THE
HEFFERNAN
GROUP

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MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

(FACTS, SKILLS AND APPLICATIONS TASK)

2001

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

This exam consists of Part I and Part II. All questions in Part I and Part II should be answered.
Part I consists of 27 multiple-choice questions, which should be answered on the detachable answer sheet which can be found on page 23 of this exam.
Part II consists of 8 short-answer questions which should be answered in the spaces provided.
Part I begins on page 2 of this exam and is worth **27 marks**.
Part II begins on page 16 of this exam and is worth **23 marks**.
All questions in Part I and Part II should be answered.
There is a total of 50 marks available.
Students may bring up to two A4 pages of pre-written notes into the exam.

PART 1

Question 1

The graph of a function has a horizontal asymptote of $y = 2$ and a vertical asymptote of $x = -1$.
The equation of the function could be

A. $y = \dfrac{1}{x-2} - 1$

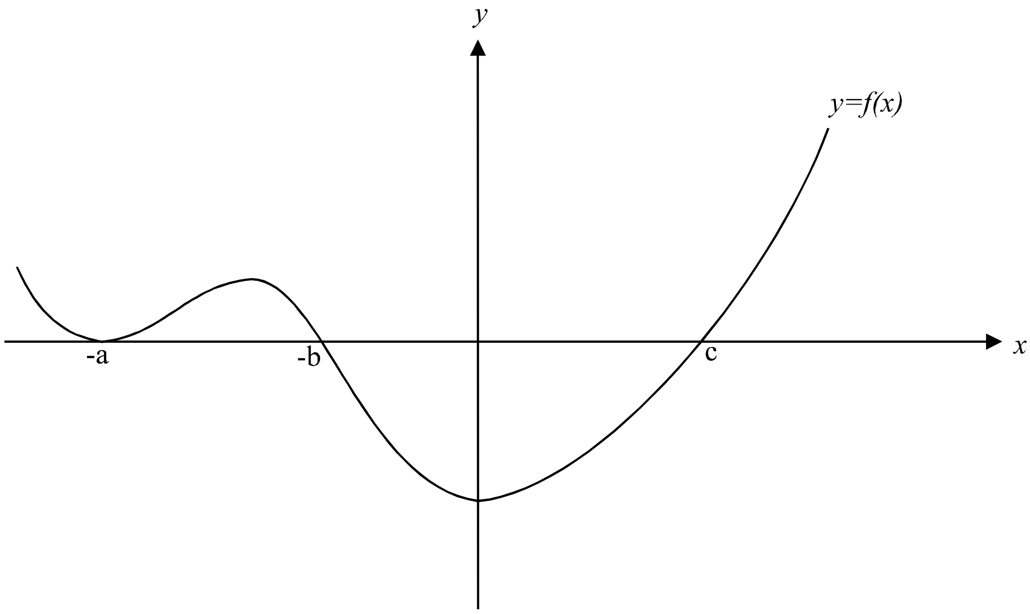
B. $y = \dfrac{2}{x-1} + 2$

C. $y = \dfrac{1}{x+1} + 2$

D. $y = \dfrac{2}{x+1} - 2$

E. $y = \dfrac{2}{x+2} - 1$

Question 2



The rule for the function *f*, shown in the graph above, would be

A. $y = (x-a)(x-b)(x+c)$

B. $y = (x-a)^2(x-b)(x+c)$

C. $y = (x+a)^2(x+b)(x-c)$

D. $y = (x+a)(x+b)(x-c)^2$

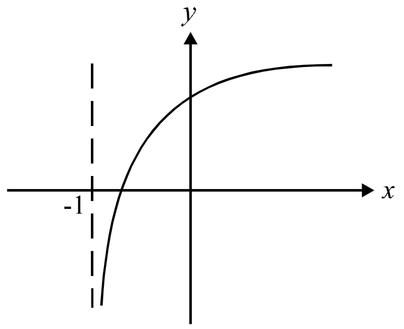
E. $y = (ax+b)^2(x-c)$

Question 3

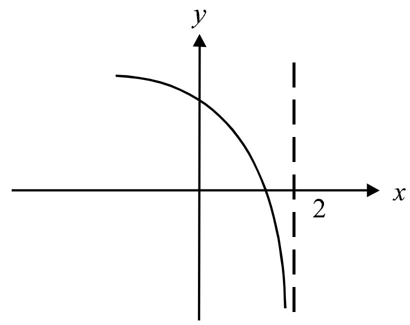
The graph of the function *f* with the rule $y = \log_e(x+1)$ is translated 1 unit left and 2 units up before being reflected in the *y*–axis.

Which one of the following could be the resulting graph?

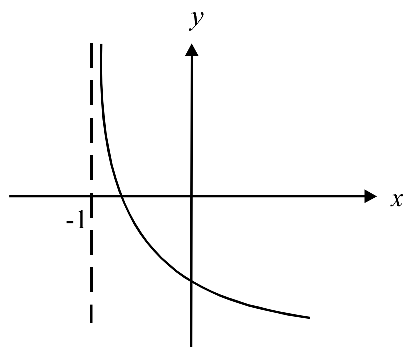
A.



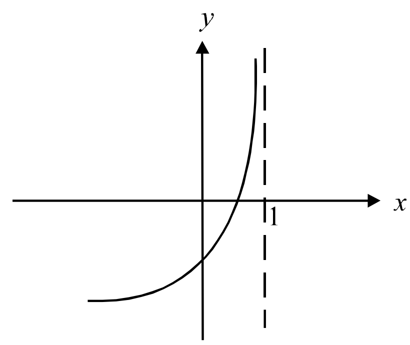
B.



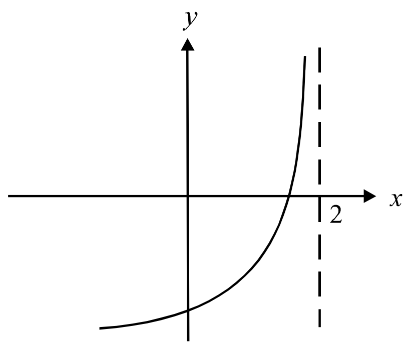
C.



D.



E.



Question 4

Data relating the variables x and y is given in the table below

x	y
1	1.4
2	3.1
3	5.2
4	5.5
5	5.7
6	5.5
7	5.6

If a is a positive constant, then the rule relating x and y could be best modelled by the function

A. $y = ax$

B. $y = -(x-a)^2$

C. $y = \dfrac{1}{ax}$

D. $y = \log_e ax$

E. $y = \cos(ax)$

Question 5

The domain and range of the function g with the rule $y = \dfrac{\sqrt{3}}{x^2}$ are respectively

A. $d_g = (0,\infty)$ $\quad r_g = (-\infty,0) \cup (0,\infty)$

B. $d_g = (-\infty,0) \cup (0,\infty)$ $\quad r_g = (0,\infty)$

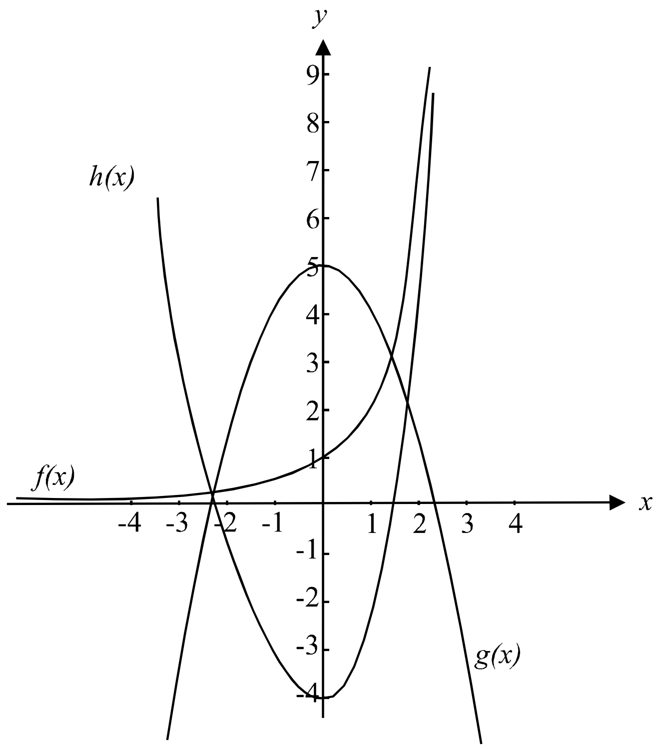
C. $d_g = (0,\infty)$ $\quad r_g = R$

D. $d_g = R$ $\quad r_g = (0,\infty)$

E. $d_g = R$ $\quad r_g = R$

Question 6

The diagram below shows the graphs of the functions $f(x), g(x)$ and $h(x)$.



Which one of the following statements is true?

A. $f(x) = g(x) + h(x)$
B. $h(x) = g(x) + f(x)$
C. $g(x) = f(x) + h(x)$
D. $g(x) = 2f(x)$
E. $h(x) = -(g(x+2))$

Question 7

The function $g:[1,\infty) \to R$ where $g(x) = 3e^{x-1} + 2$ has an inverse function $g^{-1}(x)$.
The function $g^{-1}(x)$ is given by

A. $g^{-1}:[5,\infty) \to R,\quad g^{-1}(x) = 3e^{x-1} + 2$
B. $g^{-1}:[1,\infty) \to R,\quad g^{-1}(x) = \log_e\left(\frac{x-2}{3}\right)$
C. $g^{-1}:[5,\infty) \to R,\quad g^{-1}(x) = 1 + \log_e\left(\frac{x-2}{3}\right)$
D. $g^{-1}:[1,\infty) \to R,\quad g^{-1}(x) = 1 + \log_e\left(\frac{x-2}{3}\right)$
E. $g^{-1}:R \to R,\quad g^{-1}(x) = \frac{x+2}{3} - e^{-1}$

Question 8

The coefficient of the x^3 term in the expansion of $(ax+b)^6$ is

A. $6a^6b$
B. $15a^5b$
C. $15(ab)^3$
D. $20ab^3$
E. $20(ab)^3$

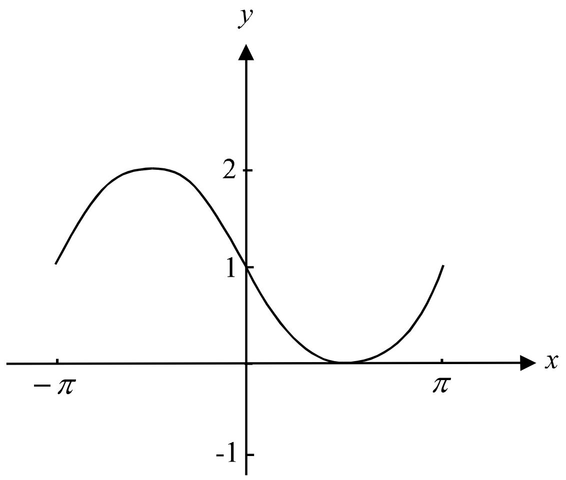
Question 9

If $\frac{1}{2}\log_2 x + 2\log_2 \sqrt{x} - 4\log_2 x = -5$ then x is equal to

A. $-\frac{1}{2}$
B. $\sqrt{2}$
C. 2
D. 4
E. $\sqrt{32}$

Question 10

One cycle of the graph of a circular function is shown in the diagram below.



The amplitude, period and range of the function are, respectively

	amplitude	period	range
A.	1	2π	$[0,2]$
B.	1	π	$[-\pi,\pi]$
C.	2	1	$[0,2]$
D.	2	2π	$[-\pi,\pi]$
E.	2	π	$[-\pi,\pi]$

Question 11

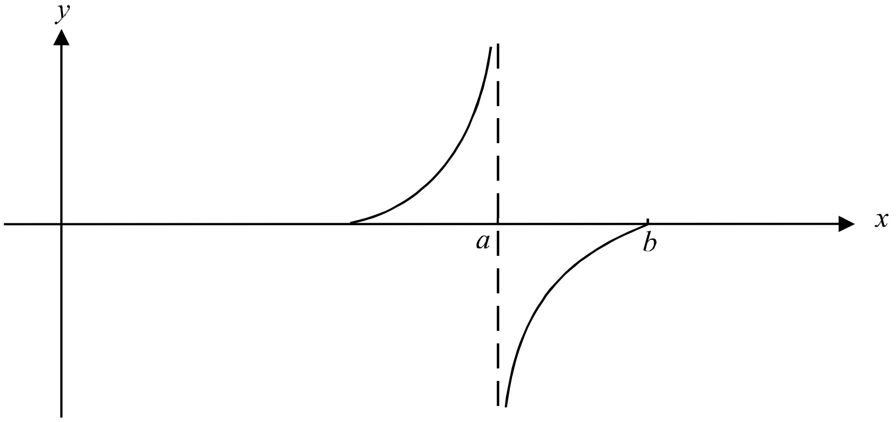
For the function $f:\left[\frac{-\pi}{6},\frac{11\pi}{6}\right]\to R,\ f(x)=a\sin(x-b)+c$ where *a, b* and *c* are positive constants.

The minimum and maximum values of *f* are respectively

	minimum	maximum
A.	$\frac{-\pi}{6}$	$\frac{11\pi}{6}$
B.	$\frac{-a}{2}+c$	$\frac{a}{2}+c$
C.	$c-a$	$a+c$
D.	$a-c$	$a+c$
E.	$a-b$	$b+c$

Question 12

The diagram below shows part of the graph of $y = \tan \frac{x}{2}$



The values of *a* and *b,* respectively, could be

A. $\pi, \ \frac{3\pi}{2}$

B. $2\pi, \ \frac{5\pi}{2}$

C. $2\pi, \ 3\pi$

D. $3\pi, \ 4\pi$

E. $3\pi, \ \frac{7\pi}{2}$

Question 13

The solution(s) to the equation $\sqrt{2}\cos\frac{x}{3} = -1$ in the interval $[0, 3\pi]$ is/are given by

A. $\frac{3\pi}{4}$

B. $\frac{9\pi}{4}$

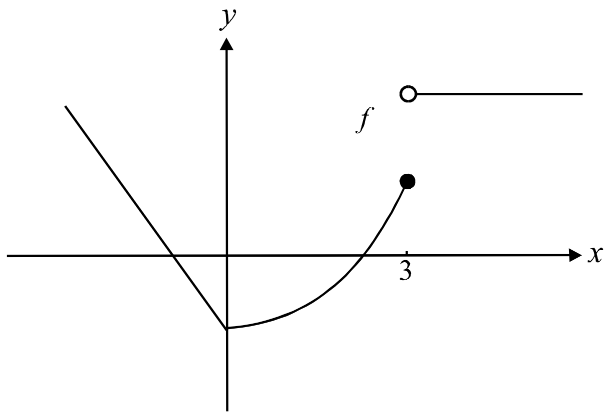
C. $\frac{15\pi}{6}$

D. $\frac{9\pi}{4}, \frac{15\pi}{4}$

E. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

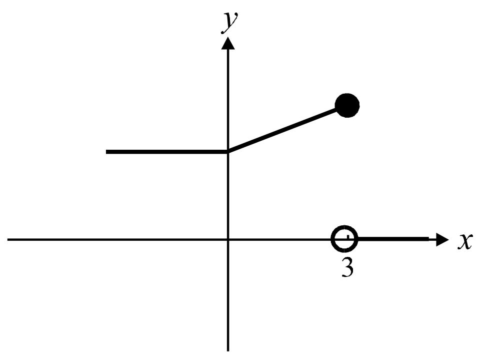
Question 14

The graph of the function *f* is shown below

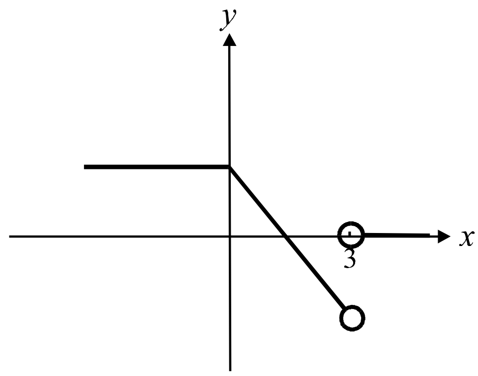


The graph of the function $y = f'(x)$ could be

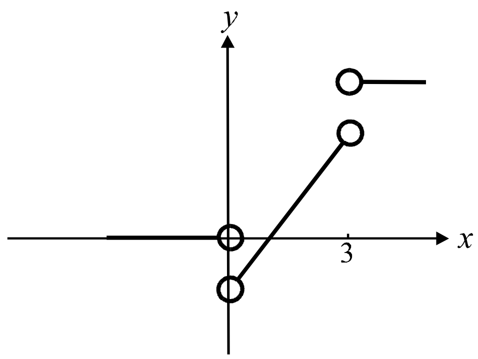
A.



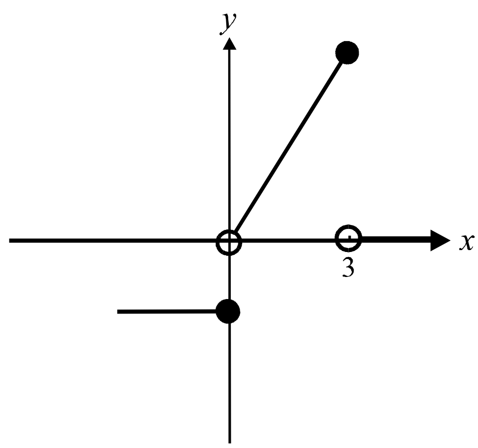
B.



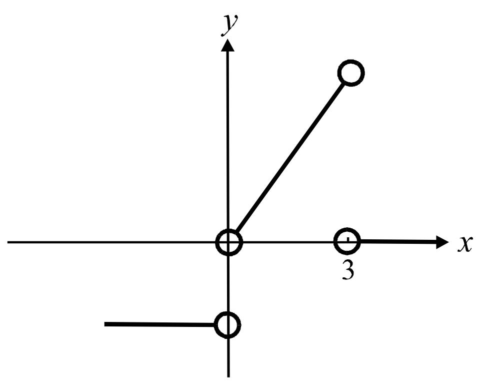
C.



D.



E.



Question 15

If $y = \sqrt{\tan 2x}$ then $\frac{dy}{dx}$ is equal to

A. $\sqrt{2\sec^2 2x}$
B. $\sec 2x$
C. $\frac{1}{2\sqrt{\tan 2x}}$
D. $\frac{\sec^2 2x}{\sqrt{\tan 2x}}$
E. $\frac{\sec^2 2x}{2\sqrt{\tan 2x}}$

Question 16

If $f(x) = 3x^2 \log_e(2x)$ then $f'(x)$ is equal to

A. 6
B. $6x\log_e(2x)$
C. $12x\log_e(2x)$
D. $6x\log_e(2x) + 3x$
E. $6x\log_e(2x) + \frac{3x}{2}$

Question 17

If $f(x) = e^{\cos(2x)}$ then $f'(\pi)$ would equal

A. -1
B. 0
C. 1
D. e
E. e^{π}

Question 18

The temperature T (degrees Celsius), of a liquid at time t (hours) is given by

$$T = 2\sqrt{t}\left(t^3 - 5t\right) + 12 \qquad t \in [0,3]$$

The minimum temperature of the liquid in degrees Celsius is closest to

A. 1.4
B. 1.5
C. 1.8
D. 1.9
E. 2.0

Question 19

The area of the region bounded by the x-axis, the line with equation $x = 4$ and the curve with equation $y = \log_e x$ can be approximated using the left endpoint method.

Using rectangles with a width of 1 unit, this approximate area in square units would be

A. $\log_e 2 + \log_e 3$
B. $\log_e 3 + \log_e 4$
C. $\log_e 2 + \log_e 3 + \log_e 4$
D. $1 + \log_e 2 + \log_e 3$
E. $1 + \log_e 2 + \log_e 3 + \log_e 4$

Question 20

$\int \left(\sin(3x) + e^{-x}\right)dx$ is equal to

A. $-\frac{1}{3}\cos 3x - e^{-x} + c$
B. $\frac{1}{3}\cos 3x + e^{-x} + c$
C. $-3\cos 3x - e^{x} + c$
D. $\cos 3x - e^{-2x} + c$
E. $3\cos 3x - e^{-x} + c$

Question 21

An antiderivative of $\dfrac{x^3+1}{\sqrt{x}}, x \neq 0$ is

A. $\dfrac{x^4+4x}{2x^2}$

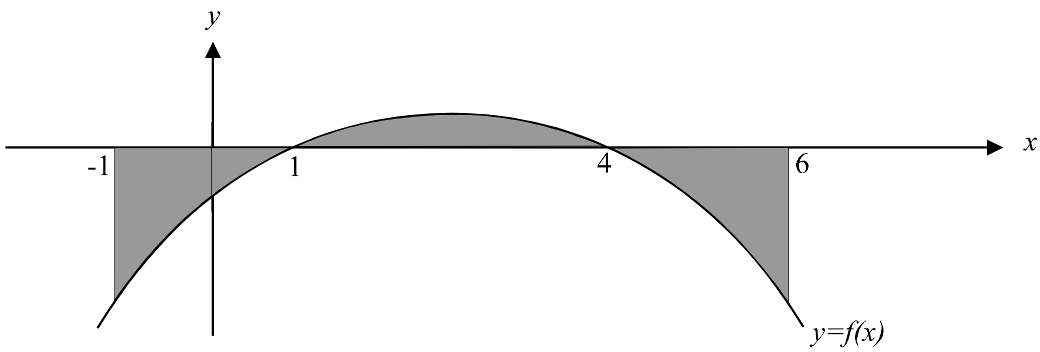
B. $\dfrac{2}{7}x^{\frac{7}{2}}+2x^{\frac{1}{2}}$

C. $\dfrac{2}{5}x^{\frac{7}{2}}-2x^{\frac{1}{2}}$

D. $\dfrac{2}{3}x^{\frac{3}{2}}-\dfrac{2}{3}x^{\frac{-3}{2}}$

E. $\dfrac{5}{2}x^{\frac{3}{2}}-\dfrac{1}{2x^{\frac{3}{2}}}$

Question 22



The total shaded area shown in the diagram above is given by

A. $\int_{-1}^{6} f(x)dx$

B. $-\int_{-1}^{0} f(x)dx + \int_{0}^{6} f(x)dx$

C. $-\int_{-1}^{1} f(x)dx + \int_{1}^{4} f(x)dx - \int_{4}^{6} f(x)dx$

D. $\int_{-1}^{1} f(x)dx - \int_{1}^{4} f(x)dx + \int_{4}^{6} f(x)dx$

E. $\int_{-1}^{0} f(x)dx - \int_{0}^{1} f(x)dx + \int_{1}^{4} f(x)dx - \int_{4}^{6} f(x)dx$

Question 23

The probability distribution for the discrete random variable X is given by

x	0	1	2	3
$\Pr(X = x)$	$2k$	k	$3k$	$4k$

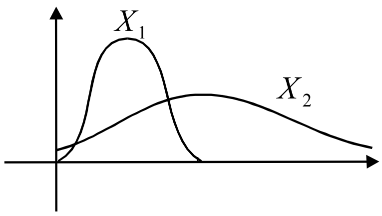
$\Pr(X = 1)$ is

A. 0.05
B. 0.1
C. 0.3
D. 5
E. 10

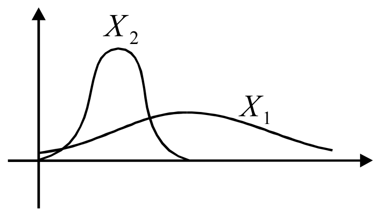
Question 24

The mean of the normal distribution X_1 is twice that of the normal distribution X_2.
The variance of the normal distribution X_1 is half that of the normal distribution X_2.
Which one of the following diagrams could show the distributions of X_1 and X_2?

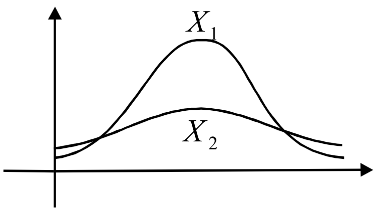
A.



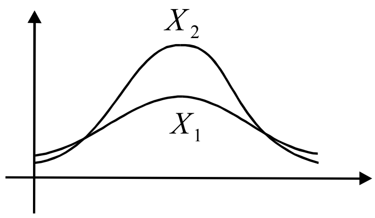
B.



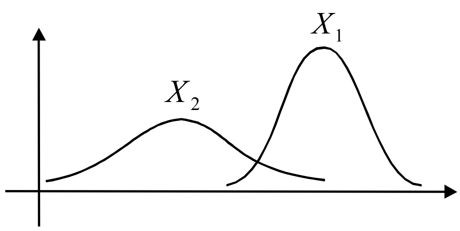
C.



D.



E.



Question 25

A pharmacist's assistant accidentally mixes 100 panadeine tablets with 50 panadeine forte tablets and places them in a bottle. The probability that of the first 10 tablets taken by the purchaser of the bottle, 5 of them will be panadeine forte tablets is given by

A. $\left(\frac{5}{150}\right)^{10}$

B. $C_{10}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5$

C. ${}^{10}C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5$

D. $\frac{{}^{50}C_5\,{}^{100}C_5}{{}^{150}C_{10}}$

E. $\frac{{}^{50}C_5\,{}^{100}C_5}{{}^{100}C_{10}}$

Question 26

Jack has three basketball commitments each week. He has two training sessions and 1 basketball match. The probability that he is late to exactly 2 of these in a week is given by

$$^{3}C_2(0.4)^2(0.6)^1$$

The mean and variance of the number of times in a week that Jack is late for his basketball commitments are given respectively by

	mean	variance
A.	0.8	0.48
B.	1.2	0.48
C.	1.2	0.72
D.	1.8	0.72
E.	3	0.4

Question 27

A lagoon contains just carp and eels. It is generally accepted that there are about 3 carp for every 1 eel in the lagoon.
Two boys with a small net, which can only hold 1 carp or 1 eel sit at the end of a pontoon and run their net through the water. They return any carp or eels that they trap in their net back to the lagoon before trying again.
The probability that in 5 successive attempts the boys will net exactly 2 eels is given by

A. $\left(\frac{1}{3}\right)^2$

B. ${}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$

C. ${}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3$

D. $\dfrac{{}^5C_2\,{}^5C_3}{{}^5C_5}$

E. $\dfrac{{}^{75}C_2\,{}^{25}C_3}{{}^{100}C_5}$

PART II

Question 1

The number of goals scored by Max in matches during the 2000 season are given in the table below.

Number of goals (x)	0	1	2	3
Number of matches during which x goals are scored (f)	2	8	4	6

a. Construct the corresponding probability distribution for X, the number of goals scored by Max in a match during the 2000 season

Number of goals (x)				
Pr (X = x)				

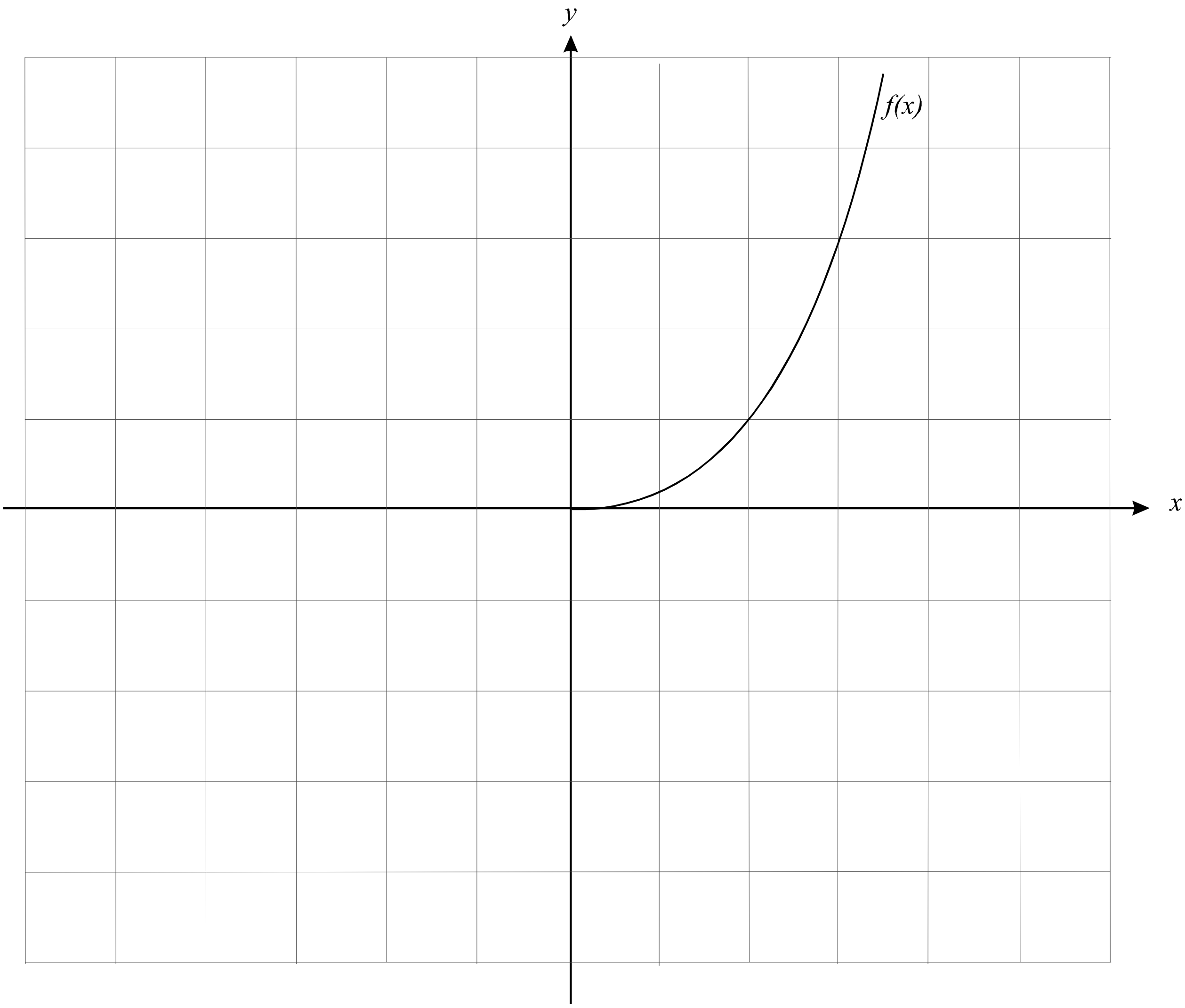
1 mark

b. Calculate $E(X)$, the expected value of X.

1 mark
Total 2 marks

Question 2

The graph of $f(x)$ is shown below



On the same set of axes, sketch

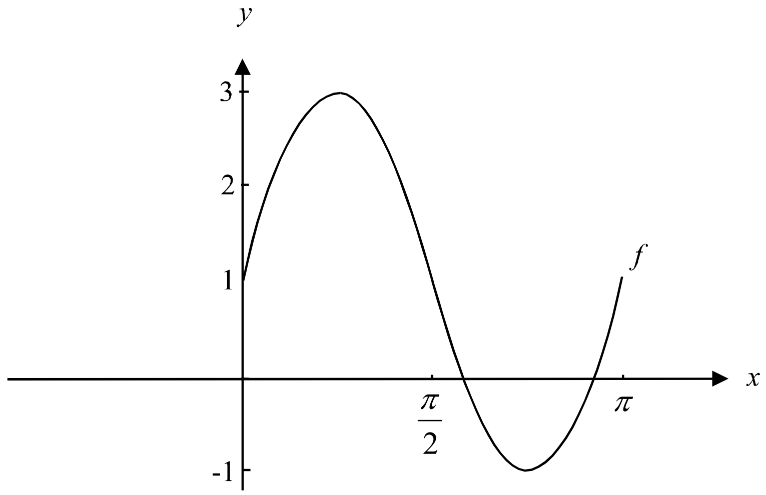
i. $-f(x)$
ii. $f(-x)$
iii. $f^{-1}(x)$

Label each graph clearly.

3 marks

Question 3

One complete cycle of the graph of the function *f* is shown in the diagram below.



Write down the rule for *f* given that *f* is a

i. sin function

ii. cos function

1 + 2 = 3 marks

Question 4

The depth, D (metres) of a diver, t (seconds) after he enters the water is given by

$$D\,(t) = (t^2 - 5t)e^{0.1t} \qquad t \in [0,5]$$

a. Find the average rate of change of depth of the diver during the interval $t \in [1,2]$. Express your answer correct to 2 decimal places.

1 mark

b. Find the instantaneous rate of change of depth at time $t = 2$ seconds. Express your answer to 2 decimal places.

2 marks

Question 5

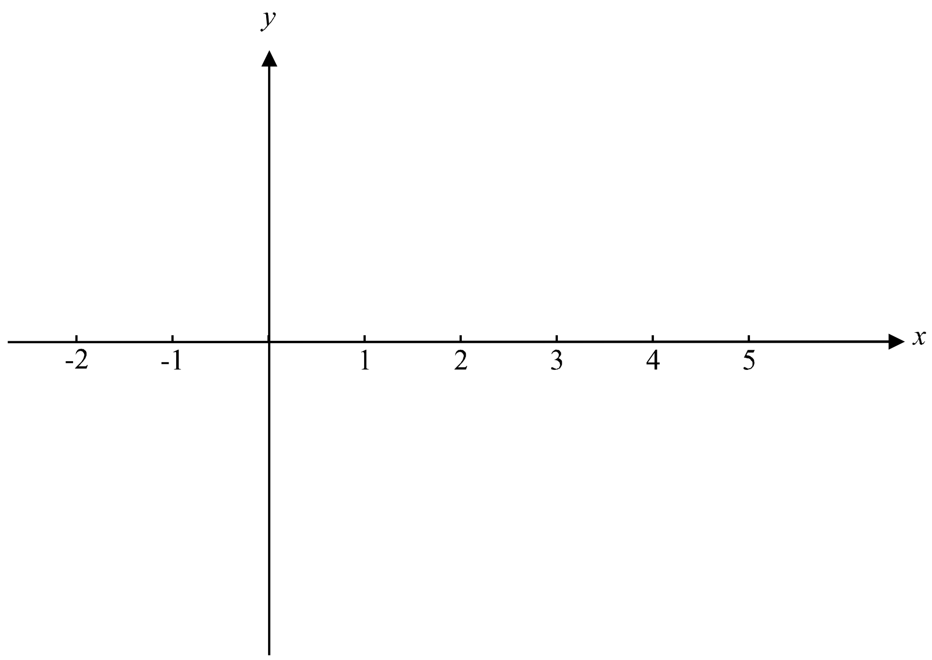
On the set of axes below sketch the graph of the discontinuous function, which has the following properties:

$f(0)=0$ $\quad$ $f'(2)=0$

$f(4)=0$ $\quad$ $f'(x)=0$ for $x \in (-\infty,0)$

$f(-1)>0$ $\quad$ $f'(x)<0$ for $x \in (0,2)$

$f'(x)>0$ for $x \in (2,\infty)$



3 marks

Question 6

The points $P(2, \log_e 5)$ and $Q(2.01, Y)$ lie very close to each other on the curve with equation $y = \log_e(x^2 + 1)$.

Use the approximation $f(x+h) \approx f(x) + hf'(x)$ to find the value of Y.

2 marks

Question 7

The weight in grams of Number 16 size frozen chickens is normally distributed with a mean of 1600g and a standard deviation of 12 grams.
A commercial freezer at the Farmwide chicken factory contains only Number 16 size frozen chickens.

a. Calculate the probability that a randomly chosen frozen chicken from this freezer will weigh between 1582g and 1642g. Express your answer correct to 3 decimal places.

2 marks

b. Each hour a worker randomly selects a Number 16 chicken from this freezer, weighs it and returns it to the freezer. What is the probability that, after 3 hours, the worker will have found 2 chickens that weigh less than 1612g. Express your answer correct to 4 decimal places.

2 marks

Question 8

Use calculus to find the area enclosed by the x-axis, the lines with equations $x = 1$ and $x = 2$ and the graph of the equation $y = \dfrac{1}{(3x-2)^{\frac{3}{2}}}$.

3 marks

MATHEMATICAL METHODS

TRIAL EXAMINATION 1

2001

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:..

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ■ C D E

The answer selected is B. Only one answer should be selected.

1.	A	B	C	D	E	10.	A	B	C	D	E	19.	A	B	C	D	E
2.	A	B	C	D	E	11.	A	B	C	D	E	20.	A	B	C	D	E
3.	A	B	C	D	E	12.	A	B	C	D	E	21.	A	B	C	D	E
4.	A	B	C	D	E	13.	A	B	C	D	E	22.	A	B	C	D	E
5.	A	B	C	D	E	14.	A	B	C	D	E	23.	A	B	C	D	E
6.	A	B	C	D	E	15.	A	B	C	D	E	24.	A	B	C	D	E
7.	A	B	C	D	E	16.	A	B	C	D	E	25.	A	B	C	D	E
8.	A	B	C	D	E	17.	A	B	C	D	E	26.	A	B	C	D	E
9.	A	B	C	D	E	18.	A	B	C	D	E	27.	A	B	C	D	E