Mathematical Methods Examination 2
Solutions

Question 1.

a.

Initial population = 500

$\Rightarrow A = 500$ **1A**

b.

$P = 500e^{kt}$

When $t = 5$, $P = 746$

$\Rightarrow \ 746 = 500e^{5k}$ **1M**

$\Rightarrow \ e^{5k} = \frac{746}{500}$

$\Rightarrow \ 5k = \log_e \frac{746}{500}$ **1M**

$\Rightarrow \ k = \frac{1}{5}\log_e \frac{746}{500}$

≈ 0.08 **1A**

c.

$P(t) = 500\, e^{\,0.08t}$ **1A**

$t \in [0, 8]$ **1A**

d.

$P(8) = 500\, e^{0.64}$

≈ 948 kangaroos **1A**

e.

$K(t) = 948\,(0.9)^{(t-8)}$ **1A**

$t \in [8, \infty)$ **1A**

f. i.

When $t = 11$, $K(t) = 948\,(0.9)^3$

≈ 691.09

$\therefore$ 692 kangaroos (round up) **1A**

ii.

$500 = 948(.9)^{(t-8)}$

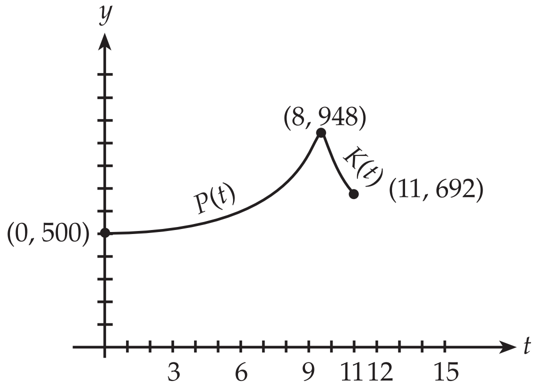
$\log_e \frac{500}{948} = (t-8)\log_e(.9)$

$t - 8 \approx 6.07$

$t \approx 14.07$

$\therefore$ during 2004 **1A**

g.



Shape **1**

Labels **1**

End pts **1**

(14 marks)

Question 2. a.

$\Pr(X \le k) = 0.9$, $\mu = 30$, $\sigma = 7$

inv Norm (0.9, 30, 7)

$k \approx 38.97$

≈ 39 **1A**

b.

$\Pr(X = 2)$

$= {}^{15}C_2(0.1)^2(0.9)^{13}$ where $n = 2$, $p = 0.1$ **1M**

≈ 0.2669

≈ 0.27 **1A**

c. i. $\Pr(X > 35)$

≈ 0.2375 **1A**

norm cdf (35, 1E99, 30, 7)

ii. $\Pr\big((X \ge 39) \mid (X > 35)\big)$

$\approx \frac{0.1}{0.2375}$ **1M**

≈ 0.42105

≈ 0.42 **1A**

d. $E(X) = \frac{nD}{N}$

$= \frac{6 \times 13}{52}$

$= 1.5$ **1A**

e. i.

Probability of a win

$$= \frac{\binom{13}{5}\binom{39}{1}}{\binom{52}{6}} + \frac{\binom{13}{6}\binom{39}{0}}{\binom{52}{6}}$$ **1M**

≈ 0.0025497

≈ 0.0025 **1A**

ii.

$\Pr(X \geq 1)$

$= 1 - \Pr(X = 0)$

$= 1 - \binom{n}{0}(0.002549)^0(0.997451)^n = 0.0127$ **1M**

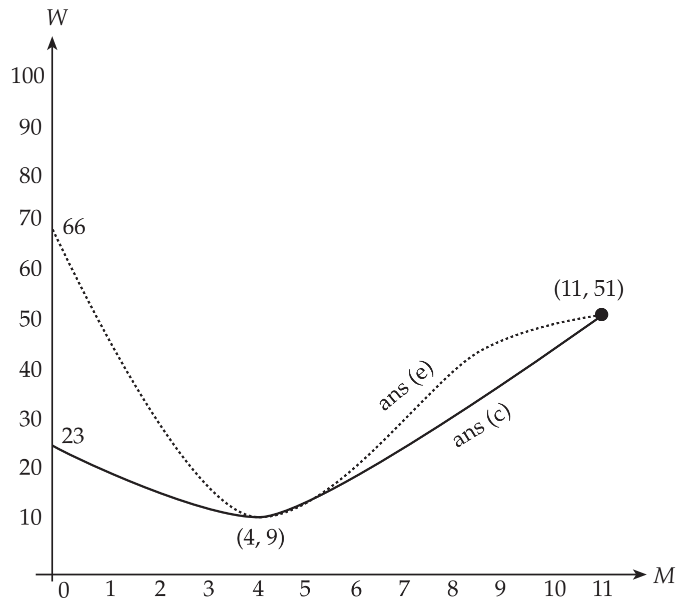
$0.997451^n = 0.9873$

$n = \dfrac{\log 0.9873}{\log 0.997451}$ **1M**

$= 5$ **1A**

(12 marks)

Question 3



a.

Mark and label (4, 9) & (11, 51) **1 A**

b.

Turning point at (4, 9)

$\Rightarrow W = K(M-4)^2 + 9$

Find K using point (11, 51)

$\Rightarrow 51 = K(7)^2 + 9$

$\Rightarrow 42 = 49K$

$\Rightarrow K = \dfrac{6}{7}$ **1A**

$W = \dfrac{6}{7}(M-4)^2 + 9$ (or expanded version) **1A**

$M \in [0,11]$ **1A**

c.

Intercept $(0, 22\frac{5}{7}) \rightarrow (0, 23)$ **1A**

Shape and points **1A**

d. i.

S.P. when M = 4, 11

$\Rightarrow W'(M) = K(M-4)(M-11)$ **1A**

ii.

$W'(M) = K(M^2 - 15M + 44)$

$W(M) = K\left(\dfrac{M^3}{3} - \dfrac{15M^2}{2} + 44M\right) + C$ **1M**

Substitute (4, 9) and (11, 51)

$\Rightarrow 9 = \dfrac{232}{3}K + C$ …(1)

$51 = \dfrac{121}{6}K + C$ …(2) **1M**

Subtract $-42 = \dfrac{343}{6}K$

$\Rightarrow K = \dfrac{-36}{49}$; $C = \dfrac{3225}{49}$ (≈ 66) **1A**

$W(M) = \dfrac{-36}{49}\left(\dfrac{M^3}{3} - \dfrac{15M^2}{2} + 44M\right) + \dfrac{3225}{49}$

$M \in [0,11]$ **1A**

e.

Intercept $\approx (0, 66)$ **1A**

Shape **1A**

f. i.

Do not take cyclic nature into acount. **1A**

ii.

A sin/cos model ∴ have cyclic nature. **1A**

(15 marks)

Question 4.

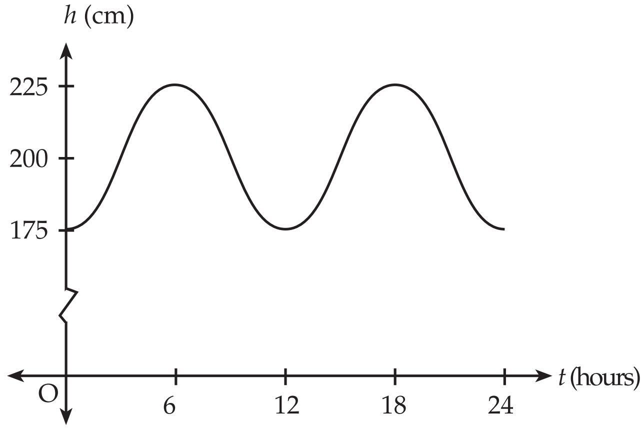
a.

$$\text{Period} = \frac{2\pi}{n} = 12 \text{ hours} \qquad \textbf{1M}$$

$$n = \frac{2\pi}{12} = \frac{\pi}{6}$$

b.

$$b = \frac{400}{2} = 200 \text{ cm} \qquad \textbf{1A}$$



$$h = 25\sin\frac{\pi}{6}(t-3) + 200$$

$$= 25\sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) + 200$$

$$e = -\frac{\pi}{2} \qquad \textbf{1A}$$

c.

$A = -25$ **1A**

$n = \frac{\pi}{6}$ **1A**

d.

$$h = -25\cos\left(\frac{\pi}{6}t\right) + 200$$

$$\frac{dh}{dt} = \frac{25\pi}{6}\sin\left(\frac{\pi}{6}t\right) \qquad \textbf{1A}$$

Maximum value of $\frac{dh}{dt}$ occurs when

$\sin\left(\frac{\pi}{6}t\right) = 1$, hence maximum rate of change **1M**

is $\frac{25\pi}{6}$ cm per hour. **1A**

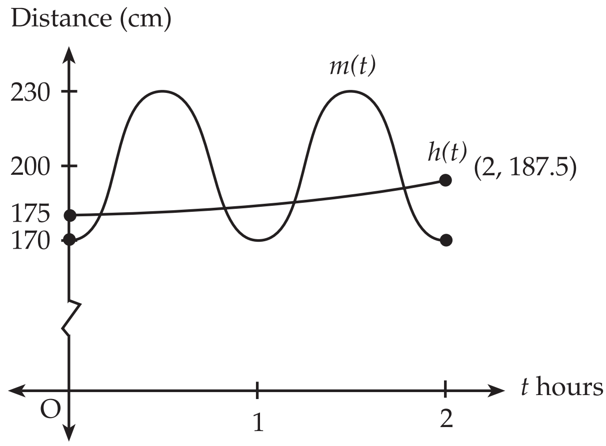
e.

$$\text{Period} = \frac{2\pi}{n} = 1 \text{ hour}$$

$$n = 2\pi$$

$$m = 30\sin\left(2\pi t - \frac{\pi}{2}\right) + 200 \qquad \textbf{2A}$$

f.



shape **1**
end points **1**
2A

g.

2 times per hour
$= 2 \times 24$ times per day
$= 48$ times per day **1A**

h.

Using graphics calculator

$t \approx 0.09350184$ hours
$= 0.09350184 \times 60$ minutes
≈ 5.61 minutes
≈ 6 minutes after noon or 12:06 p.m. **1A**

(14 marks)