The Mathematical Association of Victoria

2001

MATHEMATICAL METHODS

Trial Examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ____

Directions to students

This examination consists of four questions.

Answer all questions.

All working and answers should be written in the spaces provided.

The marks allotted to each part of each question appear at the end of each part.

There are **55 marks** available for this task.

A formula sheet is attached.

These questions have been produced to assist students in their preparation for the 2001 Mathematical Methods Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority (VCAA) Assessing Panels. The Association gratefully acknowledges the permission of the VCAA to reproduce the formula sheet.

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Mathematical Methods Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ | $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ | $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x}$ | $\int \frac{1}{x} dx = \log_e x + c, \text{for } x > 0$

$\frac{d}{dx}\left(\sin ax\right) = a\cos ax$ | $\int \sin ax \, dx = -\frac{1}{a}\cos ax + c$

$\frac{d}{dx}\left(\cos ax\right) = -a\sin ax$ | $\int \cos ax \, dx = \frac{1}{a}\sin ax + c$

$\frac{d}{dx}\left(\tan ax\right) = \frac{a}{\cos^2 ax} = a\sec^2 ax$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2 p(x)$ $= \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x(1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\, {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed $\mathrm{N}(0, 1)$.		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

END OF FORMULA SHEET

Question 1

The population of kangaroos in a National Park has been increasing exponentially from 1990 for eight years. This population growth can be modelled by a function $P(t) = Ae^{kt}$ where t represents the time in years after the beginning of 1990 and k is a constant.

At the beginning of 1990 the population was 500, and by the beginning of 1995 it had increased to 746.

a. What is the value of A?

1 mark

b. Use algebra to find the value of k correct to two decimal places.

3 marks

c. Hence, define the function $P(t)$ over an appropriate domain.

2 marks

d. Calculate the number of kangaroos in the park by the beginning of 1998.

1 mark

The kangaroo population decreased from the beginning of 1998 at a rate of 10% per year. This population can be modelled by an exponential function $K(t)$, where t represents the time in years after the beginning of 1990.

e. Fully define $K\,(t)$ over an appropriate domain.

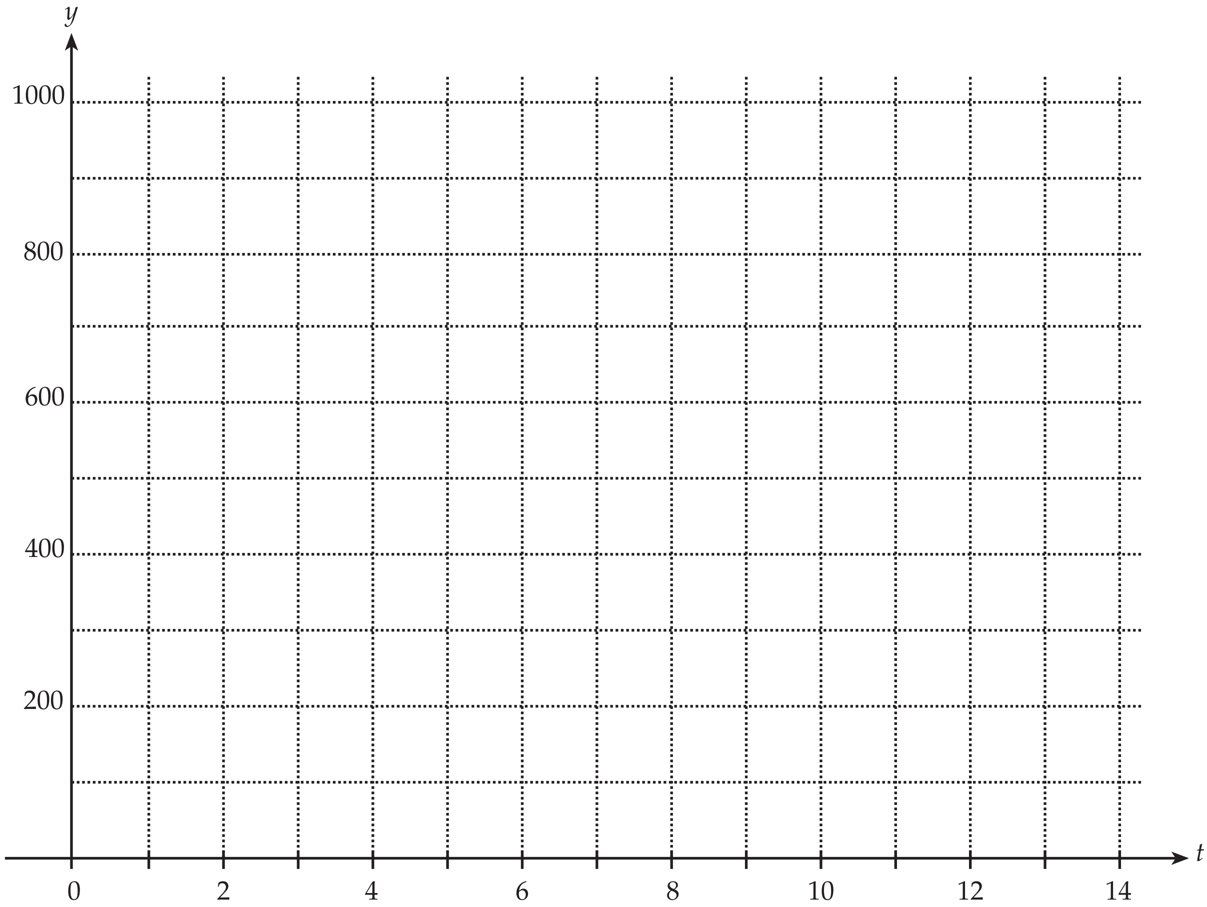
2 marks

f. **i.** Calculate the kangaroo population at the beginning of 2001.

ii. During which year after 1990 would the population again be 500?

1 + 1 = 2 marks

g. On the axes provided sketch a curve representing the kangaroo population from the beginning of 1990 to the beginning of 2001, labelling all the relevant points.



3 marks
Total 14 marks

Question 2

A casino decided to run a Mathematics Games Day for Victorian Year 12 students. Students have to sit a mathematical aptitude test which gives scores that are normally distributed with a mean of 30 and standard deviation of 7, in order to be selected for the day. Assume that all students have an equally likely chance of being selected.

a. Only students who achieve a test score within the top 10 per cent of scores are selected to compete on the day. What is the minimum score, to the nearest whole number, a student must achieve to be selected?

1 mark

b. If 15 students selected at random sit the test, what is the probability, correct to two decimal places, that two students will be chosen?

2 marks

c. **i.** Determine the probability, correct to four decimal places, that a student will score more than 35.

ii. Given that a student has scored more than 35, what is the probability, correct to two decimal places, of that student being selected?

1+ 2 = 3 marks

After the aptitude test, the successful students were allowed to play games at the casino. In one particular game, a student was dealt a hand of six cards from a pack of 52 cards.

d. What is the expected number of hearts in the hand?

1 mark

e. In this game, a player wins if they get more than four hearts in a hand.

i. Calculate the probability, correct to four decimal places, of winning.

ii. How many hands were played if the probability of winning at least one hand was 0.0127?

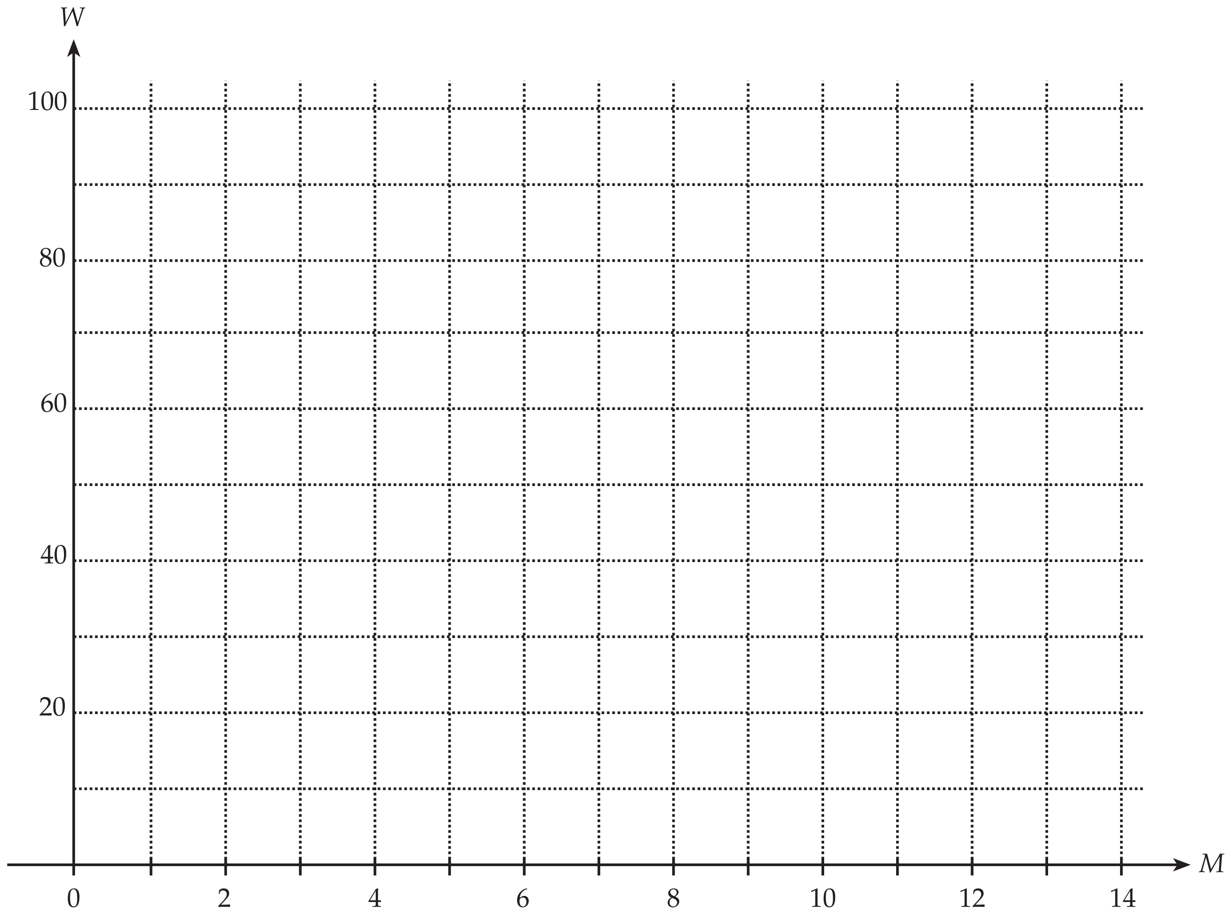
2 + 3 = 5 marks

Total 12 marks

Question 3

The amount of water in Eildon reservoir (W) is described as a percentage of total capacity. At the beginning of April 2000 the water reached a level of 9% of total capacity, and by the beginning of November 2000 it reached a level of 51% of total capacity. The water level in April 2000 was the lowest for the reservoir in the year 2000.
Use M as the number of months after December 1999 so that $M = 1$ represents the beginning of January 2000.

a. Plot this information on the axes provided.



1 mark

b. Define a **quadratic** function that models the water level in the reservoir against time measured in months. State the domain of your function. Use exact values throughout.

3 marks

c. Sketch your quadratic function on the grid provided in part **a**. Show intercepts and endpoints to the nearest whole number.

2 marks

d. Subsequent measuring showed that the November level was the **maximum** level for the summer.

i. Explain why $W'(M) = K(M - 4)(M - 11)$, could be the rule of the gradient function**.**

1 mark

ii. Hence determine a **cubic** rule, $W(M)$, that models the water level in the reservoir between December 1999 and November 2000.
State the domain of your rule.

4 marks

e. Sketch your cubic function on the grid on page 6. Show intercepts and endpoints to the nearest whole number.

2 marks

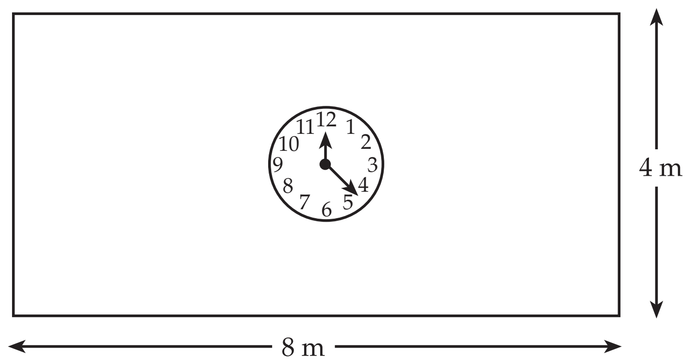
f. **i.** Why would your quadratic and cubic functions fail as models of the reservoir's water level over a longer period of time?

ii. What type of function might be a better model of the reservoir's water level over a longer period of time? Explain.
(You are not required to find this model.)

1 + 1 = 2 marks
Total 15 marks

Question 4

A clock is embedded into the middle of a rectangular flowerbed at a park. The width of the flowerbed is 4 m and the length 8 m as shown in the diagram below. The diameter of the clock is 1.2 m and the hour hand is 25 cm long. The centre of the clock is at the centre of the flowerbed.



Note: diagram not to scale

An equation which models the shortest distance, h cm, of the tip of the hour hand from the top of the flowerbed is of the form

$$h = 25\sin(nt + e) + b,$$

where t is the time in hours from 12 : 00 noon and n, e and b are constants.

a. Show that n is $\frac{\pi}{6}$.

1 mark

b. Find values for b and e.

2 marks

Another equation which models the distance is of the form

$$h = A\cos(nt) + 200$$

c. Write down the values for A and n.

2 marks

d. Using the equation from part **c**, write down the rule for the rate of change of h with respect to t. What is the maximum rate of change? Give an exact answer.

3 marks

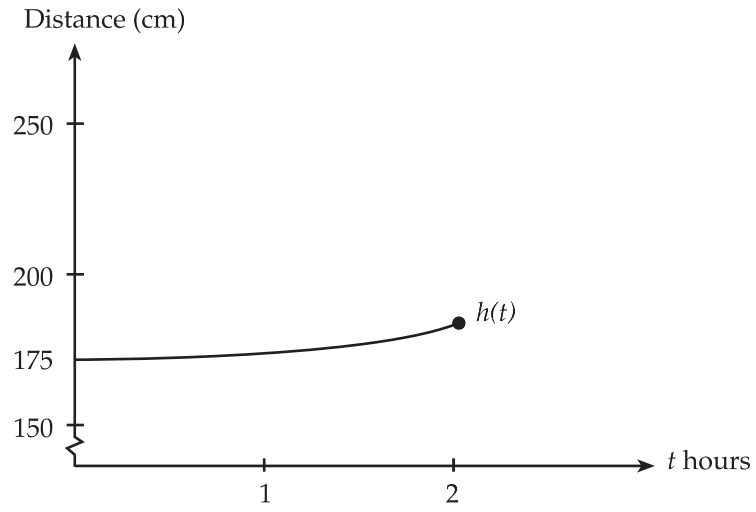
e. If m is the shortest distance, in cm, of the tip of the minute hand from the top of the flowerbed, write down a rule for this function in the form

$$m = A\sin(nt + e) + 200$$

where t is the number of hours after noon. The minute hand is 30 cm long.

2 marks

f. The graph of h for $0 \le t \le 2$ hours is shown below. Sketch the graph of m for the same time period, clearly showing the endpoint.



2 marks

g. How many times during a day is the shortest distance of the tip of the hour hand from the top of the flowerbed the same as the shortest distance of the tip of the minute hand from the top of the flowerbed?

1 mark

h. What is the first time, to the nearest minute, when the distances are the same?

1 mark
Total 14 marks