(1)

PART I Year 2000 : Maths Methods Trial Exam I

Structure of Booklet

Number of questions	Number of questions to be answered	Marks
32	32	32

Directions to students

Materials

Multiple-choice question booklet of 13 pages.

You may bring to the examination up to four pages (two A4 sheets) of pre-written notes.

You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task

Detach the formula sheet from the centre of this booklet during reading time.

Ensure that you write your **name** and your **teacher's name** in the spaces provided on the cover of the answer sheet for multiple-choice questions.

Answer **all** questions.

There is a total of 32 marks available for Part I.

Unless otherwise indicated, the diagrams in this booklet are not drawn to scale.

All written responses should be in English.

At the end of the task

Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer booklet (Part II).

Specific Instructions for Part I

This part consists of 32 questions.

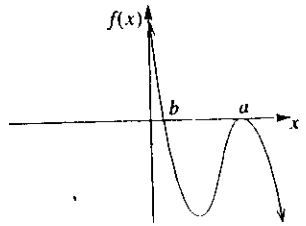
Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No mark will be given if more than one answer is completed for any question.

Question 1

a and b are positive numbers with $a > b$.



The equation of the graph shown above is most likely to be

A. $f(x) = (x - a)(x - b)$

B. $f(x) = -(x + a)^2(x + b)$

C. $f(x) = (x - a)(x - b)^2$

D. $f(x) = -(x - a)^2(x - b)$

E. $f(x) = (x - a)^2(x - b)$

Question 2

The coefficient of the term x^3 in the expansion of $(ax + b)^5$ is

A. $10a^3b^2$

B. 10

C. $10a^2b^3$

D. x^3

E. a^3b^2

Question 3

The asymptotes of the function $y = \frac{1}{(2-x)^2} + 3$ are

A. $x = \pm 2, y = 3$

B. $x = 2, y = 3$

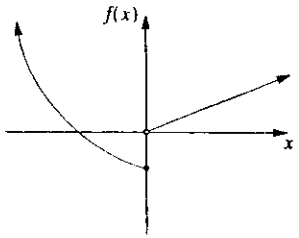
C. $x = -2, y = 3$

D. $x = 2, y = -3$

E. $x = \pm 2, y = -3$

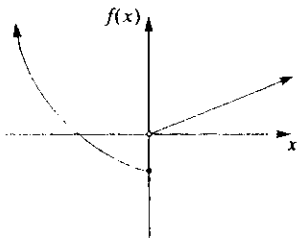
Question 4

The graph of $y = f(x)$ is shown below.

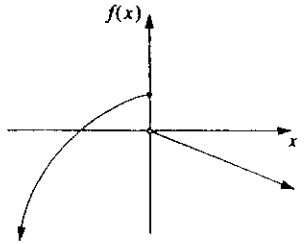


Which one of the following is the graph of $y = f(-x)$?

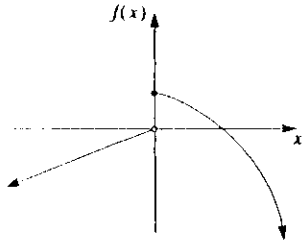
A.



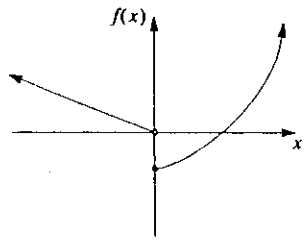
B.



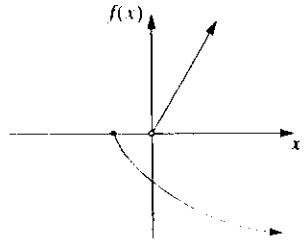
C.



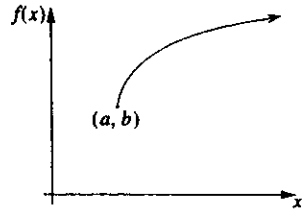
D.



E.



Question 5



a and b are positive numbers. The equation of the graph shown above is most likely to be

A. $f(x) = \sqrt{x+a} - b$

B. $f(x) = \sqrt{x-a} - b$

C. $f(x) = \sqrt{x-a} + b$

D. $f(x) = \sqrt{x-b} + a$

E. $f(x) = \sqrt{x+b} - a$

Question 6

The range of the function $f(x) = 4 - x^2, x \in [-1, 3)$ is

A. $[-1,3)$

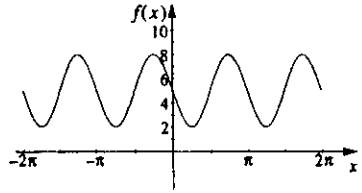
B. R

C. $(-\infty, 4]$

D. $(-5,4]$

E. $(-5,3]$

Question 7



The equation of the function for $x \in [-2\pi, 2\pi]$ shown above is

A. $f(x) = 3\cos\left(x + \frac{\pi}{4}\right) + 5$

B. $f(x) = -3\cos\left(2\left(x + \frac{\pi}{4}\right)\right) + 5$

C. $f(x) = 3\cos\left(\frac{1}{2}\left(x + \frac{\pi}{4}\right)\right) + 5$

D. $f(x) = 3\cos\left(2\left(x + \frac{\pi}{4}\right)\right) + 5$

E. $f(x) = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right) + 5$

Question 8

The number of asymptotes of the function $f(\theta) = A\tan\left(\frac{\theta}{B}\right)$, $-B\pi \le \theta \le B\pi$ is

A. 0
B. 1
C. 2
D. A
E. B

Question 9

The equation $\sqrt{3}\sin(2x) = \cos(2x)$, $-\pi \le x \le \pi$ has solutions

A. $-\frac{11\pi}{12}, -\frac{5\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}$

B. $-\frac{5\pi}{6}, \frac{\pi}{6}$

C. $-\frac{3\pi}{4}, -\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}$

D. $-\frac{11\pi}{12}, -\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}$

E. $-\frac{7\pi}{12}, -\frac{\pi}{12}, \frac{5\pi}{12}, \frac{11\pi}{12}$

Question 10

The temperature (O°C) in an oven at time t hours after it is turned on is modelled by the function $O(t) = 30 + 275\sin\left(\frac{\pi t}{30}\right)$, $0 \le t \le 24$. The time taken for the oven to reach 150°C is closest to

A. 0.05 hours.
B. 1.42 hours.
C. 4.31 hours.
D. 12.6 hours.
E. 25.9 hours.

Question 11

In an experiment, the pressure (P) in a reaction flask is modelled by the function $P = f(t)$ where t is the time in hours after mid-day. The function that models the pressure in the flask T minutes after 2 p.m. is

A. $P = f\left(\frac{T}{60} + 2\right)$

B. $P = f\left(\frac{T}{2} + 2\right)$

C. $P = f\left(\frac{T}{60} - 2\right)$

D. $P = f\left(\frac{T}{2} - 60\right)$

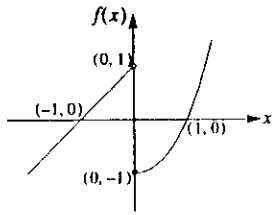
E. $P = f\left(\frac{T}{60} + 120\right)$

Question 12

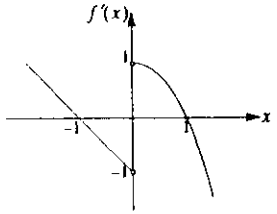
The asymptote of the graph $y = e^{-5x} - 2$ has equation

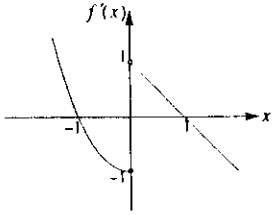
A. $y = 2$
B. $y = -2$
C. $y = 5$
D. $y = -5$
E. $x = -5$

Question 13



Given the graph of $y = f(x)$, which of the following best shows a graph of $y = f'(x)$?

A.


B.


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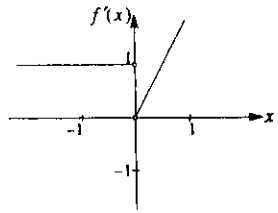
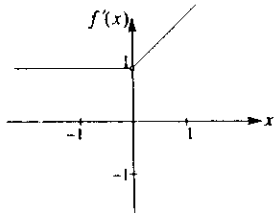
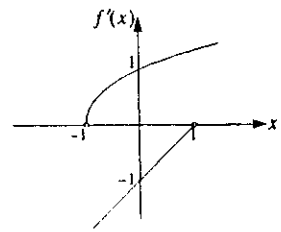
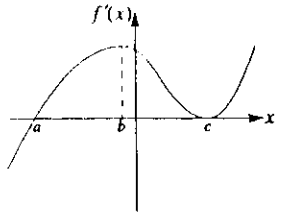


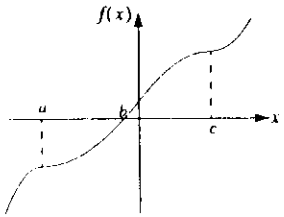
D.


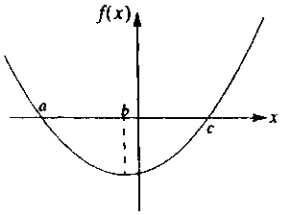
E.


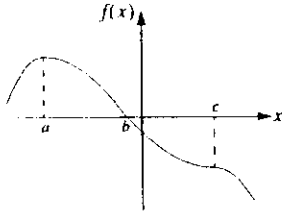
Question 14

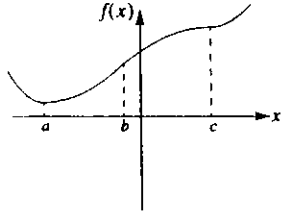


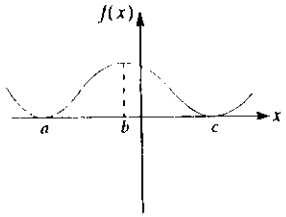
Given the above graph of the derivative function $f'(x)$, which of the following best depicts the graph of $y = f(x)$?

A.


B.


C.


D.


E.


Question 15

The derivative of $e^{2\sin 3x}$ is

A. $6\sin 3x\, e^{2\sin 3x - 1}$

B. $2\sin 3x\, e^{2\sin 3x - 1}$

C. $-2\sin 3x\, e^{2\sin 3x}$

D. $-6\sin 3x\, e^{2\sin 3x}$

E. $6\cos 3x\, e^{2\sin 3x}$

Question 16

If $y = \frac{x}{e^{-x}}$, then the rate of change of y with respect to x when $x = 1$ is

A. $2e$

B. e

C. e^2

D. 2

E. 0

Question 17

An antiderivative of $x\sqrt{x}$ is

A. $x^{3/2}$

B. $\frac{5x^{5/2}}{2}$

C. $\frac{2x^{5/2}}{5}$

D. $\frac{3x^{1/2}}{2}$

E. $\frac{3x^{5/2}}{2}$

Question 18

If $f'(x) = (2x+3)^2 - e^{-x}$ and $f(0) = \frac{7}{2}$, the constant term in $f(x)$ is equal to

A. 2

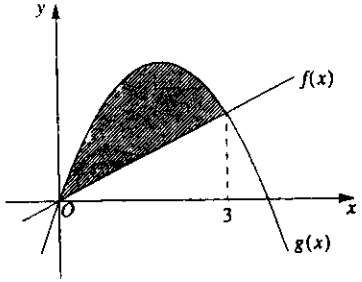
B. -2

C. 0

D. $e - 1$

E. $-e - 1$

Question 19



The shaded area shown is 5 square units, where $f(x) = ax$ and $g(x) = 4x - x^2$. The value of a is

A. $-\frac{28}{9}$

B. $-\frac{8}{9}$

C. $\frac{28}{9}$

D. $\frac{9}{8}$

E. $\frac{8}{9}$

Question 20

Which one of the following is **not** a factor of $P(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$?

A. $(x + 2)$

B. $(x + 1)$

C. $(x - 3)$

D. $(x - 4)$

E. $(x - 1)$

Question 21

The inverse of the function $f(x) = 3e^{(-2x)} + 5$ is

A. $f^{-1}(x) = \frac{1}{3e^{(-2x)} + 5}$

B. $f^{-1}(x) = \frac{1}{2}\log_e\left(\frac{x-5}{3}\right)$

C. $f^{-1}(x) = 2\log_e\left(\frac{x+5}{3}\right)$

D. $f^{-1}(x) = -\frac{1}{2}\log_e\left(\frac{x-5}{3}\right)$

E. $f^{-1}(x) = -\frac{1}{2}\log_e\left(\frac{x+5}{3}\right)$

Question 22

$2\log_{10}(a^2b) - \log_{10}(ab)$ simplifies to

A. $\log_{10}(a^2b)$

B. $\log_{10}(a^3b)$

C. $\log_{10}(a^3b^2)$

D. $\log_{10}(ab)$

E. $\log_{10}\left(\frac{a}{b}\right)$

Question 23

The equation $3 \times 10^{2x} = 7$ has a solution which is closest to

A. –0.427

B. 0.184

C. 0.424

D. 0.736

E. 1.695

Question 24

For which one of the following domains will the function $f(x) = B - (x - A)^2$ have a well defined inverse for all values of A and B?

A. R^+

B. $(-\infty, B]$

C. $[B, \infty)$

D. $(-\infty, A]$

E. R

Question 25

The probability distribution of random variable X is shown below.

x	2	3	4	5
$\Pr(X = x)$	k	$2k$	$3k$	$4k$

$\Pr(2 \leq X < 4)$ is equal to

A. 0.1

B. 0.2

C. 0.3

D. 0.4

E. 0.6

Question 26

Consider the probability distribution shown below.

x	1	2	3	4	5
$\Pr(X = x)$	0.1	0.2	0.3	0.2	0.2

The value of $E(2X + 1)$ is equal to

A. 3.2
B. 4.2
C. 6.4
D. 7.4
E. 8.2

Questions 27 and 28 refer to the following information.

A random variable X has a binomial distribution such that $E(X) = 7$ and $\text{var}(X) = 2.1$.

Question 27

The value of $\Pr(X = 7)$ is

A. 0.0090
B. 0.1029
C. 0.2001
D. 0.2334
E. 0.2668

Question 28

$\Pr(7 < X < 7 + 2\sigma)$ is closest to

A. 0.3545
B. 0.5003
C. 0.6213
D. 0.6495
E. 0.7004

Question 29

A bag contains 4 black discs and 5 white discs all of a similar size. A sample of 3 discs is randomly selected without replacement. The probability that it contains 2 white discs is

A. $^9C_3\left(\frac{5}{9}\right)^3\left(\frac{4}{9}\right)^6$

B. $^5C_2 \times {}^4C_1$

C. $^3C_2\left(\frac{2}{5}\right)^2\left(\frac{1}{4}\right)$

D. $\frac{^4C_2 \times {}^5C_1}{^9C_3}$

E. $\frac{^4C_1 \times {}^5C_2}{^9C_3}$

Question 30

A sample of globes is being selected from a **small** batch which is known to have some defectives. If X is the number of defectives in the sample when the sample is taken **without** replacement and Y is the number of defectives in the sample when the sample is taken **with** replacement, which of the following statements is correct?

A. $E(X) = E(Y)$ and $\text{var}(X) < \text{var}(Y)$
B. $E(X) < E(Y)$ and $\text{var}(X) = \text{var}(Y)$
C. $E(X) = E(Y)$ and $\text{var}(X) > \text{var}(Y)$
D. $E(X) > E(Y)$ and $\text{var}(X) = \text{var}(Y)$
E. $E(X) = E(Y)$ and $\text{var}(X) = \text{var}(Y)$

Question 31 and 32 refer to the following information.

The weight of a box of breakfast cereal is normally distributed with a mean of 750 g and a variance of 20 g.

Question 31

The probability that a randomly selected box of this cereal contains more than 740 g is

A. 0.0127
B. 0.1634
C. 0.4873
D. 0.6915
E. 0.9873

Question 32

95% of the boxes of cereal weigh more than

A. 92.07 g
B. 782.89 g
C. 717.10 g
D. 742.60 g
E. 757.37 g

END OF PART I MULTIPLE-CHOICE QUESTION BOOKLET

Specific Instructions for Part II

Answer all questions in this part in the spaces provided.

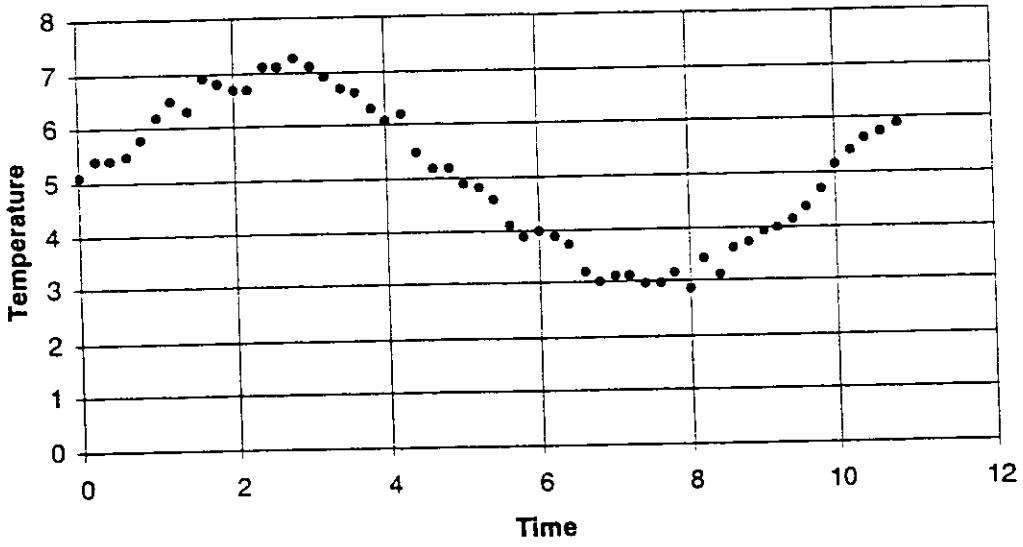
Question 1

Find the inverse of the function $y = \frac{1}{x+2} - 3, x \neq -2$, stating its domain.

2 marks

Question 2

The diagram below shows a graph of some experimental data.



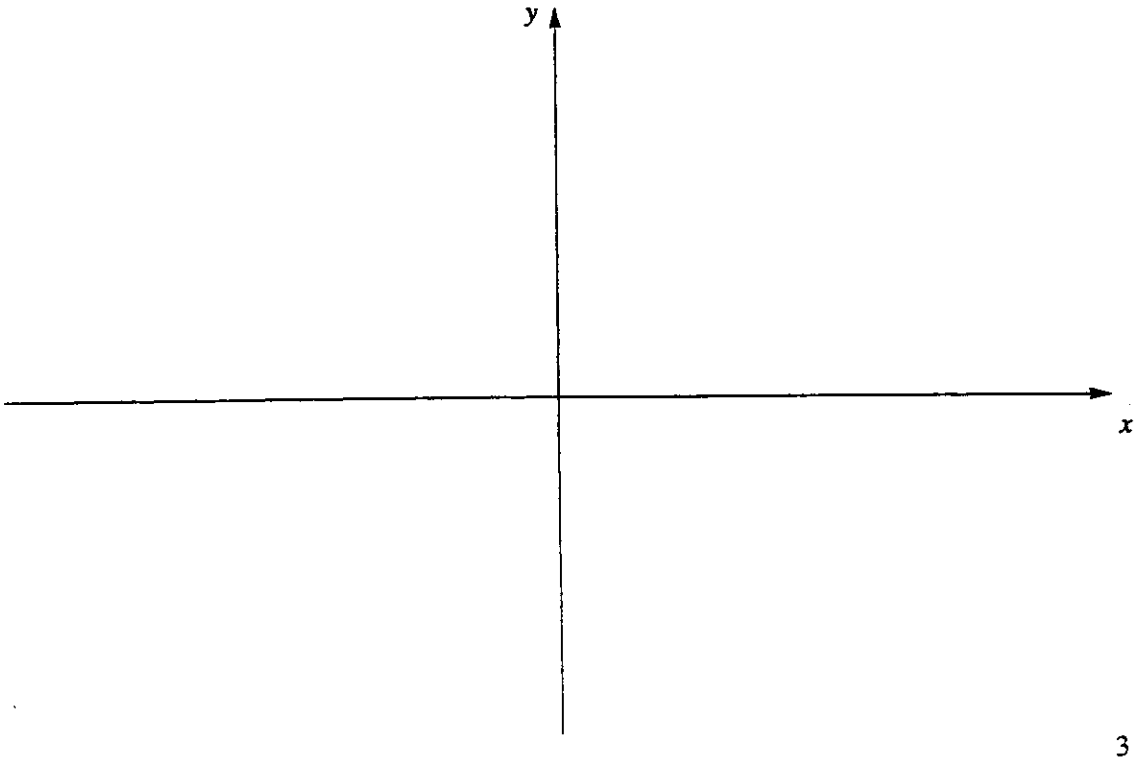
Find the period and amplitude of the data and suggest a suitable function that might be used to model the temperature in terms of time.

3 marks

Question 3

Sketch the graphs of $y = \frac{1}{(x-3)^2}$, $x \neq 3$ and $y = x - 1$.

Hence sketch the graph of $y = \frac{1}{(x-3)^2} + x - 1$, $x \neq 3$, labelling any asymptotes.



3 marks

Question 4

The volume $V(t)$ litres of water in a bathtub at time t minutes is given by

$$V(t) = 300\sin\left(\frac{\pi t}{20}\right)$$

Calculate:

a. the volume when $t = 5$ minutes, to the nearest litre.

1 mark

b. the **exact** rate of change (in surd form) of volume when $t = 5$ minutes.

2 marks

Total 3 marks

Question 5

Consider the function

$$f(x) = \tan 2x \cdot \log_e x^2$$

a. Calculate $f'(x)$.

2 mark

b. Calculate the exact value of the gradient of this function when $x = \pi$.

2 marks

Total 4 marks

Question 6

Three balls are to be randomly selected from a container which holds 6 black and 4 white balls all of equal size.

a. Calculate the probability that the sample has exactly 2 white balls

i. if each ball selected is **not** replaced.

1 mark

ii. if each ball selected is replaced before the next selection is made.

1 mark

b. Calculate the mean number of white balls in the sample.

1 mark

Total 3 marks

END OF PART II QUESTION AND ANSWER BOOKLET