GENERAL COMMENTS

The number of students presenting for Mathematical Methods (CAS) Examination 2 in 2002 was 78. The range of marks was from 7 to 55, out of a total of 55. Student responses showed that the paper was accessible and provided opportunities for students to demonstrate what they knew. There were excellent papers presented by several students. One student achieved a perfect score and 13% of students achieved a score of over 80%. The median and mean mark for the paper was 32; 58% of the students scored over half of the marks for the paper and 80% of the students scored over 40% of the available marks. Only one student scored less than 20% of the available marks.

Generally the symbolic facility of the CAS calculators was used well. This was shown in particular in Question 1ci where many students achieved full marks and Questions 4ei and Question 3a. Students sometimes did not have their calculator set in exact mode and achieved answers with numerical approximations. For example in Question 1aiii students were required to find the rule for the inverse of the function with rule $f(x) = 1.1 - 0.5\log_e(x)$. There are several equivalent acceptable forms for the exact answer including $f^{-1}(x) = e^{2.2-2x}$. Several students gave the answer $f^{-1}(x) = 90.2501e^{-2x}$, which was not the form expected and students should be aware of this aspect of the operation of CAS calculators when entering coefficients/constants in decimal form. Students used the solve facility of their calculators to obtain this result.

Some calculators gave unusual forms for expressions. For example in Question 4, $e^{\frac{t}{25}}$ was sometimes written with superscript $\sqrt[25]{t}$. Students should be aware of these properties of their CAS and know how to relate these to more common forms.

Graph sketching could have been done better with few students achieving full marks on Question 1ai. In some cases this could be attributed to poor use of the graphing facility of the calculator and that the graphics window was often not set well. Calculators do not deal well with asymptotes and students were expected to be able to sketch the graph with the asymptotic behaviour of the graph demonstrated. The use of the trace or similar function to find intersections or intercepts is neither accurate nor quick and this was exemplified in many answers to Question 4.

Computations and manipulations for the probability in Question 2 can be done quickly using a CAS calculator without any working being shown. If the question is worth more than 1 mark, students risk losing all the available marks if the answer is all that is written down, and it is incorrect. When a question is worth more than 1 mark, it is usual to award a 'method' mark if the student has indicated something of the way they have arrived at the answer. It makes good sense to write down something such as a specific expression to be evaluated, or some workings that indicate recognition of key elements of the question, which may gain this mark even if the subsequent answer is be incorrect.

For example, in Question 2b, the correct answer, 0.023, with working gained 2 marks. An answer of 0.024 with no working scored 0 marks. One mark out of the two was awarded if the answer was incorrect but working, such as one or more of the following, was shown:

- normal, $X < 16$
- $Z < -2$
- a normal distribution diagram with the appropriate mean and value marked, and the correct area shaded.

In this examination many students lost marks because they:

- did not answer the question
- gave decimal answers when an exact answer was required
- gave the wrong number of decimal places
- did not pay sufficient attention to detail in sketching graphs.

When students present working and develop solutions, they should use conventional mathematical expressions, symbols, notation and terminology, and this was generally the case.

SPECIFIC INFORMATION

Question	Marks	%	Response
Question 1	This was not done well. Students often did not show the asymptotic behaviour clearly and the endpoint was often omitted. Most students did know the condition for existence of an inverse in Question 1aii and could find the rule for the inverse in Question 1aiii. This was not the case for finding the domain of the inverse and the limited success in this question connected with the poor graphing in Question 1ai. A lack		

	of understanding of the graph made it difficult to determine the range of the original function and hence the domain of the inverse in Question 1aiv. This is not related to the use of a CAS calculator but to student understanding of the important concepts involved in these questions. On the whole in Question 1b students coped with the equations and there was evidence that they used their calculators well. This is also true of Question 1ci and 1cii requiring an understanding of the domain and range of the original function.	
	1ai 0/3 17 1/3 27 2/3 33 3/3 23 (Average mark 1.61)	Asymptote x = 0, endpoint $\left(5, 1.1 - \log_e(5)\right)$
	1aii 0/1 19 1/1 81 (Average mark 0.81)	Function is one to one
	1aiii 0/2 5 1/2 12 2/2 83 (Average mark 1.77)	$y = e^{2.2-2x}$
	1aiv 0/1 78 1/1 22 (Average mark 0.22)	$[1.1 - \log_e(5), \infty)$
	1av 0/2 46 1/2 36 2/2 18 (Average mark 0.71)	Asymptote y = 0, endpoint $\left(1.1 - \log_e(5), 5\right)$
	1b 0/2 9 1/2 24 2/2 67 (Average mark 1.57)	$a = 0.5$, $b = \dfrac{0.2}{\log_e(1.5)}$
	1ci 0/1 30 1/1 70 (Average mark 0.70)	$e^{\frac{11}{5} - \frac{2T}{k}}$
	1cii 0/2 87 1/2 6 2/2 8 (Average mark 0.21)	$\dfrac{2T}{2.2 - \log_e(5)}$
Question 2	Many students used the CAS facility of their calculator successfully and found this question fairly straightforward. Some of the unsuccessful students could not apply the definition involved. This was a question where it was possible to pick up half the marks with appropriate formulation. For example, in Question 2a incorporating an expression such as $\int_0^a \frac{2x^2}{a^2}dx = 150$ would be given a method mark and in Question 2b an expression such as $\int_0^{200} \frac{2x}{a^2}dx$ would suffice to receive the method mark. It was surprising that several students in Question 2c wrote down the correct answers with no working being shown. The preliminary instructions to the paper included the statement that 'appropriate working must be shown if more than one mark is available'. The last section of this question was not done well.	
	2ai 0/2 33 1/2 5 2/2 62 (Average mark 1.28)	integral = 150
	2aii	0.790

	0/2 1/2 2/2 (Average mark 1.12)	42 4 54	
	2b 0/2 1/2 2/2 (Average mark 1.77)	 9 4 87	0.023
	2c 0/4 1/4 2/4 3/4 4/4 (Average mark 1.62)	 47 12 6 1 33	μ = 23.5 mm and σ = 3.2 mm
	2di 0/2 1/2 2/2 (Average mark 0.4)	 78 4 18	0.427
	2dii 0/2 1/2 2/2 (Average mark 0.18)	 90 2 8	0.936
Question 3	Generally this question was well done. Questions 3ai and 3aii were completed successfully by many students but some surprisingly could complete part ii but not part i. Such students did not recognise that the answer to Question 3ai could be directly obtained from their calculator through factorisation over *R*. Most students scored some marks in Question 3b, but few students achieved full marks on part bii. The ‘show that’ instruction is quite frequently used in examination Analysis Task papers and students should be aware of what is required. In this case a number of methods were available for successful establishment of the result. The most frequently used was to establish that the normal met the curve again at $\left(-1, -\frac{5}{2}\right)$ and that the gradient of the curve at this point was also 1. Many students had difficulty with Question 3ci and did not answer with a single integral expression. They did not seem to be familiar with the result that for suitable functions *f* and *g* if $f(x) \geq g(x)$ for all *x* in an interval [*a*, *b*] then the area of the region between the curves and the lines $x = a$ and $x = b$ the area is given by $\int_a^b (f(x) - g(x))dx$ which gives the integral $\int_{-1}^{1} (x^4 - 0.5x^3 - 2.5x^2 + 0.5x + 1.5)dx$. Question 3cii was completed successfully by many students but others lost the mark because they did not give the answer to the required accuracy.		
	3ai 0/1 1/1 (Average mark 0.81)	 19 81	$a = 1$, $b = 1$, $c = {}^{-}1$
	3aii 0/2 1/2 2/2 (Average mark 1.76)	 4 15 81	$0, \frac{3}{2}, \frac{-\sqrt{5} - 1}{2}, \frac{\sqrt{5} - 1}{2}$
	3bi 0/3 1/3 2/3 3/3 (Average mark 2.05)	 19 15 8 58	Gradient of tangent $= {}^{-}1$ Gradient of normal is 1 When $x = 1$, $y = 0.5$
	3bii 0/4	 21	Normal meets curve again at $x = {}^{-}1$, $y = \frac{-5}{2}$. Gradient of

	1/4 2/4 3/4 4/4 (Average mark 2.19)	5 35 13 26	curve at this point is 1.
	3ci 0/2 1/2 2/2 (Average mark 0.97)	 47 9 44	$\int_{-1}^{1}\left(x^4 - 0.5x^3 - 2.5x^2 + 0.5x + 1.5\right)dx$
	3cii 0/1 1/1 (Average mark 0.41)	 59 41	1.73
Question 4	This question was quite well done. Students used their calculators effectively to obtain numerical results. There was a demonstration of understanding of period, amplitude, and solution of equations in Questions 4a and 4c. Use of the trace function caused inaccurate solutions in Question 4b and many students did not receive the possible mark for this question because of this. Question 4d was done well and showed students were competent with entries into their calculators of a quite complicated expression. When students did not use exact mode some difficulties were encountered as an approximation to $e^{\frac{1}{25}}$ was incorporated into the answer. Students should be conscious of this when entering expressions. In Question 4eii the equation $\frac{dy}{dt} = 0$ was required. There was no requirement to solve with exact values, but many students proceeded to do so with expressions involving a parameter which was not explained at all. A numerical solution of this equation was required and then substitution. Answers not given to the prescribed accuracy were frequent.		
	4ai 0/1 1/1 (Average mark 0.96)	 4 96	21 metres
	4aii 0/1 1/1 (Average mark 0.85)	 15 85	4.5 seconds
	4b 0/1 1/1 (Average mark 0.59)	 41 59	58.03
	4c 0/1 1/1 (Average mark 0.73)	 27 73	6
	4d 0/2 1/2 2/2 (Average mark 1.6)	 10 20 71	55
	4ei 0/2 1/2 2/2 (Average mark 1.75)	 8 9 83	$\frac{\pi}{3e^{0.04t}}\cos(\frac{\pi}{3}) + 0.04 \quad e^{0.04t}\cos(\frac{\pi}{3})$
	4eii 0/3 1/3 2/3 3/3 (Average mark 1.66)	 21 29 12 38	4.61
	4f 0/3 1/3	 79 4	0.953

	2/3 3/3 (Average mark 0.53)	0 17	