SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures										Letter
Words										

Victorian Certificate of Education 2000

MATHEMATICAL METHODS

Written examination 2 (Analysis Task)

Monday 6 November 2000: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*
4	4

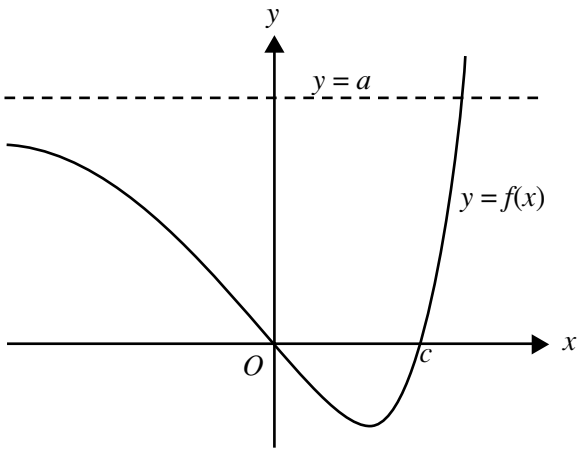
Directions to students

Materials
Question and answer book of 14 pages.
There is a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the book.
You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set square and aids for curve sketching.

The task
Detach the formula sheet from the centre of this book during reading time.
Ensure that you write your **student number** in the space provided on the front of this book.
Answer **all** questions.
The marks allotted to each part of each question are indicated at the end of each part.
There is a total of 55 marks available for the task.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
Where an exact answer is required to a question, appropriate working must be shown and calculus must be used to evaluate derivatives and definite integrals.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
All written responses should be in English.

Question 1

The graph of the function $f:R \rightarrow R, f(x) = e^{2x} - 2ke^{x} + 3$ is shown below.



The graph of f has a horizontal asymptote $y = a$.

The graph of f passes through the origin and the point $(c, 0)$.

a. Write down the exact value of a.

1 mark

b. Show that $k = 2$.

2 marks

c. Use calculus to find the exact values of the coordinates of the turning point.

4 marks

d. **i.** Find the exact value of c.

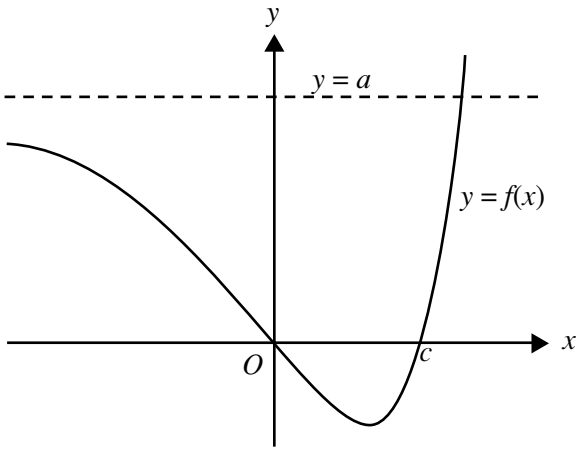
3 marks

ii. Hence use calculus to find the exact value of the area of the region bounded by the graph of f and the x-axis.

3 marks

e. Let g be the function whose graph is the reflection of the graph of f in the y-axis.

i. Sketch the graph of g on the axes below.



2 marks

ii. Write down the rule for g.

1 mark

Total 16 marks

Working space

Question 2

Pedro fishes in an area where there are large numbers of salmon. It is known that the lengths of salmon in this area are normally distributed with mean 36.0 cm and standard deviation 2.5 cm. Assume that all salmon are equally likely to be caught.

Salmon with lengths in the top 15% of lengths of salmon in the area are called gourmet salmon.

a. What is the minimum length of a gourmet salmon, in centimetres, correct to one decimal place?

2 marks

b. If Pedro catches 20 salmon, what is the probability, correct to three decimal places, that at least one will be a gourmet salmon?

3 marks

c. Pedro knows that if he catches as many salmon as he did last Tuesday, the probability of catching at least one gourmet salmon is approximately 0.68.

How many salmon did he catch last Tuesday?

2 marks

Salmon which are shorter than 33.0 cm are declared to be undersized by the government. If such salmon are caught, they must be returned to the water.

d. What is the probability, correct to three decimal places, that a salmon caught at random will be undersized?

2 marks

e. Pedro had been fishing for a week. He threw all undersized salmon back and returned with 100 salmon in his icebox. What is the expected number of gourmet salmon in his icebox? Give your answer correct to the nearest whole number.

2 marks

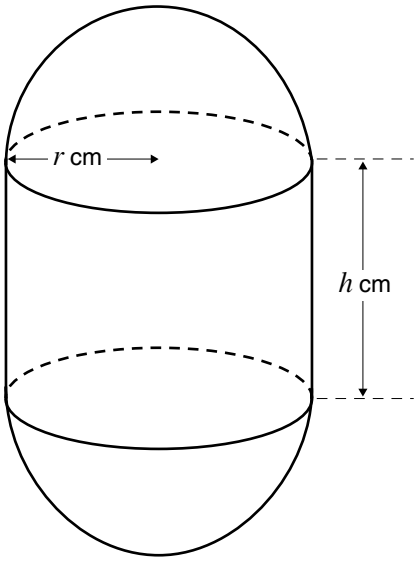
f. Pedro's friend Jose returns from fishing with 20 salmon of which 4 are undersized. The government inspector is waiting for him, and takes a random sample of three salmon from Jose's catch. If more than one of the salmon are undersized, Jose must pay a fine. What is the probability, correct to three decimal places, that Jose will be fined?

3 marks

Total 14 marks

Question 3

Victoria Jones wants to construct a time capsule in which to bury some of her treasures. The time capsule will be a right circular cylinder of height h cm, and radius r cm, with hemispherical caps of radius r cm on each end, as shown in the diagram.



Let the total volume of the capsule be V cm^3.

a. Express V in terms of r and h.

1 mark

b. The total volume of the capsule will be 8000 cm^3.

i. Show that $h = \frac{8000}{\pi r^2} - \frac{4r}{3}$.

2 marks

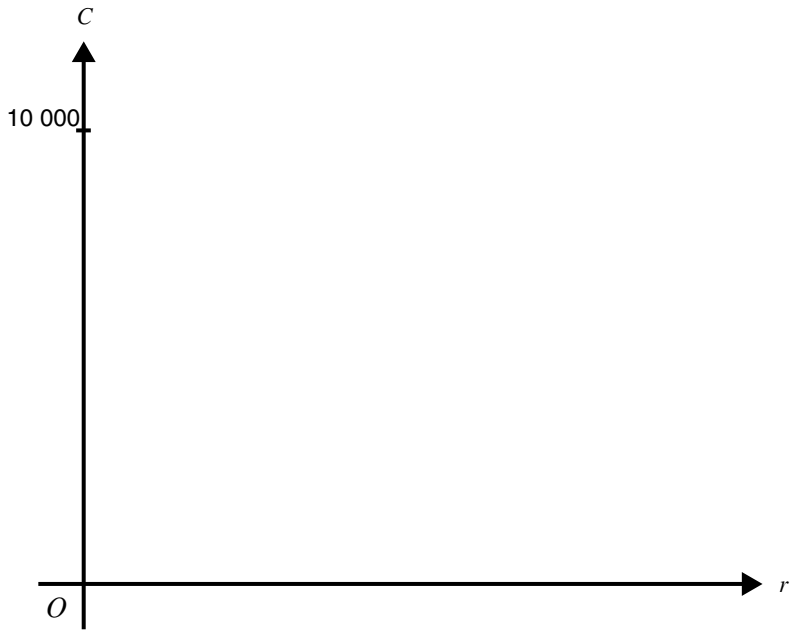
ii. The values which r may take lie in an interval. Find the end points of this interval, correct to two decimal places.

2 marks

c. The material for the cylindrical part of the capsule costs 2 cents per cm^2 of surface. The material for the hemispherical caps costs 3 cents per cm^2 of surface. [The surface area of a sphere of radius r is $4\pi r^2$.]
Find an expression for C cents, the total cost of the materials for the capsule, in terms of r.

2 marks

d. Sketch the graph of C over an appropriate domain on the axes below.
Label any horizontal or vertical asymptote with its equation.
You are not required to show the co-ordinates of any turning point.



3 marks

e. Use calculus to find the value of r, correct to two decimal places, for which C is a minimum. [You do not need to justify that the value you find is a minimum.]

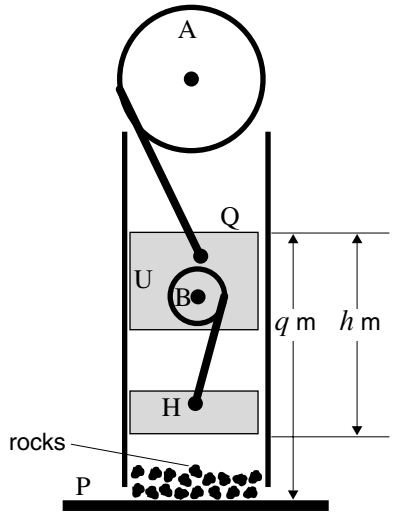
3 marks

Total 13 marks

Working space

Question 4

A device for crushing rock is shown in the diagram below. It consists of a steel platform (P) on which the rock is placed and a machine which raises and lowers a heavy 'hammer' (H). The wheel A rotates, causing the upper block U to move up and down. The other wheel B, attached to the block U, rotates independently causing the hammer H to move up and down.



Q is the top of block U. The distance, q m, between Q and the platform P is modelled by the formula

$$q = -2\cos(at) + b,$$

where t is the time in minutes and a and b are constants.

When $t = 0$, Q is at its lowest point, 3 m above the platform.

Wheel A rotates at a rate of 1 revolution per minute.

a. Show that $a = 2\pi$ and $b = 5$.

2 marks

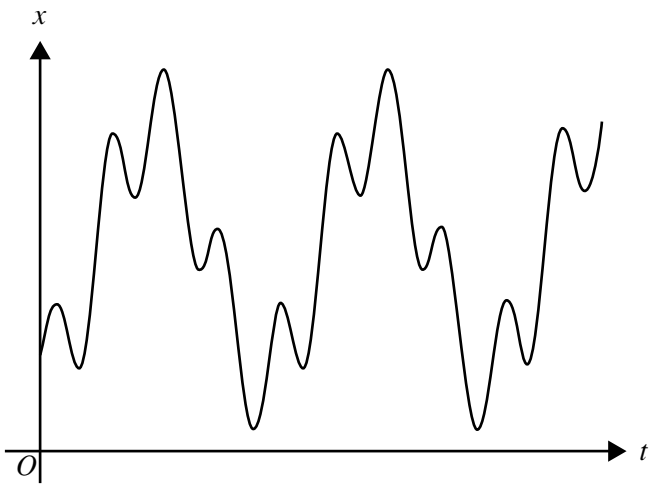
Wheel B rotates at a rate of 4 revolutions per minute. The distance, h m, between the bottom of the hammer and Q at time t minutes is modelled by the formula $h = -\sin(8\pi t) + 2$.

Let the distance between the bottom of the hammer and the platform at time t minutes be x m.

b. Show that $x = -2\cos(2\pi t) + \sin(8\pi t) + 3$.

1 mark

A section of the graph of x as a function of t is shown.



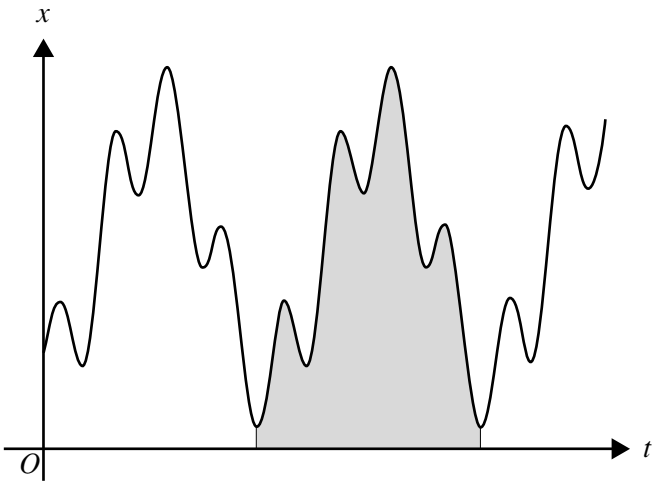
c. Use calculus to find the rate of change of x with respect to t when $t = 2$.
Give your answer correct to one decimal place.

2 marks

d. Find the first time after $t = 0$, correct to the nearest one-hundredth of a minute, when this model predicts that the hammer will be at its least distance from the platform.
Find this least distance, correct to the nearest *millimetre*.

3 marks

e. The width of the shaded region shown is the time taken for one cycle of x. Use calculus to find the exact area of the shaded region.



4 marks

Total 12 marks

END OF QUESTION AND ANSWER BOOK


Victoria
The Place To Be

MATHEMATICAL METHODS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of a triangle: $\frac{1}{2}bc\sin A$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax}dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x} \qquad \int \frac{1}{x}dx = \log_e x + c, \text{for } x > 0$$

$$\frac{d}{dx}\left(\sin ax\right) = a\cos ax \qquad \int \sin ax\ dx = -\frac{1}{a}\cos ax + c$$

$$\frac{d}{dx}\left(\cos ax\right) = -a\sin ax \qquad \int \cos ax\ dx = \frac{1}{a}\sin ax + c$$

$$\frac{d}{dx}\left(\tan ax\right) = \frac{a}{\cos^2 ax} = a\sec^2 ax$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Statistics and Probability

$\Pr(A) = 1 - \Pr(A')$ $\qquad$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$ $\qquad$ variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Discrete distributions			
	$\Pr(X = x)$	mean	variance
general	$p(x)$	$\mu = \Sigma\, x\, p(x)$	$\sigma^2 = \Sigma\,(x-\mu)^2 p(x)$ $= \Sigma\, x^2 p(x) - \mu^2$
binomial	${}^nC_x\, p^x (1-p)^{n-x}$	np	$np(1-p)$
hypergeometric	$\frac{{}^DC_x\ {}^{N-D}C_{n-x}}{{}^NC_n}$	$n\frac{D}{N}$	$n\frac{D}{N}\left(1-\frac{D}{N}\right)\frac{N-n}{N-1}$
Continuous distributions			
normal	If X is distributed $\mathrm{N}(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma}$, then Z is distributed N(0, 1).		

Table 1 Normal distribution – cdf

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7793	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

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