

YEAR 12

IARTV TEST — OCTOBER 2001

MATHEMATICAL METHODS

EXAMINATION 2 (ANALYSIS TASK)

© IARTV Test Papers are COPYRIGHT  
and may not be reproduced in whole or in part

Time: 1 hour 30 minutes

**Directions to students**

**Materials**

Question and answer booklet of 9 pages.  
An approved calculator or graphics calculator may be used.  
You may bring to this examination two A4 sheets of notes which can be written on both sides.  
A protractor, set square and aids for curve-sketching may be used.

**The task**

Time allowed: 1 hour 30 minutes.  
Answer all questions in the spaces provided in this booklet.  
There is a total of 66 marks available.

**At the end of the task**  
Hand in this booklet.

© COPYRIGHT 2001

Prepared for and published by the  
Incorporated Association of Registered Teachers of Victoria

Printed by Annette Widdell Pty Ltd, Melbourne

**Question 1. (17 marks)**

The Fast Track moving Company wishes to rent a truck and has the following alternatives:

Ajet Hire: \$30 per day plus 15 cents per kilometre.

Bravo Hire: \$25 per day plus 20 cents per kilometre

Gamma Hire: \$75 per day flat rate.

The operations manager needs to decide which scheme is best for his organisation and asks you to provide the following information on these alternatives.

Let  $C_A$  = The total daily cost in dollars of rental from Ajet

$C_B$  = The total daily cost in dollars of rental from Bravo

$C_G$  = The total daily cost in dollars of rental from Gamma

x = the distance travelled by the rental truck in kilometres.

a) Give a formula for  $C_A$ ,  $C_B$  and  $C_G$  clearly stating the domain of each function.

---

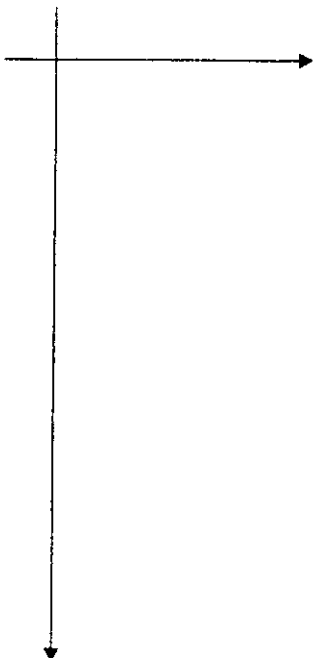


---



---

b) On the one set of axes, sketch and label the three functions given above. (Use the total daily cost as the vertical axis.)



c) State the total daily cost of each scheme if the truck were to travel 150km in the day. (3 marks)

---



---



---

d) For what distance would the daily costs from Ajet and Bravo be the same? (2 marks)

---



---



---

(1 mark)

e) When would Gamma's deal be the cheapest?

---



---



---

(2 marks)

The company wishes to know the average daily cost per kilometre, which is:

$$\text{Average daily cost} = \frac{\text{Cost}}{\text{distance}}$$

f) Write down the expressions for the average daily costs of hiring from Ajet and Bravo. (Specify the domains for each function.)

---



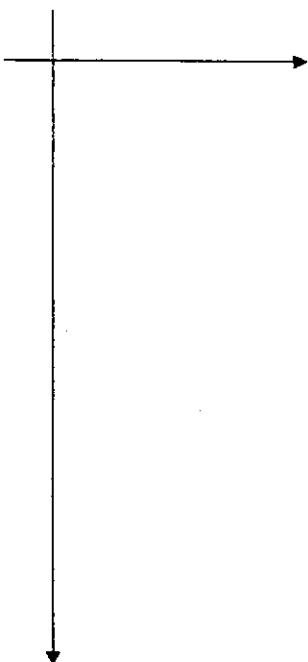
---



---

(2 marks)

g) Sketch and label the graphs of Average Cost versus distance. (Show asymptotes and point(s) of intersection.)



h) Compare the average cost for Ajet and Bravo for different distances and decide which company is cheaper to hire (and for what distances.) (3 marks)

---



---



---

(2 marks)

Question 2. (16 marks)

A group of explorers were investigating Central Australia when they stumbled upon a most unusual mountain range. It consisted of parallel ridges and valleys -- all the ridges were the same height and the valleys the same depth. One of the team had a mathematical background and decided to model the vertical cross section of this range.

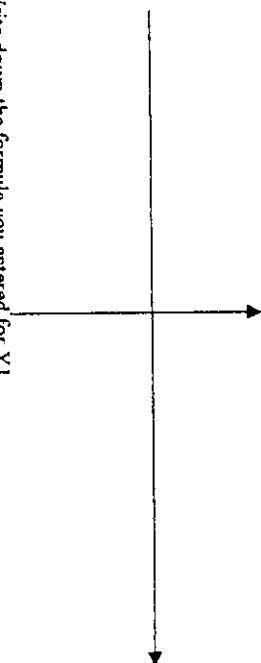
The first model that was suggested to sketch the mountain range was

$$y = \sin^2 x \cos x + 2, \quad -2\pi \leq x \leq 2\pi.$$

where  $y$  = height of the range in hundreds of metres,

$x$  = horizontal distance in hundreds of metres.

- a) Use your graphics calculator to help you draw a sketch of the outline of the mountain range on the given domain. (show co-ordinates of end points and the y-intercept.)



- b) Write down the formula you entered for Y1. (3 marks)

\_\_\_\_\_ (1 mark)

- c) Find, correct to three decimal places, the coordinates of the first maximum turning point and the first minimum turning point for  $x > 0$ .

\_\_\_\_\_ (1 mark)

\_\_\_\_\_ (1 mark)

\_\_\_\_\_ (1 mark)

- d) Use your graphics calculator to find the gradient of the mountain range when  $x = \pi/2$ .

\_\_\_\_\_ (1 mark)

- e) This model produces a periodic graph. state the period in the exact form and the amplitude of the graph to two decimal places.

\_\_\_\_\_ (2 marks)

\_\_\_\_\_ (2 marks)

\_\_\_\_\_ (2 marks)

- f) Find the derivative (ie  $\frac{dy}{dx}$ )

\_\_\_\_\_ (2 marks)

\_\_\_\_\_ (2 marks)

\_\_\_\_\_ (2 marks)

- g) Using the above derivative find the values for the seven stationary points on the interval  $[0, 2\pi]$

\_\_\_\_\_ (3 marks)

\_\_\_\_\_ (3 marks)

\_\_\_\_\_ (3 marks)

Another explorer suggested that there may be a simpler way to model the mountain range and he considered the functions:

$$Y_2 = \cos x - \cos 3x \quad -2\pi \leq x \leq 2\pi$$

$$Y_1 = \sin^2 x \cos x \quad -2\pi \leq x \leq 2\pi$$

- h) There is a simple transformation which maps  $Y_2$  onto  $Y_1$ . Describe the transformation.

\_\_\_\_\_ (1 mark)

\_\_\_\_\_ (1 mark)

- i) A surveyor who is with the group wishes to estimate the volume of the valley which is located between  $[\pi/2, \pi]$ . Using the model,  $y = \sin^2 x \cos x + 2$ , and the additional information that the valley is 1 km long, estimate the volume of this valley giving your answer in  $m^3$  in scientific notation.

\_\_\_\_\_ (2 marks)

\_\_\_\_\_ (2 marks)

\_\_\_\_\_ (2 marks)

(Question 3. (17 marks))

*In this question give all answers to the nearest dollar.*

The advertising department of the Low Glow store find that  $t$  weeks after the end of a sales promotion, the volume of sales is given by a function of the form:

$$V = A + Be^{-2t}, \quad 0 \leq t \leq 15.$$

where  $A$  = the average weekly volume of sales before the promotion,  
 $B$  = constant.

a) If  $A=50000$  and the sales volume is \$90219 at the end of the first week, find the value of  $B$ .

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_ (3 marks)

b) What is the initial volume of sales after the promotion?

\_\_\_\_\_ (1 mark)

c) Using the above values find the volume of sales at the end of the third week.

\_\_\_\_\_ (1 mark)

d) How many weeks will have passed when the sales have dropped to \$55000?

\_\_\_\_\_

\_\_\_\_\_ (3 marks)

e) Show that the volume of sales is decreasing with time.

\_\_\_\_\_

\_\_\_\_\_ (2 marks)

f) Find the rate of change of sales after 3 weeks.

\_\_\_\_\_ (2 marks)

The department wishes to analyse the success of their sales promotion and decides to first look at the total sales for the 15 week period. A good estimate for this value would be the area under the "volume of sales" curve.

g) Write down an expression which will evaluate the total sales.

\_\_\_\_\_ (1 mark)

h) Use your calculator to find this estimate for the total sales.

\_\_\_\_\_ (1 mark)

i) How much additional sales has been generated by the promotion?

\_\_\_\_\_

\_\_\_\_\_ (1 mark)

j) Show that  $V$  can be written in the form:

$$V = A + B(0.6703)^t, \quad 0 \leq t \leq 15.$$

\_\_\_\_\_

\_\_\_\_\_ (2 marks)

**Question 4. (16 marks)**

*In this question give all answers to 3 decimal places.*

Impulsive Airlines is reviewing its passenger statistics to try to develop a more streamlined policy. Initially, it looks at excess baggage on its international flights. The flight contains 500 passengers of whom 460 are within the baggage limits and the other 40 have excess baggage.

a) If a random sample of 50 passengers is chosen from this flight calculate the probability that exactly five of this sample have excess baggage. (Show working.)

\_\_\_\_\_

\_\_\_\_\_ (2 marks)

- The company also wishes to look at the number of passengers who are on time for departure compared to those who miss the flight due to lateness or cancellation. On average each person has a 92% chance of being on time for departure and an 8% chance of missing the flight. There are no families or groups flying and each person will arrive on time or miss a flight independently on the others.
- b) For a flight with 500 bookings find the probability that there will be just 460 people on time for departure.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

(2 marks)

It is also found that there is a relationship between the time of arrival for the flight and the baggage weight. Of the passengers who are on time, 95% are within the baggage limits. You may use a Venn diagram to answer the following questions.

- c) State the probability that a randomly chosen passenger will be on time and will have excess baggage.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

(1 mark)

Overall there are 8% of passengers with excess baggage.

- d) Find the probability that:
- i) a passenger will be late for departure and have excess baggage

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

(1 mark)

- ii) if a passenger is late for departure they will have excess baggage.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

(2 marks)

Consider the following information for a particular flight:

Number of tickets sold = 500

X = number of passengers who are on time for the flight.

$P_r(\text{on time}) = 0.92$

- e) What is the expectation of X?

\_\_\_\_\_

(1 mark)

- f) What is the variance of X?

\_\_\_\_\_

(1 mark)

If we assume that X is normally distributed with the mean and variance given above, find the probability that:

- g) There will be at least 470 passengers who arrive in time for the flight.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

(1 mark)

- h) There will be between 450 and 470 passengers on time for the flight.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

(1 mark)

The airline decides to overbook flights to allow for people who are late or cancel. For example, they may decide to sell 520 tickets and hope that at most 500 will turn up on time for the flight. The airline wishes to sell as many tickets as possible to ensure that the aircraft is full (i.e. 500 passengers) and to also guarantee that 99% of the passengers who arrive on time will be seated.

i) What is the maximum number of tickets that the airline should sell to meet these requirements?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

(4 marks)

END OF BOOKLET