No. 23

END OF BOOKLET



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No. 23

# YEAR 12

# IARTV TEST — OCTOBER 2001

# MATHEMATICAL METHODS Units 3 and 4

# EXAMINATION 1 (FACTS, SKILLS AND APPLICATIONS TASK)



Total time allowed for task: 1 hour 30 minutes

**QUESTION AND ANSWER BOOKLET**

**Structure of Booklet**

| Section | Number of Questions | Number of questions to be answered | Number of marks |
|---|---|---|---|
| A | 32 | 32 | 32 |
| B | 6 | 6 | 18 |

**Directions to students**

**Materials**
Question booklet of 14 pages.
A separate sheet of miscellaneous formulas and Normal distribution cdf table.
Multiple-choice answer sheet.
An approved calculator or graphical calculator may be used.
You may bring to this task two A4 sheets of notes which can be written on both sides.

**The task**
This task has two parts: *Section A* (multiple choice questions) and *Section B* (short answer questions).
Answer all questions from *Sections A* and *B*. You must complete **both** parts in the time allotted.
All questions from *Section A* should be answered on the multiple-choice answer sheet provided.
All questions from *Section B* should be answered in this question and answer booklet.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

**At the end of the task**
Please ensure that you write your name on the multiple choice answer sheet and on this booklet.
You should hand in the *Section A* multiple choice answer sheet and your answers to *Section B*.
Place the *Section A* multiple-choice answer sheet inside the front cover of this question and answer booklet and hand them in.

## SECTION A

**Specific Instructions for Section A**

*Section A* consists of 32 questions.
Answer **all** questions in this section on the multiple-choice answer sheet provided.
A correct answer scores 1, an incorrect answer scores 0.
No credit will be given for a question if two or more letters are marked for that question.
Marks will not be deducted for incorrect answers.
You should attempt every question.

**Question 1**

The values of $m$ for which the equation $2x^2 + mx + 2 = 0$ has **no** real solutions are

A. $m = 4, m = 0$ B. $-4 \le m \le 4$ C. $m > 4$ or $m < -4$ D. $-4 < m < 4$ E. $m = \pm 4$

**Question 2**

The function $f: [0, \infty) \to R$, $f(x) = 2 - 5\log_e(x+1)$ has an **inverse function** $f^{-1}$ whose rule is

A. $y = -1 + e^{\frac{2-x}{-5}}$ B. $x = e^{\frac{2-y}{5}} - 1$ C. $y = e^{\frac{2-x}{5}}$ D. $y = \dfrac{1}{2 - 5\log_e(x+1)}$ E. $y = e^{\frac{2-x}{5}} - 1$

**Question 3**

The function which has an implied domain of $[3, 5]$ is

A. $f(x) = \dfrac{3}{\sqrt{x-5}}$ B. $g(x) = \dfrac{1}{(x+3)(x-5)}$ C. $h(x) = \sqrt{5-x} + \sqrt{x-3}$

D. $i(x) = \sqrt{x^2 - 16}$ E. $j(x) = \dfrac{x+3}{5}$

**Question 4**

The temperature inside an igloo $T$°C at $t$ hours after noon is given by $T = a\cos(bt) + c$. Inside the igloo the temperature is 12°C at noon and it reaches a minimum of 6°C at midnight. The values of $a$ and $b$ respectively are

A. 6, 15 B. 3, 15 C. 3, $\frac{\pi}{6}$ D. 3, $\frac{\pi}{12}$ E. 6, $\frac{\pi}{12}$





**Question 5**

For which of the following functions is $f(-x) = f(x)$ ?

A. $f(x) = x^2 + 3$ B. $f(x) = \log_e(x+2)$ C. $f(x) = \sin x$ D. $f(x) = \sqrt{x}$ E. $f(x) = \dfrac{1}{x}$

**Question 6**

The equations of the asymptotes for the function $y = \dfrac{1}{x(x-3)^2} + 2$

A. $x = \pm 3$, $y = -2$ B. $x = \pm 3$, $y = 2$ C. $x = 2$, $y = 0$, $y = 3$ D. $x = 0$, $x = 3$, $y = 2$ E. $x = 0$, $x = 3$

**Question 7**

If $\log_2 y = 3 - 2\log_2 x$, then $y$ is equal to

A. $3x^2$ B. $2^{3-2\log_2 x}$ C. $\dfrac{8}{x^2}$ D. $\dfrac{\log_2 3}{x^2}$ E. $\dfrac{3}{x^2}$

**Question 8**

The independent term in the expansion of $(3-2x)^4$ is

A. $-216x$ B. -2 C. $16x^4$ D. 3 E. 81

**Question 9**

If $\tan\theta = x$ then $\tan\left(\dfrac{\pi}{2} - \theta\right)$ is equal to

A. $\dfrac{\pi}{2} - x$ B. $\dfrac{1}{x}$ C. $x$ D. $\tan^{-1} x$ E. $-x$

**Question 10**

The values between 0 and $2\pi$ for which $k\sin x + 1 = 0$ are

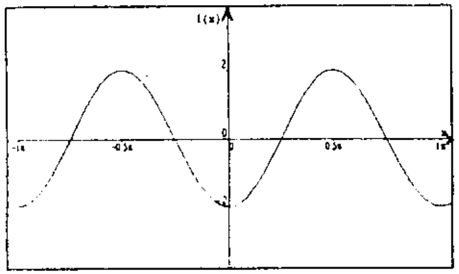
A. $\pi + \sin^{-1}\left(\dfrac{-1}{k}\right)$ and $2\pi - \sin^{-1}\left(\dfrac{-1}{k}\right)$ B. $\sin^{-1}\left(\dfrac{-1}{k}\right)$ and $\pi - \sin^{-1}\left(\dfrac{-1}{k}\right)$ C. $\sin^{-1}\left(\dfrac{-1}{k}\right)$

D. $-\sin^{-1}\left(\dfrac{1}{k}\right)$ and $2\pi - \sin^{-1}\left(\dfrac{1}{k}\right)$ E. $\pi + \sin^{-1}\left(\dfrac{1}{k}\right)$ and $-\sin^{-1}\left(\dfrac{1}{k}\right)$

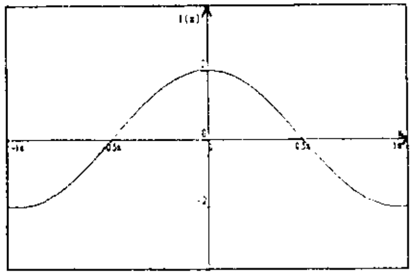
**Question 11**

Let $f:[-\pi,\pi]\rightarrow R$ where $f(x)=-2\cos(x-\pi)$. Which one of the following graphs best represents the graph of $f(x)$?

A.



B.



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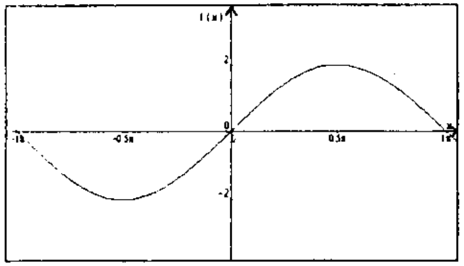

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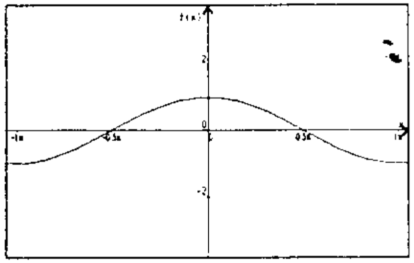
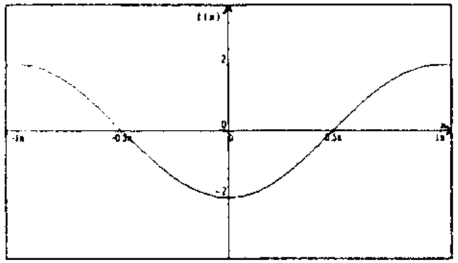


E.







**Question 12**

If $y=e^{(1-\cos x)}$ then $\dfrac{dy}{dx}$ is equal to

A. $(1-\sin x)e^{(1-\cos x)}$ B. $(1+\sin x)e^{(1-\cos x)}$ C. $(1-\cos x)e^{(1-\cos x)}$

D. $(\sin x)e^{(1-\cos x)}$ E. $(\sin x)e^{\sin x}$

**Question 13**

If $f(x)=\dfrac{1}{\sqrt{x}}+\sqrt{2x}$ then $f'(x)$ is equal to

A. $\dfrac{-1}{2\sqrt{x}}+\dfrac{1}{\sqrt{2x}}$ B. $\dfrac{\sqrt{2}x-1}{2x^{\frac{3}{2}}}$ C. $\log_e\sqrt{x}+\dfrac{1}{\sqrt{2x}}$ D. $\dfrac{-\sqrt{x}+(2x)^{\frac{3}{2}}}{2}$ E. $\dfrac{1+\sqrt{2}x}{2x^{\frac{3}{2}}}$

**Question 14**

If $f(x)=\log_e(2x)-x$ then the value of $\dfrac{f(1+h)-f(1)}{h}$ is

A. $\dfrac{\log_e(2h)-h}{h}$ B. $\dfrac{1}{x}-1$ C. $\dfrac{\log_e 2(1+h)-1-h}{h}$ D. $\dfrac{\log_e(1+h)-h}{h}$ E. $\dfrac{2}{x}-1$

**Question 15**

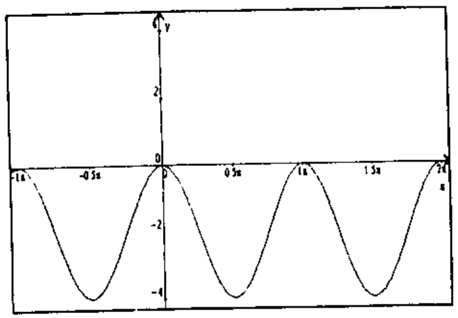
The line $y=kx-2$ is tangent to the curve $y=(x+5)(x-1)$. The $x$ *co-ordinate*, of the point where the tangent touches the curve, has a value of

A. $\dfrac{k}{2}-2$ B. $\dfrac{-6}{2-k}$ C. $\dfrac{(k-4)\pm\sqrt{(4-k)^2+12}}{2}$ D. $\dfrac{k-5}{2}$ E. 1

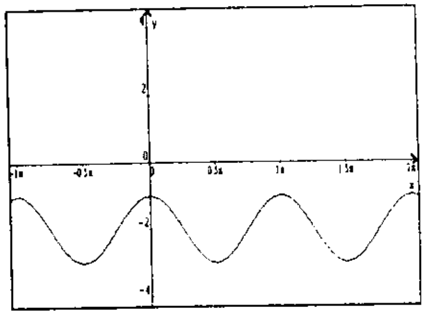
**Question 16**

For the function $f: R \rightarrow R$, $f(x) = \sin 2x - 2$ the graph of the ***derived*** function is most likely to be

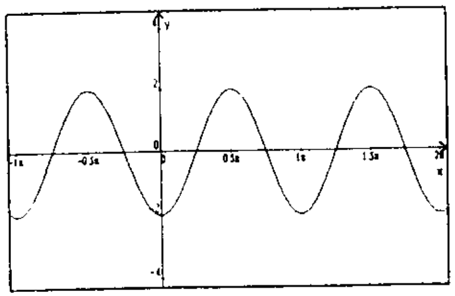
**A.**



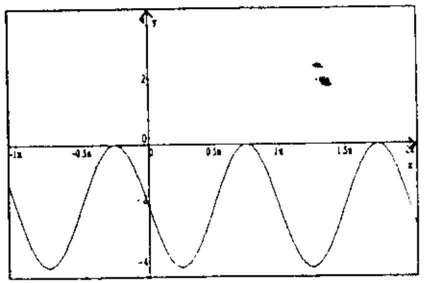

**B.**



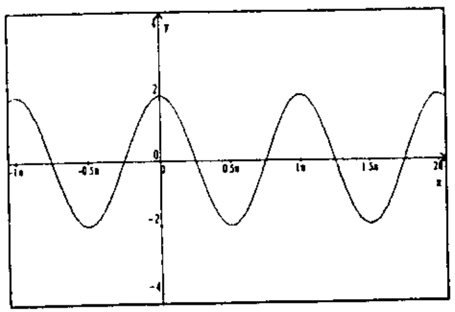

**C.**




**D.**




**E.**








**Question 17**

For the function $f(x) = -(x^2 + 7x + 10)(x+2)^2$, which one of the following is ***not*** true.

**A.** The gradient of the tangent to the curve at $x = -4.25$ is zero.
**B.** There are two turning points.
**C.** $f'(x) = 0$ at $x = -2$
**D.** There is only one local maximum point.
**E.** $f(-2) = 0$

**Question 18**

If $y = \frac{-4}{x} + x$ and the value of $x$ increases from 2 by a small amount $\frac{p}{2}$ then the *approximate* change in y is

**A.** $\frac{16}{p^2} + 1$ **B.** $\frac{4}{p}$ **C.** $p$ **D.** $\frac{p^2 - 16}{2p}$ **E.** $\frac{5p}{8}$

**Question 19**

The equation of the tangent to the curve $f(x) = \log_e(x^2) + e$ at the point $(e, 2+e)$ is

**A.** $f(x) = \log_e(e^2) + e$ **B.** $f(x) = \frac{2}{e}$ **C.** $f(x) = \frac{x}{e^2} + 2 + e - \frac{1}{e}$ **D.** $f(x) = \frac{2x}{e} + e$ **E.** $f(x) = \frac{1}{e^2}$

**Question 20**

With the introduction of a new insect repellent, the population $M$ thousands of a particular type of mosquito is expected to decrease with time ($t$) years according to the formula $M = \frac{M_0}{e^{0.1t}}$. The average rate of decrease of mosquitos in 3 years is

**A.** $\frac{M_0 - M_0 e^{-0.3}}{3}$ **B.** $\frac{M_0 e^{-0.3} - M_0}{3}$ **C.** $0.1M_0 e^{-0.3}$ **D.** $0.3M_0 e^{-0.3}$ **E.** $-0.1M_0 e^{-0.3}$

**Question 21**

The expression which evaluates the area of the shaded region, for the function $y = x(x-2)$, is

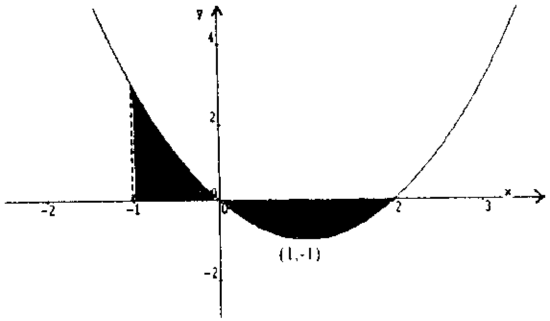
A. $\int_{-1}^{2} x(x+2)dx$

B. $\left[\frac{x^3}{3} - x^2\right]_{-1}^{2}$

C. $\left[x^2 - 2x\right]_{-1}^{2}$

D. $\left[\frac{x^3}{3} - x^2\right]_{-1}^{0} - \left[\frac{x^3}{3} - x^2\right]_{0}^{2}$

E. $\int_{2}^{-1} x(x-2)dx$




**Question 22**

If $\int_0^2 f(x)dx = 4$ then $\int_0^2 (f(x)+3)dx$ has a value of

A. 10 B. -2 C. 7 D. 14 E. -7

**Question 23**

The velocity of a particle $t$ seconds after a force is applied is given by

$\frac{dx}{dt} = 3t^3 - 2t^2 + 10,\ 0 \le t \le 5$. The velocity of the particle will be a minimum when the time is

A. $t = 0, \frac{2}{3}$ B. $t = \frac{4}{9}$ C. $t = 0, \frac{4}{9}$ D. $t = \frac{2}{3}$ E. $t = 0$

**Question 24**

$\int \frac{2}{(1-x)^2}dx$ is equal to

A. $-\log_e(1-x)^2 + c$ B. $\frac{-2}{3(1-x)^3} + c$ C. $\frac{-2}{(1-x)} + c$ D. $2\log_e(1-x)^2 + c$ E. $\frac{2}{(1-x)} + c$

**Question 25**

For the following probability distribution of the random variable $X$, the **most likely** value of $X$ is

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $Pr(X = x)$ | 0.2 | 0.15 | $k$ | 0.35 | 0.10 |

A. 3 B. 2 C. 0.2 D. 0.35 E. 4



No. 23



**Question 26**

A young accountant put together a business plan for a client which included the following. The probability of making a profit of \$1000 is 0.2, a profit of \$4,000 is 0.45 and a profit of \$15,000 is 0.35. The profit standard deviation is

A. \$2,687.50 B. \$51.84 C. \$7,250 D. \$5,795.47 E. \$6,666.67

**Question 27**

The probability of winning a prize at the games in *Fun Park* is 0.3. If a child plays 8 games the probability of winning exactly 5 games is

A. $\binom{8}{5}(0.3)^3(0.7)^5$ B. $1-(0.7)^3$ C. ${}^8C_5(0.3)^5(0.7)^3$

D. $(0.3)^5$ E. ${}^8C_3(0.3)^3(0.7)^5$

**Question 28**

A fair coin is tossed 10 times. The probability of obtaining at least 8 heads is

A. 0.9893 B. 0.0547 C. 0.0107 D. 0.9453 E. 0.0439

**Question 29**

The probability of surviving the *Smybitosis* disease is 35%. If 120 people contract the disease the expected number and variance of people who will die from the disease respectively, are

A. 78, 50 B. 42, 50 C. 78, 5 D. 78, 27 E. 42, 27

***Question 30*** and ***31*** refer to the following information.

The time taken to complete mathematics homework, in a given class, is normally distributed with a mean of 25 minutes and a standard deviation of 9 minutes.

**Question 30**

The probability that a randomly chosen student will take more than 12 minutes to complete their homework is

A. 0.9257 B. 0.0044 C. 0.0743 D. 0.9956 E. 0.743

**Question 31**

The fastest 10% of students are rewarded with a chocolate *Freddo* frog. To receive the reward the time the students would have to complete their homework in is closest to

A. 15 mins B. 13 mins C. 39 mins D. 14 mins E. 40 mins

**Question 32**

A chartered catamaran heading to the *Great Barrier Reef* has 5 new and 10 old wetsuits on board. Upon reaching the reef 4 wetsuits are chosen at random from the storage cabin. The probability that less than 2 old wetsuits are selected is

**A.** $\dfrac{\binom{10}{0}\binom{5}{4}+\binom{10}{1}\binom{5}{3}+\binom{10}{2}\binom{5}{2}}{\binom{15}{4}}$

**B.** $\dfrac{\binom{10}{0}\binom{5}{4}\times\binom{10}{1}\binom{5}{3}}{\binom{15}{4}}$

**C.** $1-\dfrac{\binom{10}{4}\binom{5}{0}}{\binom{15}{4}}$

**D.** $1-\dfrac{\binom{10}{0}\binom{5}{4}+\binom{10}{1}\binom{5}{3}}{\binom{15}{4}}$

**E.** $\dfrac{\binom{10}{0}\binom{5}{4}+\binom{10}{1}\binom{5}{3}}{\binom{15}{4}}$

**END OF SECTION A**





## SECTION B

**Specific Instructions for Section B**

Section B consists of 6 questions. There are a total of 18 marks available.
Answer all six questions in the spaces provided in this question and answer booklet.
You need not give numerical answers as decimals unless instructed to do so.
Alternative forms may involve, for example, $\pi$, $e$, surds or fractions.
Full marks may not be given for answers which do not show appropriate working or do not clearly state answers.

**Question 1**

a) For the function $f: [5, \infty) \to R$, $f(x) = \log_e(x-5)$, state the domain of the inverse function $f^{-1}$.

(1 mark)

b) For the function $f: [-1, \infty) \to R, f(x) = 5 - x^2$ state the range.

(1 mark)
**Total 2 marks**

**Question 2**

*Bizyboard* manufactures square cardboard boxes which have a base length of $x$ m. The boxes are assembled such that they have a surface area of $6\text{m}^2$. Let $h$ m be the height of a box.

a) Write an expression for the area $A$ of cardboard required for one box. Hence, express $h$ in terms of $x$.

(1 mark)

b) Write an expression for the volume $V$ of the cardboard box in terms of $x$.

(1 mark)

c) Calculate the maximum box volume.

(1 mark)
**Total 3 marks**

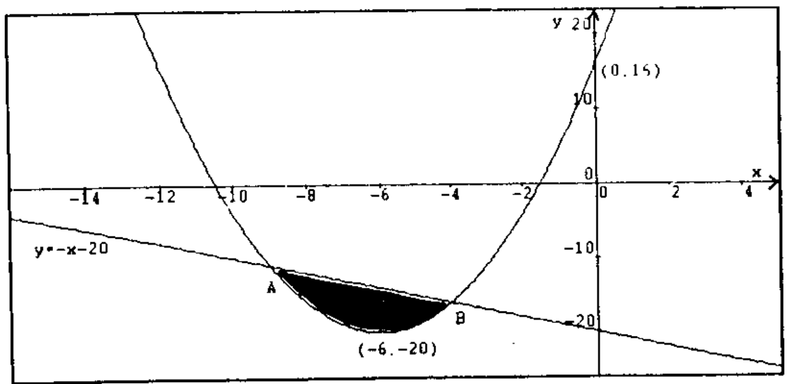
**Question 3**

Determine the $\theta$**-axis** intercepts for the function $f(\theta) = 2\sin\left(2\theta + \frac{\pi}{3}\right) + 1$ in the interval $0 \le \theta \le 2\pi$. Show working and give all answers in the exact form.

(3 marks)
**Total 3 marks**

**Question 4**

In the diagram below, a parabola with a vertex of (-6,-20), is intersected twice by the line $y = -x - 20$.




**i)** Find the equation of the parabola.

(1 mark)





ii) Determine the co-ordinates of the points of intersection **A** and **B**.

(1 mark)

**iii)** Evaluate the area shaded.

(2 marks)
**Total 4 marks**

**Question 5**

The random variable $X$ represents the number of red *Smarties* in a "fun-size" packet of *Smarties*.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| p($x$) | $\frac{m}{3}$ | $2m^2$ | $\frac{m}{2}$ | $\frac{2}{3}$ |

i) Find the value of $m$.

(2 marks)

ii) Find the expected value of $X$.

(1 mark)

iii) Find the $\Pr\left(X \ge 2 \mid X \le 2\right)$

(1 mark)
**Total 4 marks**

**Question 6**

Computer prices at *Computer World* are normally distributed with a mean price of $3,000 and a standard deviation of $900.

a) Find the percentage of computers that would cost less than $2,500 (to the nearest whole number).

(1 mark)

b) From what price is a computer considered to be amongst the 20% most expensive computers in the store?

(1 mark)
**Total 2 marks**

**END OF SECTION B**
**END OF BOOKLET**



No. 23