = 0$.

B. It passes through the point (2, 0).

C. The slope of the tangent at any point on the graph is positive.

D. It has domain R^+.

E. It has range R.

Question 3

Dylan drew the graph of the function $f: R \to R, f(x) = \dfrac{x^3 + 1}{x}$ by adding the ordinates of the graphs of two functions g and h.

The rules for g and h that Dylan could have used are

A. $g(x) = x^3$ and $h(x) = \dfrac{1}{x}$

B. $g(x) = x^2$ and $h(x) = \dfrac{1}{x}$

C. $g(x) = x^3 + 1$ and $h(x) = x$

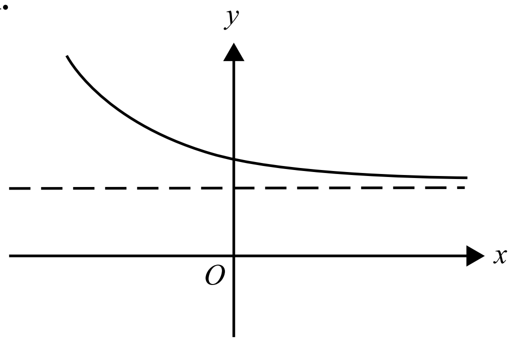
D. $g(x) = x^3 + 1$ and $h(x) = \dfrac{1}{x}$

E. $g(x) = x^2$ and $h(x) = 1$

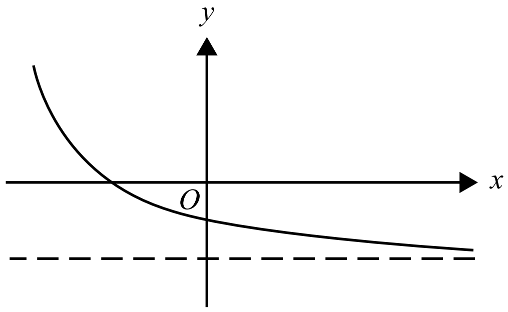
Question 4

If k and P are positive real numbers, which one of the following graphs is most likely to be the graph of the function with equation $y = e^{kx} + P$?

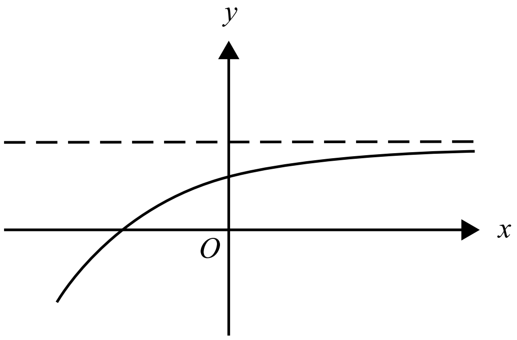
A.



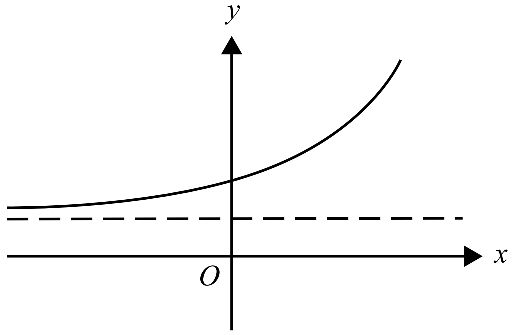
B.



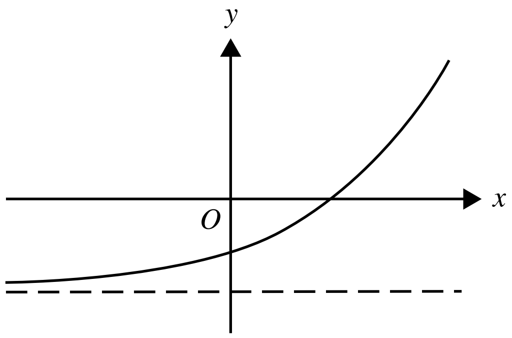
C.



D.



E.



Question 5

The graph of the function f is obtained from the graph of the function with equation $y = \sqrt{x}$ by a reflection in the y-axis followed by a dilation of 2 units from the x-axis.

The rule for f is

A. $f(x) = -2\sqrt{x}$

B. $f(x) = \sqrt{-2x}$

C. $f(x) = \sqrt{-0.5x}$

D. $f(x) = -0.5\sqrt{x}$

E. $f(x) = 2\sqrt{-x}$

Question 6

The number of solutions of the equation $0.5\cos(2x) = 1$, for $x \in [-\pi, \pi]$, is

A. 0

B. 1

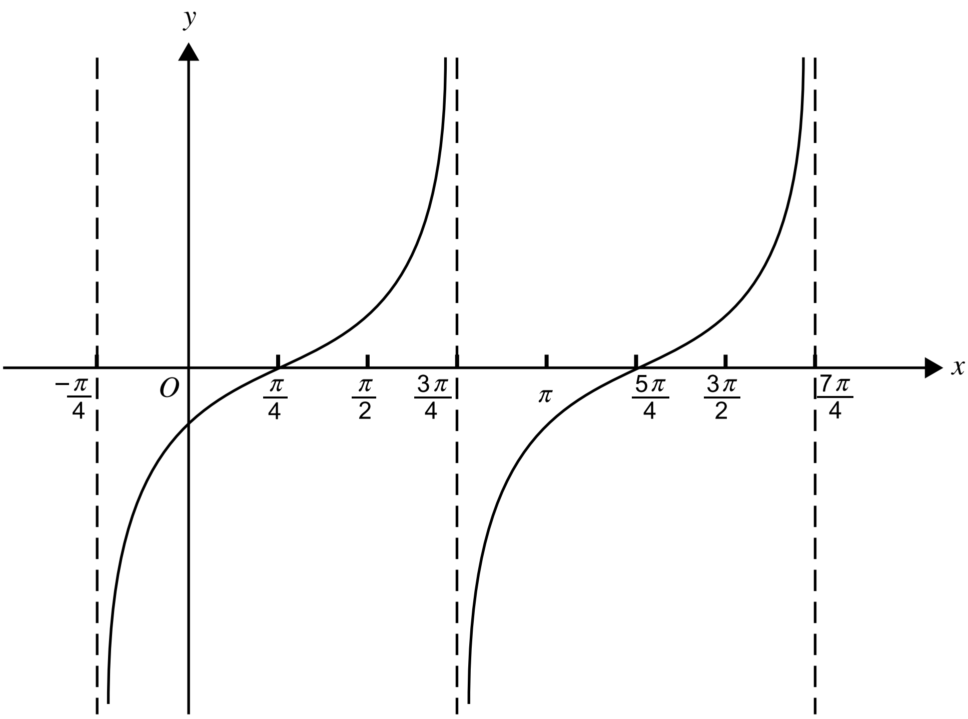
C. 2

D. 3

E. 4

Question 7

The diagram shows two cycles of the graph of a circular function.



The period of the circular function is

A. $\frac{\pi}{2}$

B. $\frac{3\pi}{4}$

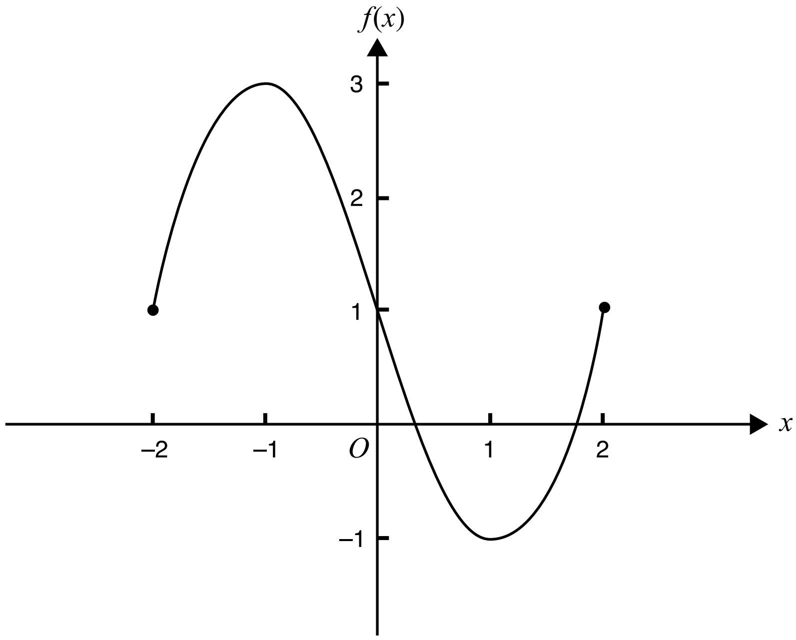
C. π

D. $\frac{7\pi}{4}$

E. 2π

Question 8

The graph of the function f: $[-2, 2] \rightarrow R$, $f(x) = P \sin(k\pi x) + Q$ is shown below.



The values of P, k and Q respectively are

	P	k	Q
A.	2	0.5	1
B.	2	2	1
C.	–2	2	–1
D.	–2	0.5	1
E.	–2	0.5	–1

Question 9

If $y = 2\tan(2x)$, then $\frac{dy}{dx}$ is equal to

A. $\dfrac{1}{\cos^2(2x)}$

B. $\dfrac{2}{\cos^2(2x)}$

C. $\dfrac{4}{\cos^2(x)}$

D. $\dfrac{4}{\cos^2(2x)}$

E. $\dfrac{4}{\cos^2(4x)}$

Question 10

If u is a function of x and if $y = u(x)\ \log_e(x)$, then the rate of change of y with respect to x when $x = 2$ is equal to

A. $u(2)\ \log_e(2)$

B. $u'(2)\ \log_e(2) - \dfrac{u(2)}{2}$

C. $u'(2)\ \log_e(2) + \dfrac{u(2)}{2}$

D. $\dfrac{u'(2)}{2}$

E. $u'(2)\ \log_e(2)$

Question 11

Let $f: R \to R$ be a function such that $f'(-1) = 0$

and $f'(x) > 0$ when $x < -1$

and $f'(x) > 0$ when $x > -1$.

At $x = -1$, the graph of f has a

A. local minimum.

B. local maximum.

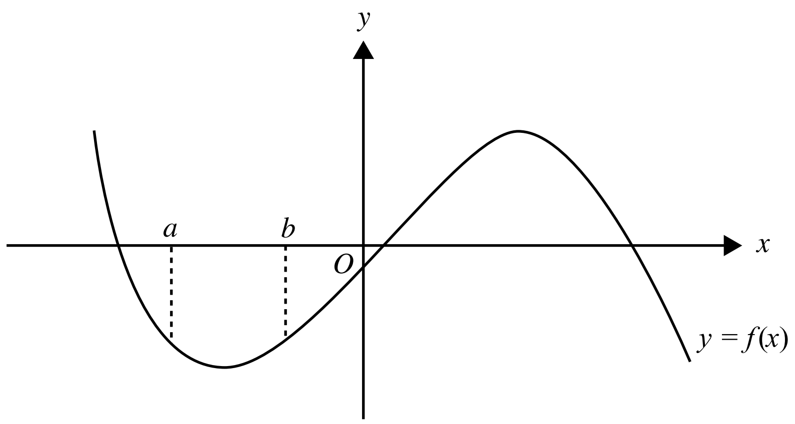
C. stationary point of inflection.

D. point of discontinuity.

E. gradient of -1.

Question 12

Part of the graph of the function f is shown below.



Let g be a function such that $g'(x) = f(x)$.

On the interval (a, b), the graph of g will have a

A. negative gradient.

B. positive gradient.

C. local minimum value.

D. local maximum value.

E. zero gradient.

Question 13

If $f'(x) = 4e^{2x}$, then $f(x)$ could be equal to

A. $2e^{2x} + 3$

B. $4e^{2x} + 5$

C. $8e^{2x} + 2$

D. $4\log_e(2x) - 4$

E. $\log_e(8x) + 5$

Question 14

If $\int_1^4 f(x)dx = 2$, then $\int_1^4 (2f(x) + 3)dx$ is equal to

A. 2

B. 4

C. 7

D. 10

E. 13

Question 15

Let g be any continuous function on the interval $[0, 5]$, and f a function such that $f'(x) = g(x)$, for all $x \in [0, 5]$.

Then $\int_0^5 g(x)dx$ is equal to

A. $g'(5) - g'(0)$

B. $f(5)$

C. $f'(5) - f'(0)$

D. $g(5) - g(0)$

E. $f(5) - f(0)$

Question 16

The total area of the regions enclosed by the graph of the function with equation $y = \sin(2x)$, and the x-axis between $x = 0$ and $x = 2\pi$, is equal to

A. 1

B. 2

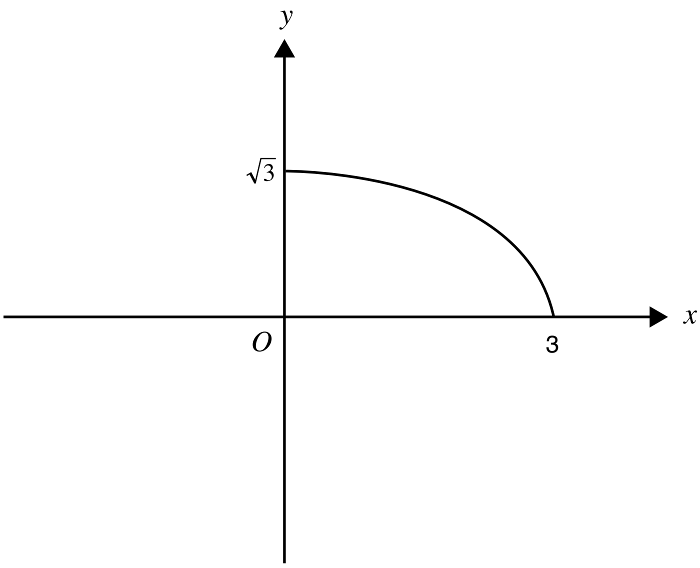
C. 4

D. 8

E. 16

Question 17

The diagram below shows part of the graph with equation $y = \sqrt{3 - x}$.



Using the left-rectangle approximation with rectangles of width 1 unit, an approximation to $\int_0^3 \sqrt{3-x}\ dx$ is

A. $\frac{3\sqrt{3}}{2}$

B. $\sqrt{2} + \sqrt{3}$

C. π

D. $2\sqrt{3}$

E. $1 + \sqrt{2} + \sqrt{3}$

Question 18

Given that $f(x) = 2\left[(x+3)^2 - 4\right]$, the coordinates of the turning point of the graph of f are

A. (–3, –8)
B. (–3, –4)
C. (3, –8)
D. (3, –4)
E. (–3, 8)

Question 19

The first seven lines of Pascal's Triangle are given below.

						1						
					1		1					
				1		2		1				
			1		3		3		1			
		1		4		6		4		1		
	1		5		10		10		5		1	
1		6		15		20		15		6		1

The coefficient of x^2 in the expansion of $(2x - 3)^5$ is equal to

A. 1080
B. 540
C. –10
D. –540
E. –1080

Question 20

Let $p(x) = (x^2 + a)(x + b)(x - c)$, where a, b and c are three distinct positive real numbers.
The number of real solutions to the equation $p(x) = 0$ is exactly

A. 0
B. 1
C. 2
D. 3
E. 4

Question 21

Let $f(x) = \dfrac{2}{x-3} + 1$.

The equations of the asymptotes of the graph of the inverse function f^{-1} are

A. $x = 1$ and $y = 3$
B. $x = 1$ and $y = -3$
C. $x = 3$ and $y = 1$
D. $x = -3$ and $y = -1$
E. $x = -1$ and $y = -3$

Question 22

If $2\log_e(x) - \log_e(x+2) = 1 + \log_e(y)$, then y is equal to

A. $\dfrac{x^2}{10(x+2)}$

B. $\dfrac{2x}{x+2} - 1$

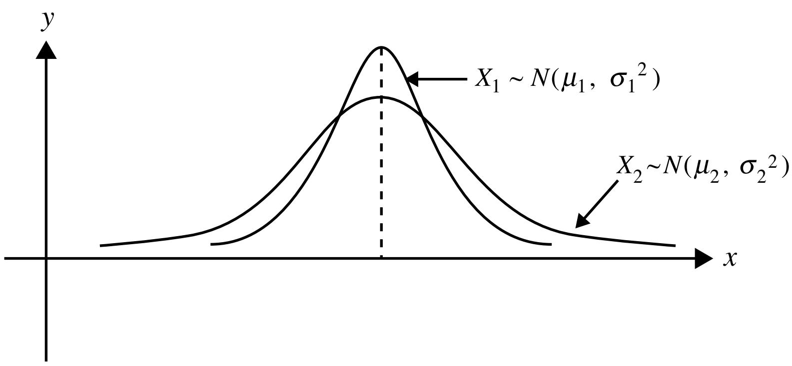
C. $\dfrac{x^2}{x+2}$

D. $\dfrac{2x}{x+2}$

E. $\dfrac{x^2}{e(x+2)}$

Question 23

The diagram below shows the graphs of two normal distribution curves with means μ_1 and μ_2 and standard deviations σ_1 and σ_2 respectively.



Which one of the following statements is true?

A. $\mu_1 > \mu_2$ and $\sigma_1 = \sigma_2$

B. $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$

C. $\mu_1 = \mu_2$ and $\sigma_1 > \sigma_2$

D. $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$

E. $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$

Question 24

Which of the following tables could represent the probability distribution of a discrete random variable?

I

v	2	3	4	5
$\Pr(V = v)$	0.1	0.2	0.4	0.5

II

w	–2	–1	0	1
$\Pr(W = w)$	0.2	0.3	0.3	0.2

III

x	10	20	30	40
$\Pr(X = x)$	0.4	0.3	0.2	0.1

IV

y	0	1	2	3
$\Pr(Y = y)$	–0.1	0.4	0.5	0.2

V

z	1	2	3	4
$\Pr(Z = z)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$

A. I and III

B. I and IV

C. II and IV

D. II, III and V

E. I, II and V

Question 25

The random variable X has the following probability distribution, where $0 < p < 1$.

x	0	1
$\Pr(X = x)$	p	$1 - p$

The standard deviation of X is

A. $1 - p$

B. $\sqrt{p(1-p)}$

C. $p(1 - p)$

D. $\sqrt{p^2 + (1-p)^2}$

E. $p^2 + (1 - p)^2$

Question 26

60 per cent of all tickets sold at a racecourse are Adult tickets and the remaining 40 per cent are Concession tickets. A random sample of 20 tickets is taken.

The probability that this sample contains exactly twelve Adult tickets is equal to

A. $\dfrac{{}^{60}C_{12} \times {}^{40}C_8}{{}^{100}C_{20}}$

B. ${}^{20}C_{12}\,(0.4)^8 \times (0.6)^{12}$

C. ${}^{20}C_{12}\,(0.4)^{12} \times (0.6)^8$

D. $(0.4)^8 \times (0.6)^{12}$

E. $(0.4)^{12} \times (0.6)^8$

Question 27

A bag contains 12 bread rolls, of which 8 are white and the remainder multigrain. Tony takes 2 bread rolls at random from the bag to eat.

The probability that at least one is a multigrain roll is

A. $1 - \dfrac{2^{12}}{3^{12}}$

B. $1 - \dfrac{{}^{8}C_2}{{}^{12}C_2}$

C. $1 - \dfrac{2^{12}}{3^{12}} - 12 \times \dfrac{1}{3} \times \dfrac{2^{11}}{3^{11}}$

D. $1 - \dfrac{{}^{8}C_2}{{}^{12}C_2} - \dfrac{{}^{8}C_1 \times {}^{4}C_1}{{}^{12}C_2}$

E. $\dfrac{{}^{8}C_1 \times {}^{4}C_1}{{}^{12}C_2}$

END OF PART I MULTIPLE-CHOICE QUESTION BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2003

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures										Letter
Words										

MATHEMATICAL METHODS

Written examination 1

(Facts, skills and applications)

Friday 7 November 2003

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)

PART II
QUESTION AND ANSWER BOOK

This examination has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of a separate question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
6	6	23

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphic calculator (memory must be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 7 pages.

Instructions
- Detach the formula sheet from the centre of the Part I book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions (Part I) inside the front cover of this question and answer book (Part II).

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

This page is blank

Instructions for Part II

- Answer **all** questions in the spaces provided.
- A decimal approximation will not be accepted if an **exact** answer is required to a question.
- Where an **exact** answer is required to a question, appropriate working must be shown.
- In questions where more than 1 mark is available, appropriate working must be shown.
- Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or antiderivative.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Use the remainder theorem to determine if $2x^4 - 3x^3 + 7x + 11$ is exactly divisible by $(x + 1)$.

2 marks

Question 2

a. Find the coordinates of the point P on the curve with equation $y = x^2 - 2x - 1$ at which the tangent is parallel to the line $y = 3x - 5$.

b. Find the equation of the normal to the curve at the point P.

3 + 2 = 5 marks

Question 3

Find the **exact** solutions of the equation $\sin(2\pi x) = -\sqrt{3}\cos(2\pi x),\ 0 \le x \le 1$.

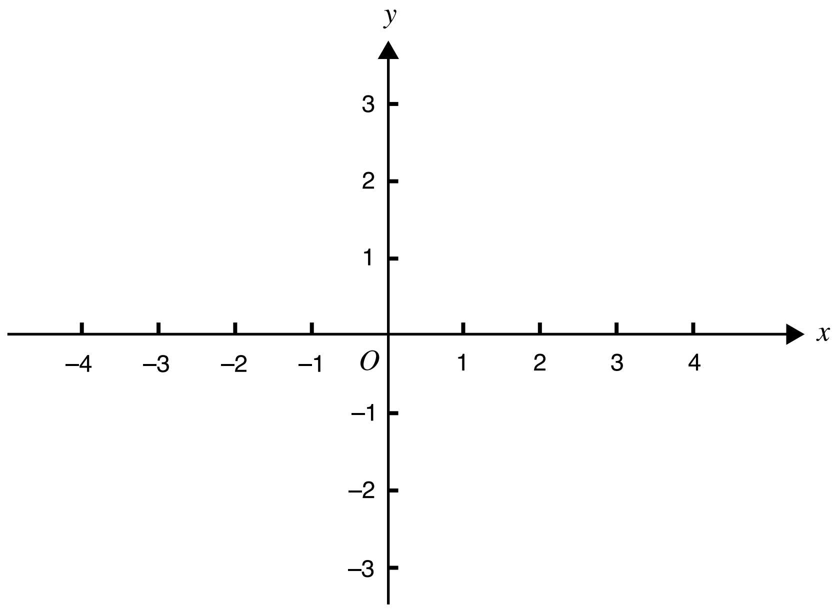
2 marks

Question 4

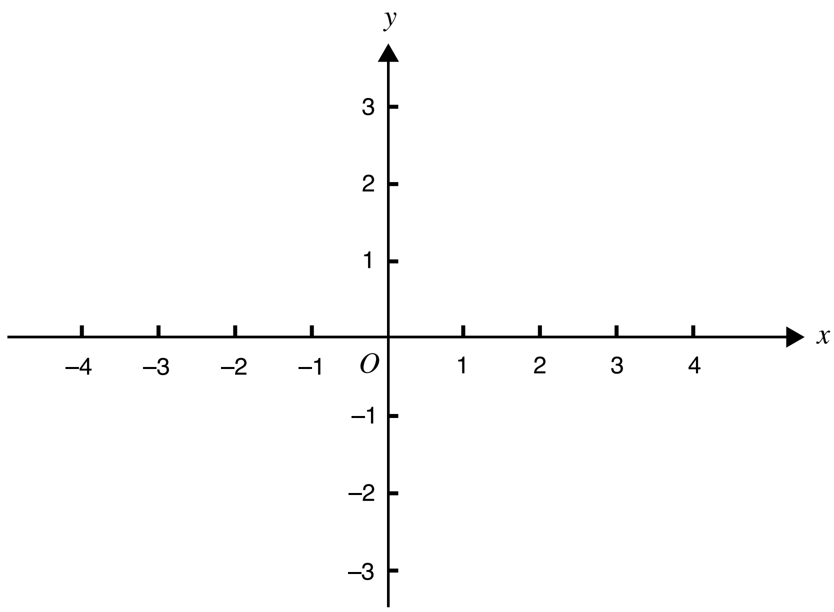
a. The graph of the function f with rule $f(x) = 2\log_e(x + 3) + 1$ intersects the axes at the points $(a, 0)$ and $(0, b)$.

Find the **exact** values of a and b.

b. Hence sketch the graph of the function with rule $f(x) = 2\log_e(x+3) + 1$ on the axes below. Label any asymptote with its equation.



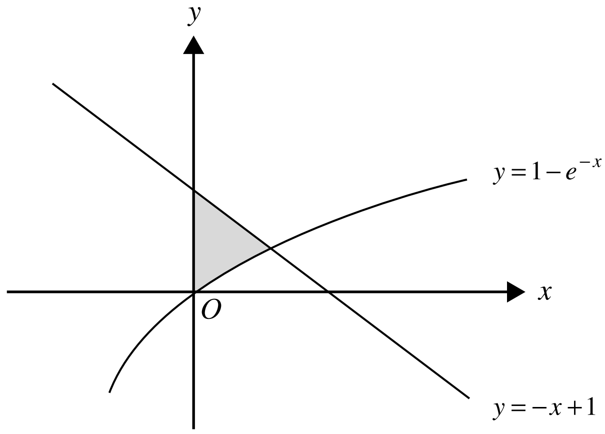
c. On the axes below, sketch the graph of f', the derivative of f. Label any point(s) of intersection with the axes with its coordinates. Label any asymptote with its equation.



2 + 2 + 2 = 6 marks

Question 5

The graphs with equations $y = -x + 1$ and $y = 1 - e^{-x}$ are shown below.



a. Find the solution to the equation $-x + 1 = 1 - e^{-x}$, correct to three decimal places.

b. Use calculus to find the area of the shaded region, correct to two decimal places.

1 + 3 = 4 marks

Question 6

The height of plants sold by a garden nursery supplier are normally distributed with a mean of 20 cm and a standard deviation of 5 cm.

a. Complete the following table by finding the proportions for each of the three plant sizes, correct to four decimal places.

Description of plants	**Size of plants (cm)**	**Cost ($)**	**Proportion of plants**
Small	less than 10	1.50	
Medium	10 – 30	2.50	
Large	above 30	4.00	

b. Find the expected cost, to the nearest dollar, for 100 plants chosen at random from the garden nursery supplier.

2 + 2 = 4 marks

END OF PART II QUESTION AND ANSWER BOOK