Mathematical Methods GA 3: Written examination 2

GENERAL COMMENTS

The number of students presenting for Mathematical Methods Examination 2 in 2000 was 16 875, a decrease of about 2.7% from the 17 327 who sat in 1999.

While changes to content in the revised course were generally minor refinements from the previous study (except in the Probability Area of study), there was a significant change in the expected use of graphics calculators in parts of some questions. One of the new aspects of this examination was for students to know when to use the calculator and when not to. In this respect, students should be aware of instructions in the wording of questions, such as 'Use calculus to ...' and 'Find the exact value of ...' 'Use calculus to ...' requires that if the question involves a derivative, that derivative must be shown or, if it involves an antiderivative, even in a definite integral, that antiderivative must be shown. If an exact answer is required, the calculator is unlikely to produce it. There was evidence that some students were not able to use their calculator efficiently to produce an answer to the required number of decimal places; for example, there are better ways to find the minimum required in Question 4d than using a 'trace' procedure.

Total marks awarded for the exam ranged from zero to 55. The paper provided opportunities for good students to show what they knew and while there were some suggestions that the paper was harder than last year's, the raw score required for a high grade was higher than last year, indicating that good students were able to earn more marks on the paper. There were excellent papers presented by very capable students who achieved perfect or near-perfect scores. There was the usual number of students who were unable to obtain more than a few marks, despite some marks being available for work that would have been covered earlier.

Many students lost marks because they:

- did not answer the question asked
- gave decimal answers when an exact answer was required
- gave the wrong number of decimal places
- misread the question in other ways
- were not sufficiently careful with their working.

For example:

- Question 3d. Many students ignored the instructions to graph the function *over an appropriate domain* and to label asymptotes *with their equations*.
- Question 2b. It was common to see 1–0.15 evaluated as 0.75.

The questions where students were required to *show* a given result were generally handled poorly. This is a skill which should be taught and practised, and has been commented on in previous reports. A reasonable answer to Question 1b, would be something like the following:

$f(0) = 0 = e^0 - 2ke^0 + 3$
so $4 - 2k = 0$
and $k = 2$.

Similarly, in Question 4a:

Period = 1, $\frac{2p}{a} = 1$ so $a = 2p$.
When $t = 0$, $q = 3$, so $3 = -2\cos(0) + b$
Then $3 = -2 + b$
So $b = 5$

Too often, in Question 4a, students substituted in the given values for a and b, then showing that 0 = 0 without a word of explanation. In fact, it is ***not*** possible to show that $a = 2p$ by this method, only that a multiple of 2p gives the correct value for q. In Question 3bi, poor algebra was a frequent cause of error, some students even suggesting the question was wrong or juggling their answer to Question 3a so that their poor algebra on the wrong expression gave the correct answer for 3bi.

It was good to observe that more students were able to take up a question part way through when they were unable to start it. Teachers should emphasise to students that the questions are designed this way and they should note information given in the early parts of the question and use this to help answer later parts.

There were many places throughout the paper where a correct answer would obtain full marks without showing any working, even where the part of the question was worth more than one mark. Students should be aware that they can get full marks for these questions where a particular method is not required (e.g.

‘Use calculus to ...’). However, they should also be aware that they would get ***no marks*** if the answer is wrong and no working is shown. Where the answer is obtained from a calculator, the ‘working’ may consist of showing what input was given to the calculator. For example, in Question 2a, the answer 38.6, with no working, would be awarded 2 marks out of 2. The answer 38.5 (the student has probably truncated 38.591 instead of rounding it), with no working, would be awarded zero marks. If an expression such as INVNORM(0.85, 36, 2.5) = 38.5 or similar is given, 1 mark out of 2 may be awarded.

SPECIFIC INFORMATION

Question 1

	Average mark	Available marks
a.	0.81	1
b.	1.19	2
c.	1.96	4
di.	1.52	3
dii.	1.30	3
ei.	0.96	2
eii.	0.44	1

While most students recognised the asymptote at $y = 3$, there were many poor attempts at 1b. Some students seemed to not read the question properly, overlooking the fact that the graph passed through the origin, or made no attempt to apply the fundamental idea that the co-ordinates of a point on a graph satisfy the equation for the rule of the corresponding function. In parts c and d, while most students knew what to do, many apparently failed to recognise an expression as being quadratic in e^x, which made it difficult for them to determine exact answers. Many did not provide the exact answers, but a numerical approximation via their calculator, and were awarded no marks. While this may be a useful check, it does not provide the exact values required. In dii., many errors occurred in the antidifferentiation of e^{2x} and of 3, but errors caused by sign problems and the area being below the x-axis were few.

Part e. was poorly done on the whole, students incorrectly sketching the reflection of the graph in the y-axis or the line $y = x$. Many who drew the correct graph failed to label the x-intercept.

Question 2

	Average mark	Available marks
a.	1.01	2
b.	1.62	3
c.	0.52	2
d.	1.34	2
e.	0.34	2
f.	1.31	3

A correct answer with no working was common for parts a. and d., presumably obtained using a graphics calculator. A common error in part a. was to misread the table to give $z = 1.365$ instead of $z = 1.0365$. Parts c. and e. were handled poorly in general, not many students knowing what to do in part c and few appreciating the reduced sample space in part e. Part e. is easily solved using proportions: He returned with 100 fish which is expected to be 85% of his catch. So his catch was 100/0.85. Of these, 15% were expected to be gourmet, so $\frac{100 \cdot 0.15}{0.85} = 17$ fish were gourmet. A surprising number of answers were not in the range 0 to 100, an indication of lack of understanding of the problem. Part f. was answered reasonably well, indicating that this new section of the course has been well received.

Many incorrect answers were the result of misreading ‘at least one’ in part b. and ‘more than one’ in part f.

Question 3

	Average mark	Available marks
a.	0.75	1
bi.	1.16	2
bii.	0.45	2
c.	0.94	2
d.	0.52	3
e.	0.59	3

For a fairly standard question, requiring the use of calculus to find a minimum value from a modelled situation, this was done very poorly by many students. The main reason for this was poor algebra in parts bi., c and e. Answers (or lack of them) to part bii. demonstrated that a large number of students were not very familiar with the idea of a variable being restricted in a modelled situation; in this case both r and h had to be positive. Even in part a., there were common errors with using the formulas given on the formula sheet.

Those who correctly found end-points in part bii. often failed to use these in part d. A scale indication on the C-axis was intended to help students to establish a suitable window for the graphics calculator, but too few students were able to produce a suitable graph. Labelling of asymptotes was poorly done. Many were unable to do much with part e. because they were unable to arrive at a cost function in part c.

Question 4

	Average mark	Available marks
a.	1.03	2
b.	0.51	1
c.	0.89	2
d.	0.76	3
e.	1.36	4

In part a., few students stated that the period of 1 led to $a = 2\pi$. Just half the students were awarded the 1 mark in part b., yet only had to indicate that $x = q - h$, which is clear from the diagram.

Attempts to part c. frequently showed poor differentiation, with π and negative signs causing the major problems. It was common for the answer to be left as 8π, resulting in the loss of 1 mark because the question was not read carefully enough.

In part d., students had to observe that the minimum needed to be found using a graphics calculator, since the zeros of the derivative cannot be found analytically. While many students did this successfully, teachers should ensure that students can decide when to use the calculator and when not to. Some students used a ‘trace’ function of their calculator and did not get answers to the required accuracy. A few found the wrong minimum. A method mark was awarded if students wrote down the equation they were attempting to solve, or an estimated value (in a suitable range) to use in their calculator. There were many attempts to solve the derivative equated to zero using tangent, although the sine and cosine parts of the equation had different arguments.

A good number of students used the fact that the width of the area required in part e. was one period and so used whole number limits. Errors in antidifferentiation were common and there were many attempts to obtain marks by finding the area with a graphics calculator. Since the question instructs that calculus must be used, a correct antiderivative must be shown – no marks were awarded for the correct answer if the antiderivative was missing or incorrect.

a.

$a = 2\pi$, $1 = 2\pi$

Amplitude = 2, so $b = 3 + 2 = 5$

b.

$x = q - h = -2\cos(2\pi t) + \sin(8\pi t) + 3$

c.

$\frac{dx}{dt} = 4\pi\sin(2\pi t) + 8\pi\cos(8\pi t)$

When $t = 2, \frac{dx}{dt} = 25.1$

d.

$4\pi\sin(2\pi t) + 8\pi\cos(8\pi t) = 0$

graph indicates $t_{min} \approx 1$

$t = 0.94$

$x = 0.136$

e.

$$A = \int_0^1 \left(-2\cos(2\pi t) + \sin(8\pi t) + 3\right)dt$$

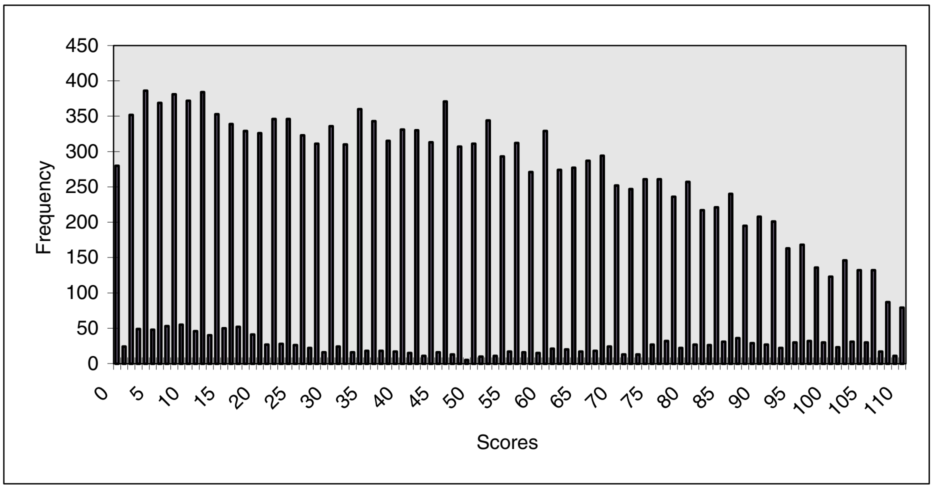
$$= \left[-\frac{\sin(2\pi t)}{\pi} - \frac{\cos(8\pi t)}{\pi} + 3t\right]_0^1$$

$$= 3$$

GA MA083 MATHEMATICS: Mathematical Methods WRITTEN EXAMINATION 2

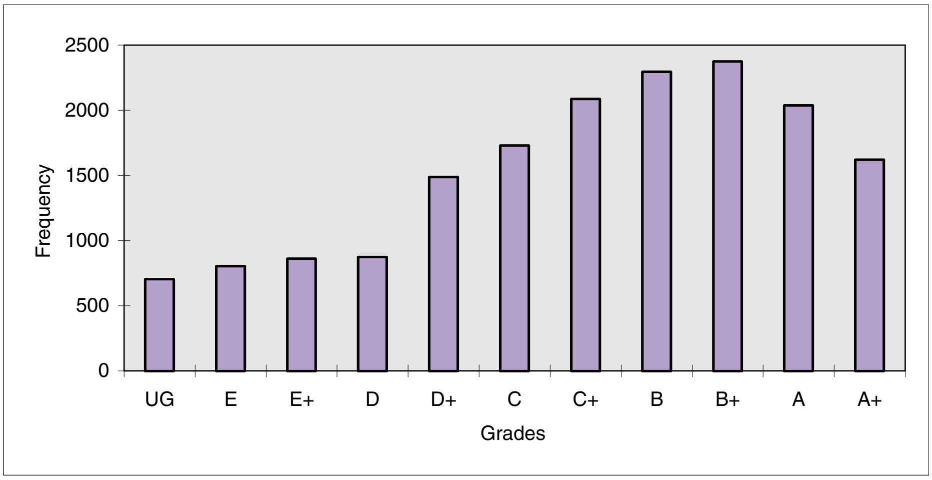
HISTOGRAM OF TOTAL SCORES 2000

Count 16875 Mean 46.77 Standard Deviation 30.19 NA Result 526



HISTOGRAM OF TOTAL GRADES 2000

Count 16875 Mean 6.04 Standard Deviation 2.78 NA Result 526



ENROLMENTS		%
Female	7922	45.5
Male	9479	54.5
Total	17401	

GLOSSARY OF TERMS

Count Number of students undertaking the assessment. This excludes those for whom NA was the result.

Mean This is the 'average' score; that is all scores totalled then divided by the 'Count'.

Standard Deviation This is a measure of how widely values are dispersed from the average value (the mean).