Multiple Choice Questions

Question 1

The function $f(x) = 3\cos(4(x - \pi))$ has a period and amplitude respectively of

A $3, \frac{\pi}{4}$

B $\frac{\pi}{4}, 3$

C $3, \frac{\pi}{2}$

D $\frac{\pi}{2}, 3$

E $4\pi, 3$

Question 2

The solution(s) of $\sin(2x) = \cos(2 - x)$, $x \in [0, \pi]$ is/are closest to

A 1.19, 3.28

B 1.19

C 1.19, 2.78

D 1.19, 3.28, 5.38, 5.83

E 2.78

Question 3

$\int_0^{\frac{\pi}{3}} (a\sin(\theta) + b\cos(\theta))d\theta$ equals

A $\frac{1}{2}(3a + \sqrt{3}b)$

B $\frac{1}{2}(3a - \sqrt{3}b)$

C $\frac{1}{2}(a + \sqrt{3}b)$

D $\frac{1}{2}(a - \sqrt{3}b)$

E $\frac{1}{2}(\sqrt{3}b - a)$

Question 4

The value of $\cos(\pi + x) + \sin\left(\frac{\pi}{2} - x\right)$ when $\cos(x) = 0.7$ is

A $\pi + 0.7$

B $\pi - 0.7$

C -1.4

D 0

E 1.4

Question 5

The coefficient of the term containing x^6 in the expansion of $(a - x^3)^5$ is

A $-a^3$

B a^3

C $10a^2$

D $-10a^3$

E $10a^3$

Question 6

If $\log_3(x - 2) + \log_3(x) - 1 = 0$ then x equals

A -1

B 1

C 3

D -1 or 3

E -3 or 1

Question 7

If $2e^x - 1 = \frac{1}{e^x}$ then x equals

A $-\frac{1}{2}$ or 1

B 0

C 1

D $\log_e \frac{1}{2}$

E $\log_e \frac{1}{2}$ or 1

Question 8

$2^{2x} + 2^x + b = 0$ has only one real solution if

A $b > 0$

B $b < 0$

C $b \leq 0$

D $b = \frac{1}{4}$ only

E $b = -\frac{1}{4}$ only

Question 9

The range of f: $(3, \infty) \rightarrow R$, where $f(x) = 2\sqrt{5x - 3} + 6$ is

A $(6, \infty)$

B $[\frac{3}{5}, \infty)$

C R

D $(4\sqrt{3} + 6, \infty)$

E $[4\sqrt{3} + 6, \infty)$

Question 10

The graph of $y = x^3e^x + 1$ is translated –1 unit parallel to the x-axis and then dilated by a factor of 2 from the x-axis. The equation of the new graph is

A $y = 2(x + 1)^3e^{(x + 1)} + 2$

B $y = 2(x + 1)^3e^{(x + 1)} + 1$

C $y = (\frac{1}{2}x + 1)^3e^{(\frac{1}{2}x + 1)} + 1$

D $y = 2(x - 1)^3e^{(x - 1)} + 2$

E $y = \frac{1}{2}(x + 1)^3e^{(x + 1)} + \frac{1}{2}$

Question 11

If f: $R\backslash\{2\} \rightarrow R$, $f(x) = \frac{5}{2 - x} + 3$ then the largest possible value of a where g: $(-\infty, a) \rightarrow R$, $g(x) = f(2x)$ is a one to one function is

A –2

B –1

C 1

D 2

E 3

Question 12

The function f defined by $f : R \to R$, where $f(x) = -e^{2(x-1)} + 4$ will have an inverse function with

A two asymptotes.

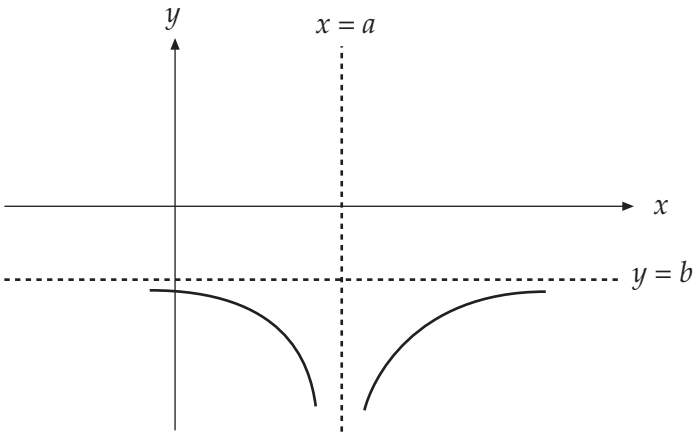
B only one asymptote, $y = 4$ and only one intercept, $(0,\ 4 - \frac{1}{e^2})$.

C only one asymptote, $x = 4$ and only one intercept, $(4 - \frac{1}{e^2}, 0)$.

D only one asymptote, $y = 4$ and intercepts at $(0,\ 4 - \frac{1}{e^2})$ and $(\frac{1}{2}\log_e(4) + 1, 0)$.

E only one asymptote, $x = 4$ and intercepts at $(4 - \frac{1}{e^2}, 0)$ and $(0, \frac{1}{2}\log_e(4) + 1)$.

Question 13



The rule for the above graph could be

A $y = \dfrac{-1}{(x-a)^2} + b$

B $y = \dfrac{-1}{(x-a)^2} - b$

C $y = \dfrac{1}{(x-a)^2} + b$

D $y = \dfrac{-1}{(x+a)^2} + b$

E $y = \dfrac{-1}{(x+a)^2} - b$

Question 14

The equation of the graph of the quartic function which passes through the points with coordinates (–1, 0), (0, 1), (1, 0), (2, 9) and (–2, 9) is

A $y = x^4 - 2x^3 + 1$

B $y = (x - 1)^2(x + 1)^2$

C $y = -x^4 + x^2 + 2x - 2$

D $y = -(x + 1)(x - 1)^3$

E $y = -(x - 1)(x + 1)^3$

Question 15

The **largest** instantaneous rate of change, correct to three decimal places, of the function $f(x) = (2x + 1)10^x$ with respect to x where $2x + 1 = 10^x$ is

A 4.303

B 4.302

C 2.719

D 0

E – 0.125

Question 16

If $h(x)$ and $g(x)$ are the tangents to the curve $f(x) = x^2 + 4x - 5$ where $f(x) = 0$, then $h(x) = g(x)$ at

A (–2, –18)

B (–2, –9)

C (–5, 1)

D (–5, –6)

E (–6, 6)

Question 17

If $g(x) = \dfrac{\log_e(\cos x)}{\tan x}$ then $g'(x)$ equals

A $\dfrac{\tan^2(x) - \sec^2(x)\log_e(\cos x)}{\tan^2(x)}$

B $\dfrac{\tan^2(x) + \sec^2(x)\log_e(\cos x)}{\tan^2(x)}$

C $\dfrac{-\tan^2(x) + \sec^2(x)\log_e(\cos x)}{\tan^2(x)}$

D $-1 - \dfrac{\log_e(\cos x)}{\sin^2(x)}$

E $-1 - \dfrac{\log_e(\cos x)}{\tan^2(x)}$

Question 18

An approximate value for $\dfrac{1}{\sqrt{99.96}}$ is

A $\dfrac{1}{10} + 0.04 \times \dfrac{-1}{2000}$

B $\dfrac{1}{10} - 0.04 \times \dfrac{-1}{2000}$

C $\dfrac{1}{10} - 0.04 \times \dfrac{-1}{20}$

D $\dfrac{1}{10} + 0.04 \times \dfrac{-1}{20}$

E $\dfrac{1}{10} - 0.4 \times \dfrac{-1}{2000}$

Question 19

If $f(x) = (x-3)^2 x$ and h is a function such that $h'(x) = f(x)$, then the largest subset of R for which the gradient of $h(x)$ is negative is

A $(-\infty, 0) \cup (3, \infty)$

B $(-\infty, 0)$

C $(1, 3)$

D $(-\infty, 1) \cup (3, \infty)$

E $(0, 3) \cup (3, \infty)$

Question 20

Which of the following rules for $f(x)$ will not give an overestimate of the area bounded by the graph of $f(x)$ and the x-axis, if the **right** rectangle rule is used between $x = 0$ and $x = 3$ using strips of width 0.5?

A $f(x) = 10^x$

B $f(x) = e^x$

C $f(x) = \sqrt{x}$

D $f(x) = (x - 3)(3 - x)$

E $f(x) = -x^2(x + 1)(x + 2)$

Question 21

An antiderivative of $\dfrac{4x^5}{x^6 + 7}$ could be

A $\log_e(x^6 + 7)$

B $4\log_e(x^6 + 7)$

C $\dfrac{2}{3}\log_e(x^6)$

D $\dfrac{3}{2}\log_e(x^6 + 7)$

E $\dfrac{2}{3}\log_e(x^6 + 7) + 3$

Question 22

If $\displaystyle\int_1^{\frac{3}{2}} a(2x - 3)^4\,dx = 10$, where a is a constant, then a equals

A –100

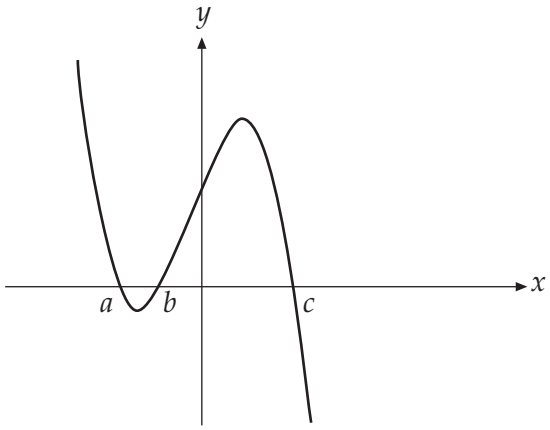
B –50

C 1

D 50

E 100

Question 23



The area enclosed by the graph $y = f(x)$ shown above and the x-axis can be determined by evaluating

A $\int_a^b f(x)dx + \int_b^c f(x)dx$

B $\int_a^b f(x)dx + \int_c^b f(x)dx$

C $\int_b^a f(x)dx - \int_b^c f(x)dx$

D $\int_b^a f(x)dx + \int_b^c f(x)dx$

E $\int_0^a f(x)dx + \int_0^b f(x)dx - \int_b^c f(x)dx$

Question 24

X is a normally distributed variable with $\mu = 3$ and $\sigma^2 = 2.56$.
If $\Pr(X < k) = 0.734$, then k equals

A 0.2

B 1.4

C 2.0

D 4.0

E 4.6

Question 25

A fire alarm has a probability of failure of 0.05. In an apartment block where there are 10 such alarms. The probability that at least one fails is given by:

A $1 - (0.95)^{10}$

B $1 - {}^{10}C_1\,(0.05)^1(0.95)^9$

C $1 - [{}^{10}C_1\,(0.05)^1(0.95)^9 + (0.95)^{10}]$

D $1 - [{}^{10}C_9\,(0.05)^9(0.95)^1 + (0.05)^{10}]$

E $1 - (0.05)^{10}$

Question 26

In a container of tulip bulbs there are 5 that have red flowers, 7 yellow and 3 orange. A group of three bulbs is chosen at random. The probability that exactly one of the flowers is orange is:

A $\frac{1}{5}$

B $\frac{2}{5 \times 14 \times 13}$

C $\frac{2 \times 3}{5 \times 14 \times 13}$

D $\frac{12 \times 11 \times 3}{5 \times 14 \times 13}$

E $\frac{12 \times 11}{5 \times 14 \times 13}$

Question 27

A normal distribution has $\mu = 5.6$ and $\sigma = 6.5$. If the variable x, has a value of 1.2, the value of the normal variable z, is closest to:

A 0.338

B – 0.786

C – 0.677

D 0.677

E – 4.40

Part II: Short Answer

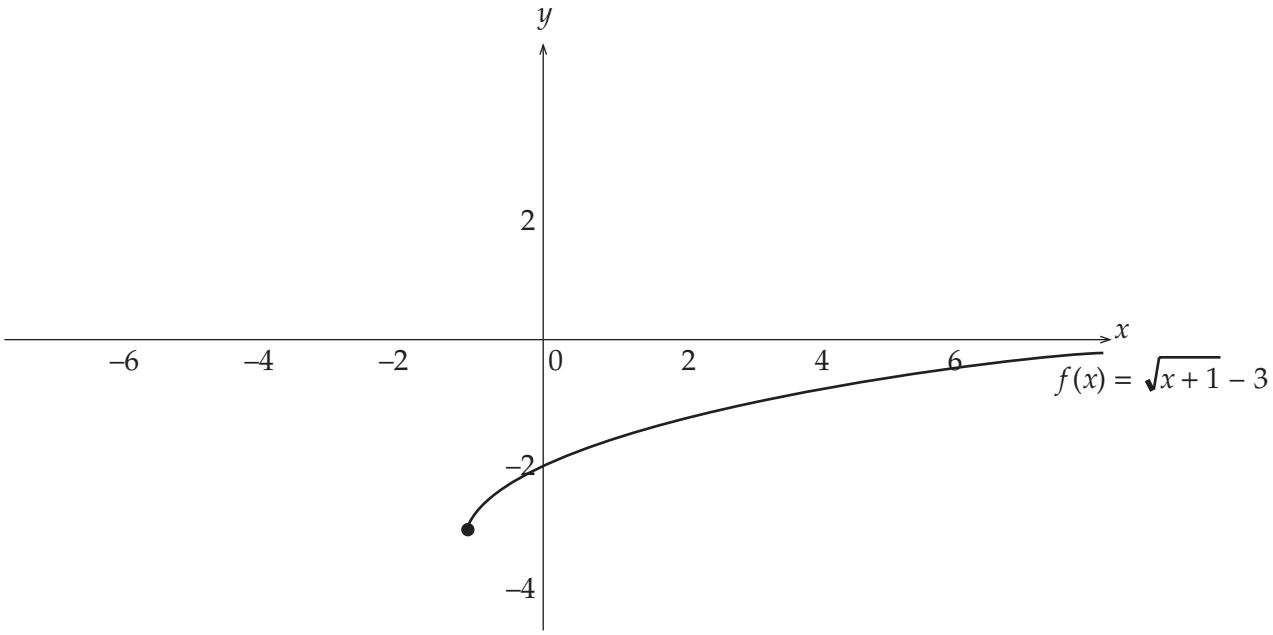
Question 1

Find exact solutions of $1 + \sin(x) = 2\cos^2(x)$ where $x \in [0, 2\pi]$.

4 marks

Question 2

The graph of f: $[-1, \infty) \rightarrow R$, $f(x) = \sqrt{x+1} - 3$ is shown below.



a Sketch g: $(-\infty, 3) \rightarrow R$, $g(x) = 3e^{(x-2)} - 3$ on the above axes, labelling all relevant features of the graph.

b Find the coordinates of the points where $f(x) = g(x)$, correct to two decimal places.

c Find the area bounded by the two curves, correct to two decimal places.

3 + 2 + 2 = 7 marks

Question 3

Let $f(x) = (2x - 1)^3(x + 2)$.

a Find $f'(x)$.

b Hence show that the stationary points occur at $x = \frac{1}{2}$ and $x = -\frac{11}{8}$.

c Find the average rate of change of $f(x)$ between the two stationary points.

1+ 2 + 3 = 6 marks

Question 4

a If $y = x^2 \log_e x$ find $\frac{dy}{dx}$.

b Hence find $\int 2x \log_e(x) dx$.

1 + 2 = 3 marks

Question 5

Two chocolate companies make 750 gram blocks of milk chocolate. Company X's blocks have a mean of 760 and a standard deviation of 7, while Company Y's blocks have a mean of 768 and a standard deviation of 14.

Determine which company has the higher probability of producing a block that weighs at least 750 grams?

3 marks