

IARTV TEST — OCTOBER 2001
MATHEMATICAL METHODS Units 3 and 4
EXAMINATION 1 — ANSWERS & SOLUTIONS

SECTION A: MULTIPLE CHOICE QUESTIONS

1. D	12. D	23. B
2. E	13. B	24. E
3. C	14. D	25. A
4. D	15. A	26. D
5. A	16. E	27. C
6. E	17. B	28. B
7. C	18. C	29. D
8. E	19. D	30. A
9. B	20. A	31. B
10. B	21. D	32. E
11. B	22. A	

SECTION B: SHORT ANSWER QUESTIONS

QUESTION 1

a) $f: [5, \infty) \rightarrow \mathbb{R}, f(x) = \log_e(x-5)$
Inverse function $f^{-1} = e^x + 5$
Domain of $f^{-1} = \mathbb{R}$

b) $f: [-1, \infty) \rightarrow \mathbb{R}, f(x) = 5 - x^2$
Range of $f(x) = (-\infty, 5]$

QUESTION 4

i) $y = (x-6)^2 - 20$

ii) Point A (-9, -11) Point B (-4, -16)

iii) Shaded Area = $\int_{-9}^{-4} (-x - 20 - ((x+6)^2 + 20)) dx$
 $= \frac{125}{6} \text{ units}^2$

QUESTION 2

a) Area $A = 2x^2 - 4xh$

b) $2x^2 - 4xh = 6 \quad h = \frac{3-x^2}{2x}$

c) Volume $V = x^2h = \frac{3x}{2} - \frac{x^3}{2}$

Max Volume: $\frac{dV}{dx} = 0, \frac{3}{2} - \frac{3x^2}{2} = 0, x = \pm 1 \quad h = 1$

Max Volume = $V(1) = 1 \text{ m}^3$

QUESTION 5

i) $\frac{m}{3} + 2m^2 - \frac{m}{2} + \frac{2}{3} = 1$

$12m^2 + 5m - 2 = 0$

$m = \frac{1}{4}$

ii) $E(X) = 2\frac{3}{8}$

iii) $\Pr(X \geq 2 | X \leq 2) = \Pr(X=2) = \frac{1}{8} = \frac{\frac{1}{e}}{\Pr(X \leq 2)} = \frac{1/e}{\frac{1}{3}} = 0.375$

QUESTION 3 Solving for θ -axis intercepts:

$2\sin\left(2\theta + \frac{\pi}{3}\right) + 1 = 0$ in the interval $0 \leq \theta \leq 2\pi$

$\sin\left(2\theta + \frac{\pi}{3}\right) = \frac{-1}{2}, \quad \text{where } \sin\frac{\pi}{6} = \frac{1}{2}$

4 solutions $\left\{ \frac{\pi}{3} \leq 2\theta + \frac{\pi}{3} \leq 4\pi + \frac{\pi}{3} \right\}$:

$2\theta + \frac{\pi}{3} = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

$2\theta = \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{17\pi}{6}, \frac{21\pi}{6}$

$\theta = \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}$

$\theta = \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{7\pi}{4}$

QUESTION 6

Mean value $\bar{x} = \$3,000$

Standard deviation $\sigma = \$900$

a) $\Pr(X < \$2,500): Z = \frac{x - \mu}{\sigma} = \frac{-500}{900} = -0.5556$

$\Pr(Z < -0.5556) = 1 - \Pr(Z < 0.5556)$
 $= 1 - 0.7107$
 $= 0.2893$
 $= 29\%$

b) $\Pr\left(Z < \frac{x - \mu}{\sigma}\right) = 0.8$

$\frac{x - 3000}{900} = 0.8416$

$x = \$3,757.46$