

Question 1

The volume of water, $V(t)$ megalitres, in a small reservoir is known to be modelled by the equation

$$V(t) = 10 \sin\left(\frac{t}{2} + 3\right) + 15 \text{ for } 0 \leq t \leq 12$$

where t is the number of months in a certain year.

- On the axes below, sketch a graph of $V(t)$ versus t for the entire year. Clearly show important features including endpoints and turning points to one decimal place. (3)
- Find the function for the rate of change of volume with respect to time. (1)
- Calculate the exact values of t when the volume is at its minimum and maximum values during the year. (4)
- Correct to the nearest 0.1 month, for how long during the year is the volume of water in the reservoir less than 7.5 megalitres? (2)

Question 2

The design of a new stadium roof is being considered by an architect. A diagram of the proposed stadium (Figure 1) and a cross-sectional view of the stadium on Cartesian axes (Figure 2) are shown.

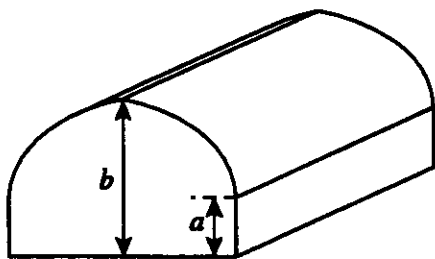


Figure 1

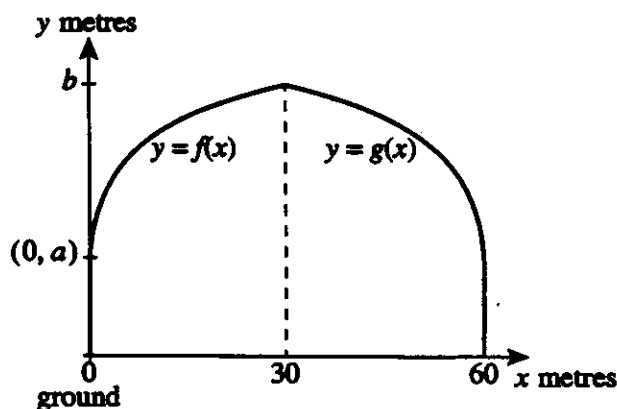


Figure 2

The equation of the left side of the curved roof ($y = f(x)$) is given by $f: [0, 30] \rightarrow \mathbb{R}$,

where $f(x) = 5 \log_e(x + 1) + 10$.

- Calculate the value of a (the height of the side wall). (1)
 - Calculate (to the nearest centimetre) the maximum height of the roof above the ground (b). (1)
- The right side of the roof is to be symmetrical to the left side about the line $x = 30$. Its equation, $y = g(x)$, can be deduced by reflecting the graph of $y = f(x)$ in the y axis and translating it 60 units to the right.
- Deduce the equation for $g(x)$. (2)
 - State the domain and range of $g(x)$. (2)

The architect needs to know the slope of the roof at various points so that roofing materials can be designed.

e. Calculate an expression in terms of x for the slope of the roof at any x value

i. when $0 < x < 30$. (1)

ii. when $30 < x < 60$. (1)

f. Calculate the exact slope of the roof when

i. $x = 15$. (1)

ii. $x = 45$. (1)

The architect considered bricking in one entire end wall enclosed under the roof span.

g. Write an expression in terms of $f(x)$ for the exact area ($A \text{ m}^2$) of one end of the stadium enclosed under the entire cross sectional span of the roof. (2)

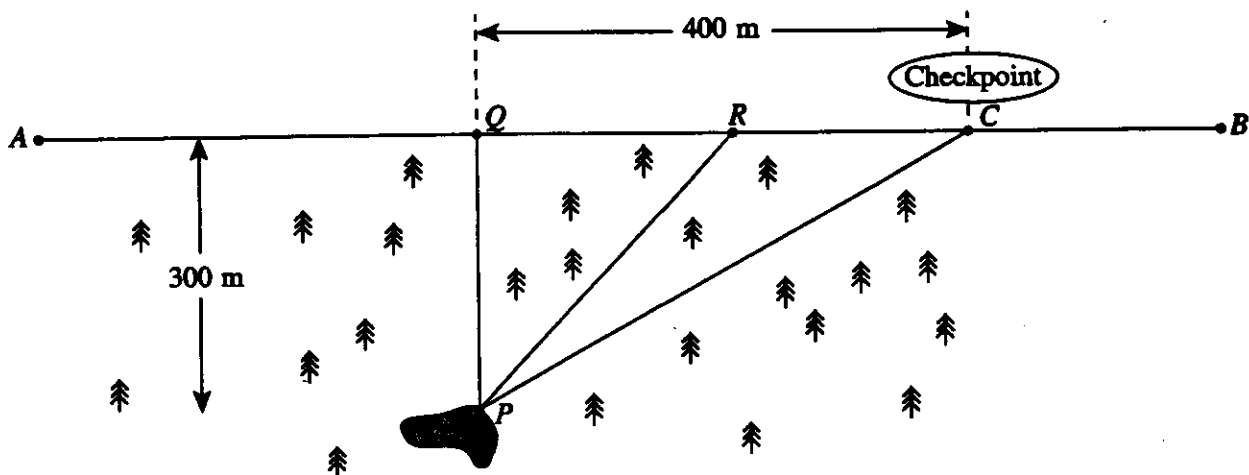
h. Use calculus to calculate this exact area given that (3)

$$\int \log_e(x+1) dx = (x+1)\log_e(x+1) - x.$$

i. Use your calculator to find this area in square metres, correct to 1 decimal place. (1)

Question 3

Paolo, an experienced orienteer, is standing by a pond P in a heavily wooded forest. He has 5 minutes to report to checkpoint C , along a straight track AB , otherwise he will be disqualified – and he must walk, not run.



Q is a point on the road directly opposite his position. The distances from P to Q and from Q to the checkpoint are as shown in the diagram. Paolo averages 12 km/h when walking along the track and 6 km/h through the forest.

a. Find the exact time, in minutes, that Paolo will take to walk in a straight line from the pond to the checkpoint. (2)

b. How long, to the nearest minute, will Paolo take to walk from the pond to the checkpoint via Q ? (1)

Paolo wishes to get to the checkpoint in the quickest time possible. He believes he can do this by walking through the forest to a point R and then along the track to the checkpoint.

- c. If R is x metres from Q , show that the time taken along this route is (2)

$$\frac{2\sqrt{90000 + x^2} + 400 - x}{200} \text{ minutes.}$$

- d. Using calculus, find the time in minutes for Paolo to get to the checkpoint as quickly as possible. Express your answer to 1 decimal place. (4)
- e. i. Find $T'(170)$ and $T'(180)$ and hence justify that the time found in d. is the quickest possible. (3)
- ii. Sketch the graph of the time function, clearly showing the endpoints, and hence verify this time to 4 decimal places. (2)
- f. Will Paolo be disqualified? Give reasons for your answer. (1)

Question 4

Isaias wants to enter his rally car into the gruelling Arnhem desert off-road race. As this race requires precision driving and fierce concentration, the organisers put the applicants through a preliminary driving test. The probability that an applicant passes this test on their first attempt is 0.6.

For your answers in a., express all probabilities to 4 decimal places.

- a. i. What is the probability that 6 out of 10 applicants pass the test on their first attempt? (1)
- ii. What is the probability that at least 6 out of 10 applicants pass the test on their first attempt? (1)
- iii. What is the probability that at most 6 out of 10 applicants pass the test on their first attempt? (1)
- b. A group of 250 candidates attempt the preliminary driving test. Calculate the mean and standard deviation of the number of candidates who pass on their first attempt. (2)

Isaias passes the preliminary driving test and is invited to participate in the rally. The probability of developing engine trouble for every day's full driving is 1% and is independent for each day.

- c. What is the exact probability that a rally car will not have engine trouble on any particular day? (1)
- d. What is the probability that a car will not have developed engine trouble during the first 2 days? Express your answer correct to 4 decimal places. (1)
- e. What is the probability that a car will have developed engine trouble during the first D days? (1)

It is discovered that engine trouble in the particular make of car that Isaias is driving is caused by the overheating of an electronic sensor. The manufacturer of the sensor has found that the temperature at which a randomly chosen sensor fails is normally distributed with a mean of 94.5°C and a standard deviation of 5.7°C .

- f. What proportion of sensors will operate at 100°C ? Express your answer correct to the nearest percentage. (3)

Due to the regular failure of the sensors in this make of car, the manufacturer decides to quote a safe operating temperature at which 99% of the sensors will work.

- g. What temperature, to the nearest degree, should the supplier quote? (3)