THE HEFFERNAN GROUP

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Student Name……………………………………

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 2

(ANALYSIS TASK)

2001

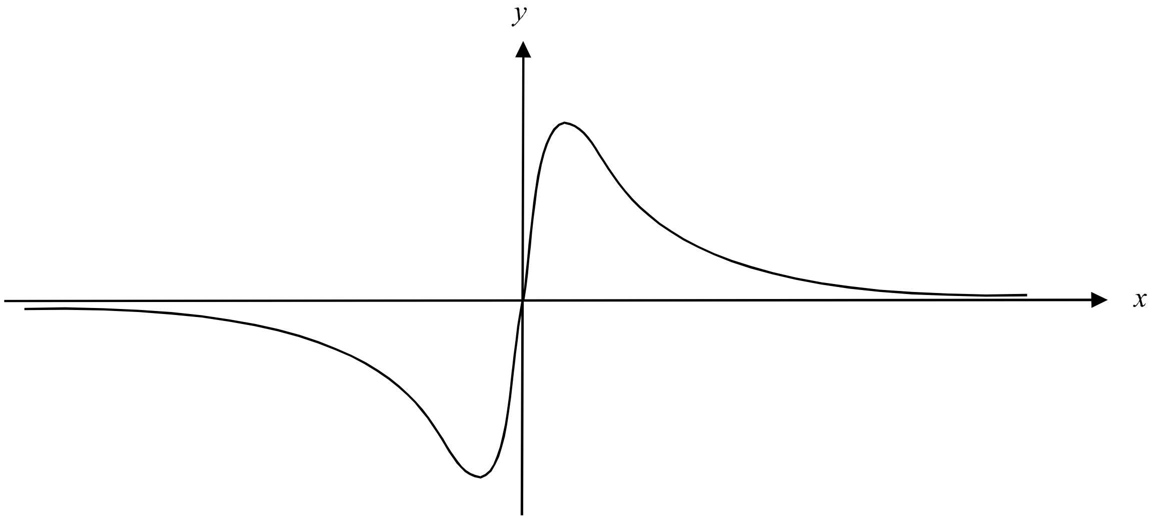
Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of 4 questions.
All questions should be answered.
There is a total of 55 marks available.
The marks allocated to each of the four questions are indicated throughout.
Students may bring up to two A4 pages of pre-written notes into the exam.

Question 1

The graph of the function $f : R \to R,\ f(x) = \dfrac{kx}{(x^2+1)^2}$ is shown below.



The graph of *f* passes through the point $(1,\ 2.5)$.

a. Show that $k = 10$.

1 mark

b. Show algebraically that the *x*-intercept is located at $x = 0$.

1 mark

c. **i.** Show that the derivative of $\frac{10x}{(x^2+1)^2}$ is $\frac{10(1-3x^2)}{(x^2+1)^3}$.

2 marks

ii. Hence find the coordinates of the turning points of the graph. Express your answers in exact form.

3 marks

iii. Hence write down the range of *f*.

1 mark

d. **i.** Find the derivative of $\frac{-5}{x^2+1}$.

1 mark

ii. Hence find the area bounded by the graph of *f*, the *x*-axis and the line $x = 2$.

3 marks

Total 12 marks

Question 2

A CD player has a 'random' function which will randomly choose a track from any of the CD's which are in the CD stacker. Each track is chosen randomly just once and when all tracks on all CD's have been played, the play button must be pushed again.
The CD stacker contains a Dean Martin CD which has 12 tracks and a Cold Chisel CD which has 18 tracks.
Marie listens to 5 tracks whilst the random function is selected.

a. What is the probability that the first track Marie hears is a Dean Martin track?

1 mark

b. What is the probability that 2 of the five tracks Marie listens to are Dean Martin tracks? Express your answer correct to 2 decimal places.

2 marks

c. What is the probability that at least one of these 5 tracks is a Cold Chisel track? Express your answer correct to 4 decimal places.

2 marks

d. Of the 5 tracks Marie listens to, what is the expected number of Dean Martin tracks?

1 mark

The CD player also has a repeat function which, when used in conjunction with the random function, means that any track from any CD in the stacker can be played more than once.

Marie listens to 8 tracks whilst the random function and the repeat function are in operation.

e. What is the probability that half the tracks she listens to are Dean Martin tracks? Express your answer correct to 4 decimal places.

1 mark

f. What is the probability that a particular track will be played twice whilst Marie listens to the 8 tracks? Express your answer correct to 4 decimal places.

2 marks

Marie bought her Cold Chisel CD and her Dean Martin CD from a discount CD shop for $5 each. She was told that the CD's were from a batch which did not have a fixed playing time. The playing time was normally distributed with a mean of 74 minutes and a standard deviation of 4 minutes. This meant that, depending on the total length of the tracks, as indicated on the CD cover, the music could just cut out. In such a case, the shop would replace the CD.
The length of the tracks on the Dean Martin CD total 69 minutes and on the Cold Chisel CD total 76 minutes.

g. Find the probability that Marie had to replace her

i. Cold Chisel CD.

1 mark

ii. Dean Martin CD.

2 marks

h. What is the probability that at least one of the 2 CD's Marie bought at the discount CD shop needed to be replaced? Express your answer to 4 decimal places.

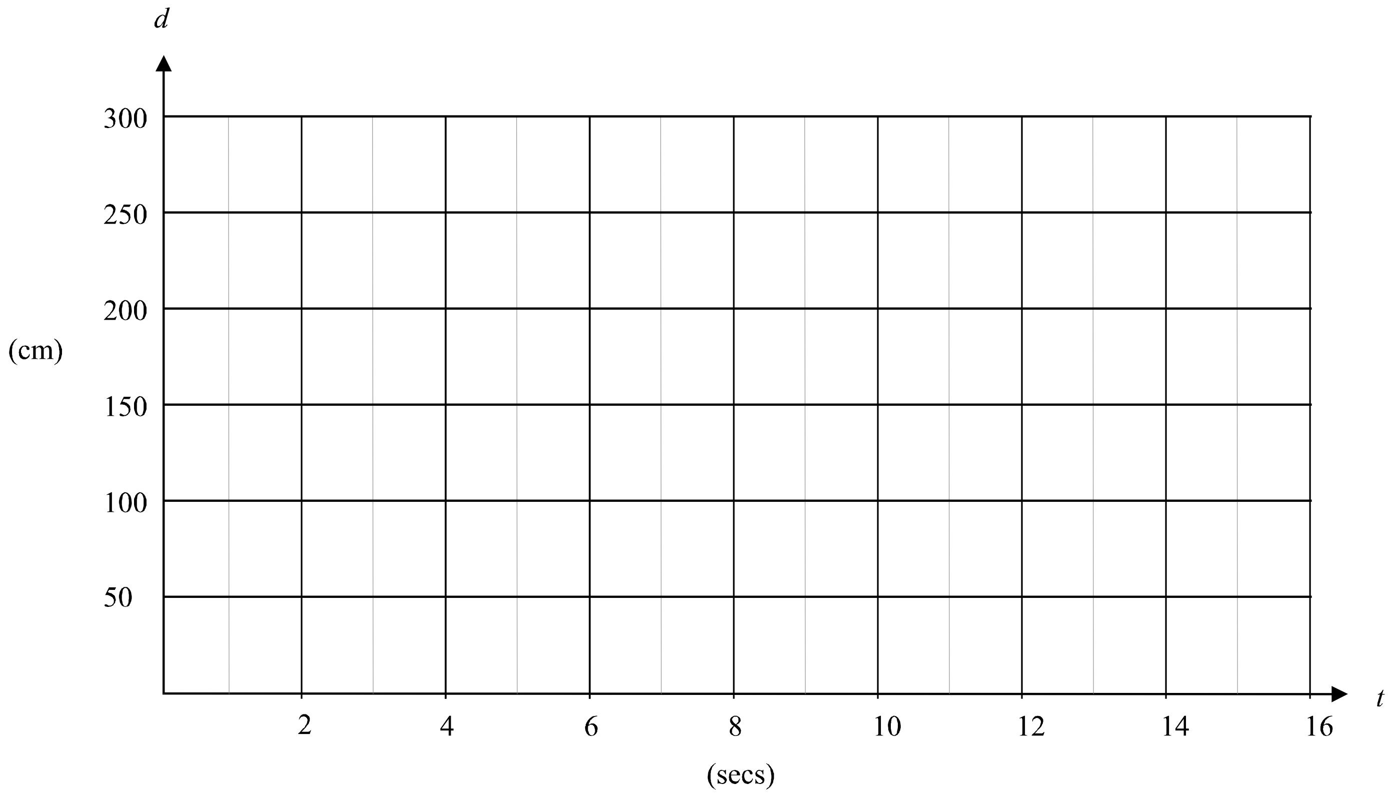
3 marks
Total 15 marks

Question 3

A group of men and women are bell ringers at an old church. Each bell ringer must hold the end of a long rope that is attached to a frame on which a heavy bell is mounted. The bell ringers must pull the rope down in a vertical line, and then, whilst still hanging on to the end of the rope, let the rope recoil before pulling it down again. This process is repeated over and over.

For one of the bell ringers, George, the distance, *d* (cm) of the end of the rope from the floor at time *t* (secs) is given by $d(t) = 100\sin\frac{\pi t}{3} + 150, \quad t \geq 0$

a. On the graph below, sketch the function *d* for $t \in [0,16]$



2 marks

b. How far from the ground is the bottom of Georges rope initially?

1 mark

c. What is the maximum distance of the bottom of the rope from the floor of George's bell?

1 mark

When the bottom of the rope of George's bell is 100 cm from the floor, George's bell rings.

d. Write down

i. the times when George's bell rings during the first 6 seconds.

2 marks

ii. the number of times George's bell rings in the first minute of his bell ringing.

1 mark

Tom is one of the other bell ringers. The distance d (cm) of the bottom of Tom's rope from the floor at time t (secs) is given by $d_T(t) = 100\sqrt{3}\cos\frac{\pi t}{3} + 150, \quad t \geq 0$

e. George and Tom start pulling on their ropes at the same time. Write down the times, during the first 12 seconds of rope pulling that the bottom of their respective ropes would be the same distance from the ground.

2 marks

f. June is another of the bell ringers and has difficulties in ringing the heavy old bell assigned to her.

The distance D (cm) between the bottom of George's rope and the bottom of June's rope at time t (secs) is given by $D(t) = 100\sin\frac{\pi t}{3} - t^2 + 20t + 150 \quad t \in [0, 20]$

i. Find the time, t, when the distance between the bottom of the two ropes is greatest. Express your answer correct to 1 decimal place.

2 marks

ii. Show that this is a maximum value.

2 marks

Total 13 marks

Question 4

The function $g:[-10,\ a] \to R,\ g(x) = 2e^{\frac{x}{2}} - e^{\frac{3x}{2}} + 10$, is used to model the cross-section of a cliff that is used for abseiling.
The *x*-axis represents sea level and each unit of measurement represents one metre.
The base of the cliff is located at the point $(a, 0)$.
The graph of *g* is shown in Figure 1 below.

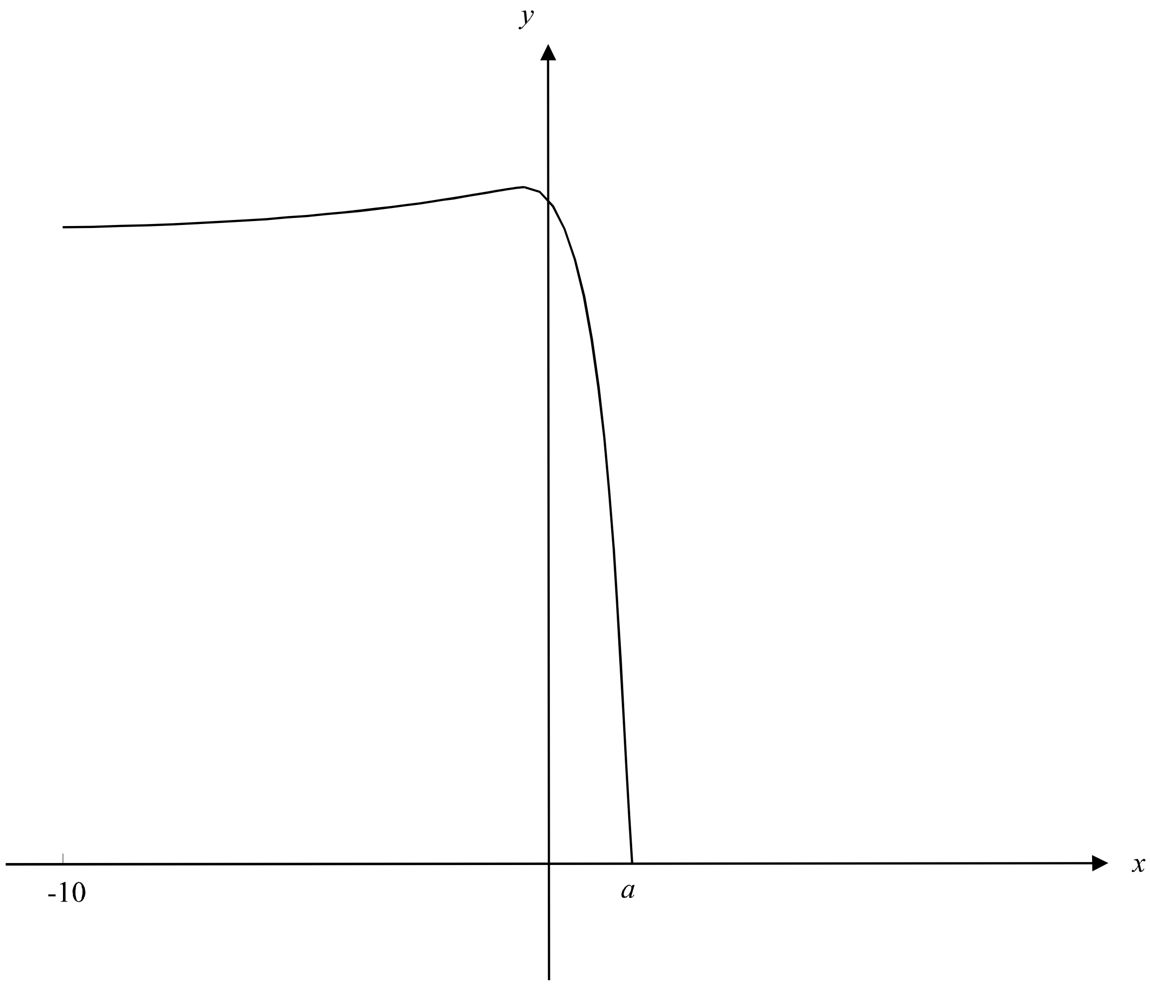


Figure 1

a. Find the coordinates of the *y*-intercept of the graph.

1 mark

b. Find the value of a correct to 1 decimal place.

1 mark

c. **i.** Use calculus to find the x-coordinate of the turning point of the graph. Express your answer as an exact value.

3 marks

ii. The y-coordinate of the turning point is given by $2\sqrt{m} - \sqrt{n} + 10$. Find the values of m and n.

2 marks

A vertical pole stands in a position where $x = -2$. A rope is tied to this pole at one end and is attached to an abseiler at the other end. The rope is held taut, in a straight line, and forms a tangent to the cliff at the point $(0, 11)$ as indicated in Figure 2 below.

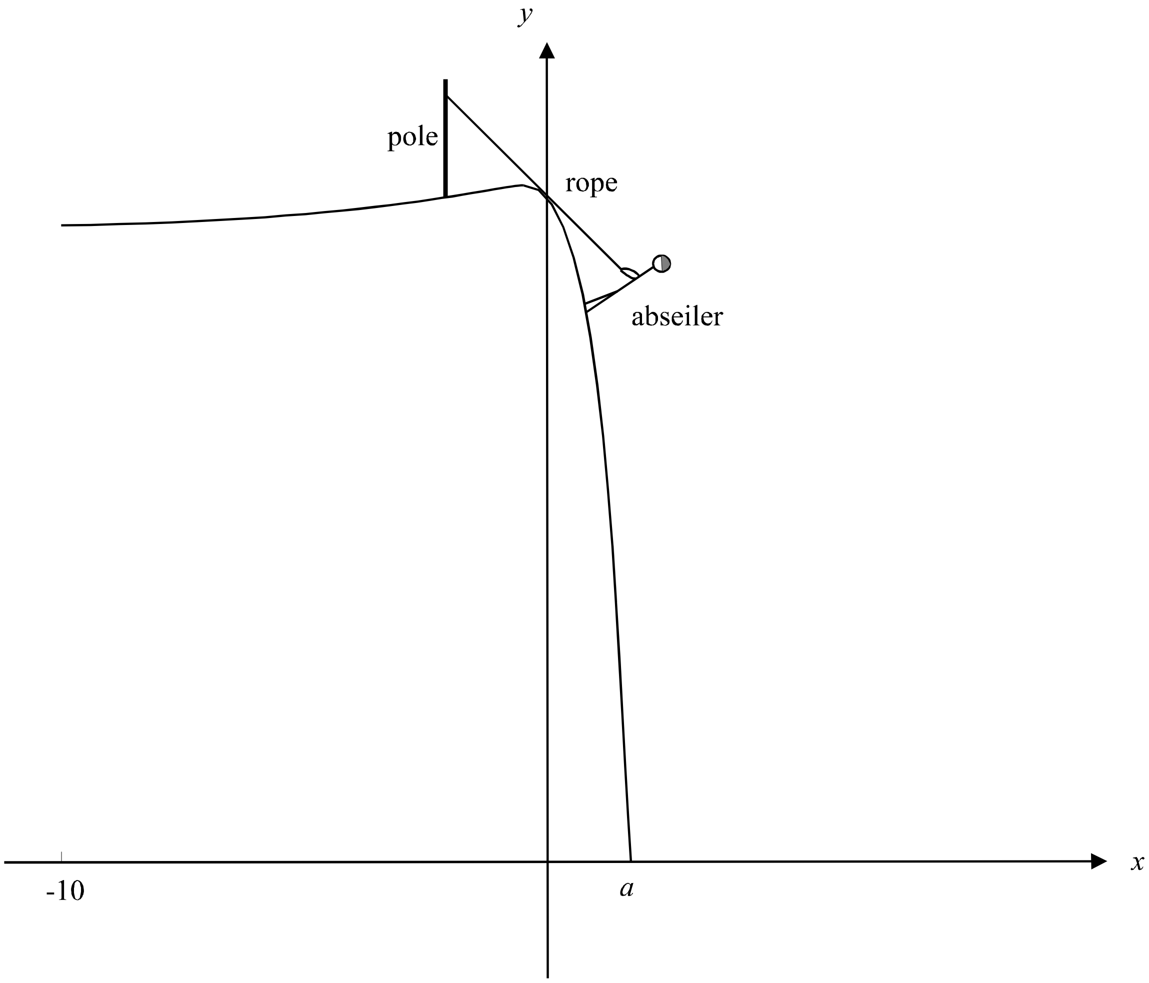


Figure 2

d. Show that the equation formed by this taut rope is given by $y = -\frac{x}{2} + 11$.

2 marks

e. What is the vertical distance from the bottom of the pole to the position on the pole where the taut rope is attached? Express your answer correct to 2 decimal places.

3 marks

f. Find the area enclosed by the pole, the rope and the top of the cliff which is indicated by the shaded area shown in Figure 3 below. Express your answer as an exact value.

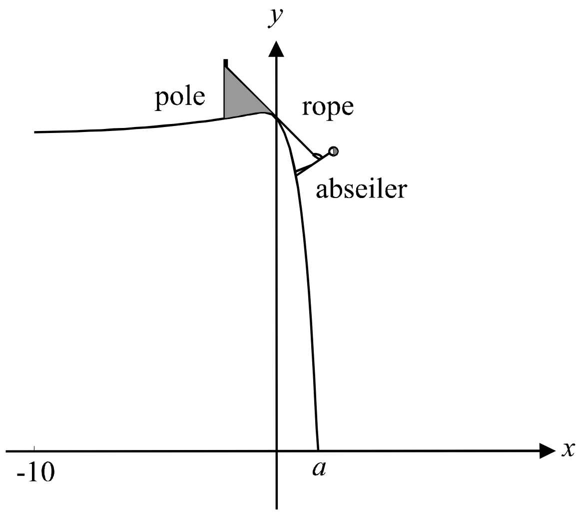


Figure 3

3 marks

Total 15 marks