Question 1

The rate of increase of the population of a certain species of insect, *Droughtus murray*, found in a district in northern Victoria is given by the rule $\frac{dA}{dt} = 8000 + e^{0.3(t+2)}$, where t is the time in days after midnight on December 31 2001.

30 days has September, April, June and November. All the rest have 31, except February which had 28 days in 2002.

a What is the rate of increase (number of insects per day) of *Droughtus murray* at midnight on January 1 2002?

1 mark

b On what date will the rate of increase first exceed 30 000 insects per day?

2 marks

c Find $\int_{31}^{59} \frac{dA}{dt}\, dt$, correct to the nearest integer.

1 mark

d What does the answer to part **c** represent?

1 mark

e Find A, as a function of t if the population of insects was 50 000 when $t = 3$ days.

3 marks

Due to the cold weather and drought, *Droughtus murray* started to decline in numbers in April 2002. The model given above only applied until the end of March. The population of insects for April 1 until December 31 2002 can be represented by the quadratic model $A_2 = a(t_2 - b)^2 + c$, where t_2 is the time in days after midnight on March 31 2002.

f If the population reached a minimum of 8 000 on July 21 2002, find a, b and c correct to the nearest integer.

3 marks

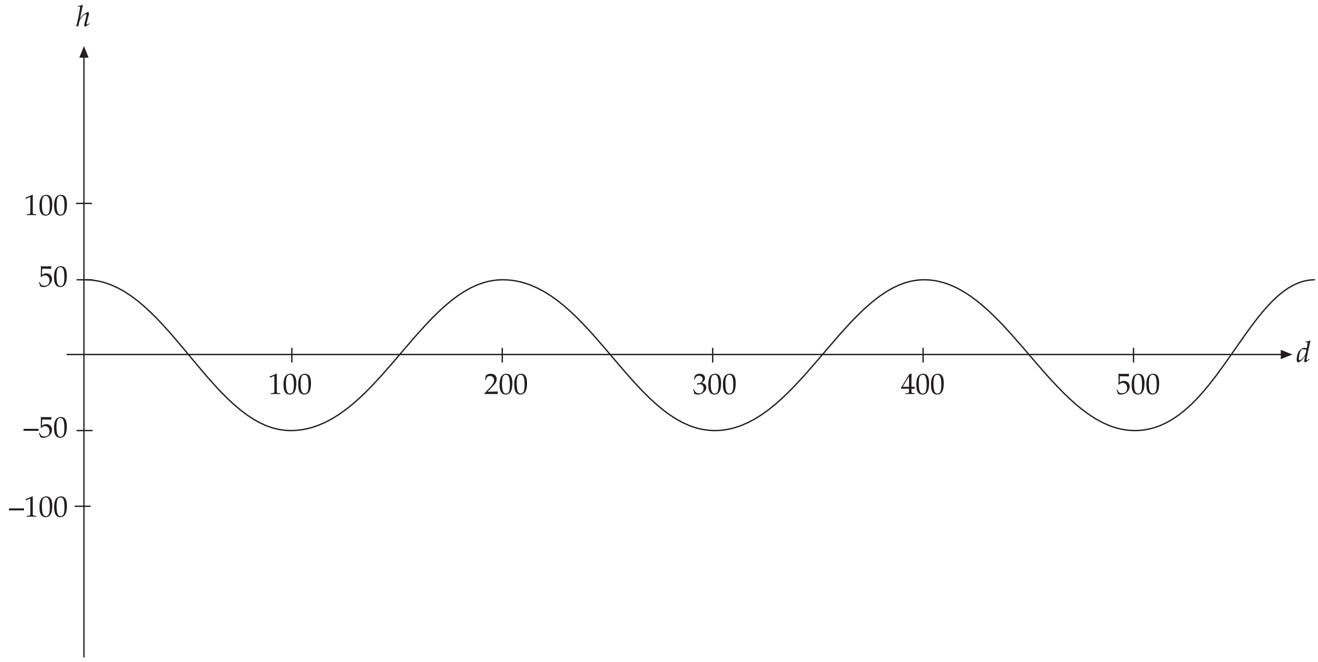
g On what dates in 2002 was the population of *Droughtus murray* half its maximum value?

3 marks

Total 14 marks

Question 2

A pavement assessment engineer measures the corrugations at a point along the Old Telegraph Road on Cape York. She finds that a cross-section can be modelled by a **sine** curve as shown below, where h mm represents the height of the surface above the original road level and d mm represents the horizontal distance along the road.



a What is the period of this curve?

1 mark

b What is the amplitude of the curve?

1 mark

c What is the horizontal translation?

1 mark

d Given the function is of the form $h = a \sin(n\ (d - b))$, write down a rule that models the corrugations on the road.

1 mark

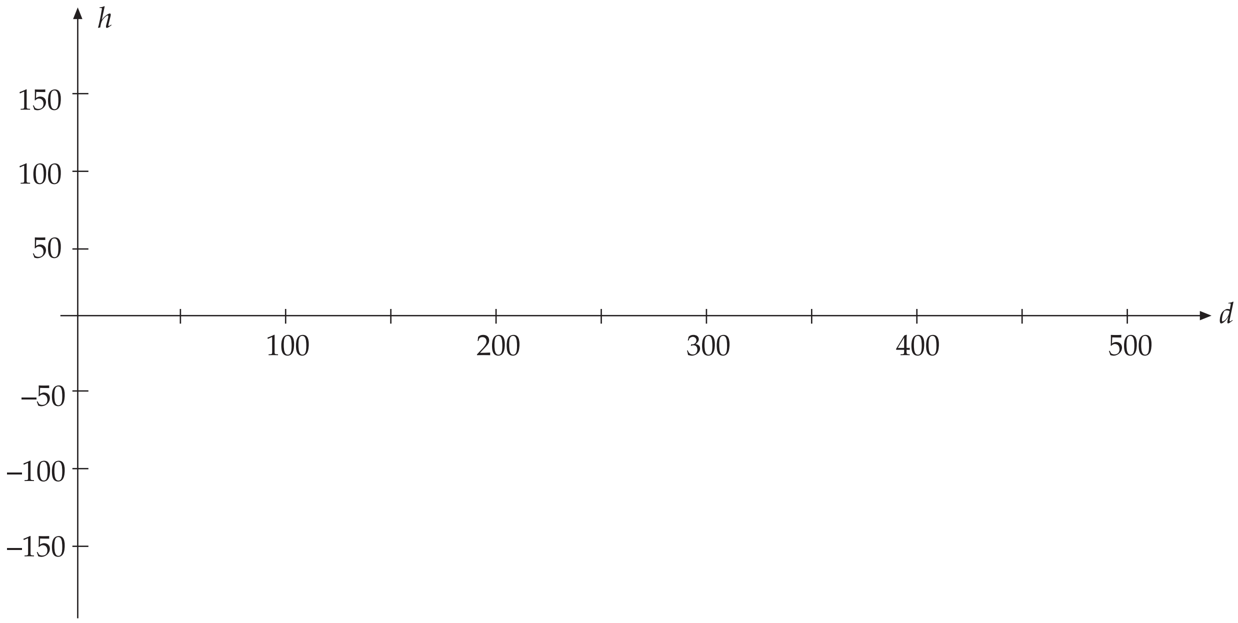
e Assuming that this pattern continues for 5 km how many troughs would a vehicle have to negotiate?

1 mark

A little further on, the track starts to bend, and as the vehicles brake and accelerate the pattern of corrugations changes. The cross-section of the road can now be modelled by the function

$h = 75\sin(\frac{\pi}{120}d) - 40\cos(\frac{\pi}{40}d)$

f Using a graphics calculator, sketch this function on the axes provided for a distance of 500 metres.



3 marks

g What is the period of this new function?

1 mark

h What are the absolute maximum and minimum values of this function? (Correct to 2 decimal places).

2 marks

i Give the coordinates, correct to two decimal places, of all local maximum points during the first cycle?

1 mark

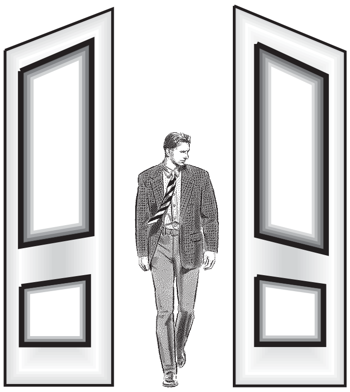
j Find the cross-sectional area of earth, correct to two decimal places, which has been displaced in the first 50 mm.

2 marks

Total 14 marks

Question 3

A competition was being run by a Victorian University for the best pattern design for the front door of their new Mathematics building. No dimensions were given.



The winning entrant created two designs using

$g: R \setminus \{C\} \to R$, where $g(x) = A + \dfrac{B}{x - C}$ and $g^{-1}(x) = x$,

where A, B and C are real constants.

The first pattern was created using $f: R \setminus \{-1\} \to R$, where $f(x) = 2 + \dfrac{3}{x+1}$. This pattern is to be used for the top panel of the door.

a Find the rule for $f^{-1}(x)$.

1 mark

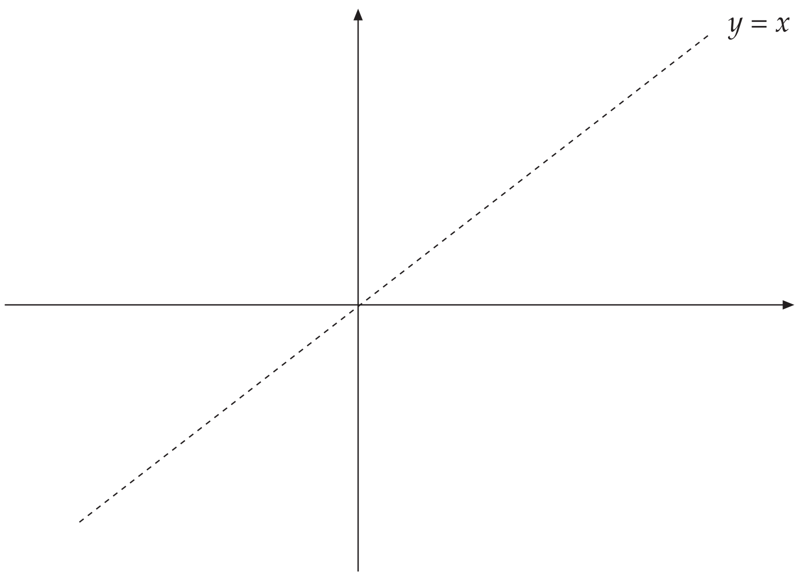
b Describe the transformations, in terms of translations, that have occurred to $f(x)$ to get $f^{-1}(x)$.

2 marks

c Use algebra to show that the coordinates of the points of intersection of $f(x)$ and $f^{-1}(x)$ are $\left(\frac{1+\sqrt{21}}{2}, \frac{1+\sqrt{21}}{2}\right)$ and $\left(\frac{1-\sqrt{21}}{2}, \frac{1-\sqrt{21}}{2}\right)$.

2 marks

d Draw the pattern created by $f(x)$ and $f^{-1}(x)$. Label all intercepts and asymptotes.



4 marks

e A plaque is to be placed on the rectangle formed by the asymptotes. What is the area of the plaque in units squared?

1 mark

f The second pattern was created for the bottom panel of the door using $g(x)$ and $g^{-1}(x)$, where A > 0, B < 0 and C < 0. A stained glass window is to be inserted in the area bounded by the two curves. Under some conditions a bounded area does not exist.

i Show that the x-coordinates of the two points of intersection of $g(x)$ and $g^{-1}(x)$ are

$$x = \frac{A + C \pm \sqrt{A^2 - 2AC + C^2 + 4B}}{2}$$

2 marks

ii Hence state the conditions for which the stained glass window would exist.

1 mark

iii Write down the rule, in terms of A, B and C used to determine the area bounded by the two curves $g(x)$ and $g^{-1}(x)$.

1 mark

Question 4

A ferry charges a fee of \$5 per vehicle (including driver) plus \$2 per passenger. The number of passengers in each vehicle varies in accordance with the following probability table.
All probability answers should be given correct to **three decimal** places.

Passengers (x)	0	1	2	3	4	5
$\Pr(X = x)$	0.18	0.33	0.17	0.22	0.08	0.02

a Find the **expected** fee per vehicle.

2 marks

b On a particular trip the ferry carries 18 vehicles. Find the probability that 4 or more have no passengers.

2 marks

c On the next trip the ferry carries 21 vehicles of which 7 are single cab utilities with no passengers. Find the probability that of the first 10 vehicles onto the ferry 5 are single cab utilities.

2 marks

d The length of the vehicles on the ferry is normally distributed with 90% of vehicles being less than 5.1 metres in length and 95% being greater than 3.6 metres.

By setting up two simultaneous equations, show that the mean length of the vehicles is 4.443m and that the standard deviation is 0.513m correct to three decimal places.

4 marks

e Find the probability that **four** of the first five vehicles in the line waiting for the next ferry are greater than 4.8 metres in length, given that all five are greater than 4.5 metres.

3 marks

Total 13 marks

