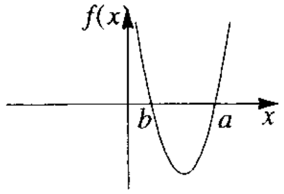
SECTION I (MULTIPLE CHOICE)

Question 1

A. $f(x) = (x - a)(x - b)$

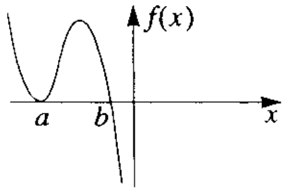
This would be a parabola.



B. $f(x) = -(x + a)^2(x + b)$

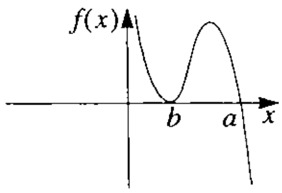
This would have intercepts on the negative x-axis.

The repeated factor would give a 'touching' intercept at $x = -a$.



C. $f(x) = (x - a)(x - b)^2$

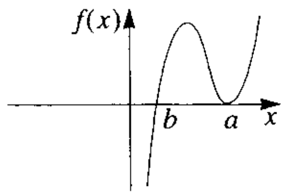
This would be:



D. $f(x) = -(x - a)^2(x - b)$ Correct.

E. $f(x) = (x - a)^2(x - b)$

This would be:



Answer D

Question 2

In full, the expansion is $(ax + b)^5 = a^5x^5 + 5a^4bx^4 + 10a^3b^2x^3 + 10a^2b^3x^2 + 5ab^4x + b^5$.

It is, of course, not necessary to write out the full expansion to get the answer $10a^3b^2$. Note that it is the coefficient that was asked for, not the term: $10a^3b^2x^3$.

Answer A

Question 3

$y = \dfrac{1}{(2 - x)^2} + 3 = \dfrac{1}{(x - 2)^2} + 3$ is the standard truncus translated two to the right and three up.

The asymptotes become $x = 2$, $y = 3$ instead of the axes.

Answer B

Question 4

A. This is $y = f(x)$

B. This is $y = -f(x)$

C. This is $y = -f(-x)$

D. Correct.

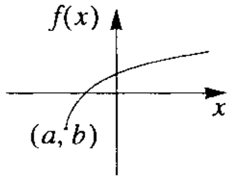
E. This is $y = f^{-1}(x)$

Answer D

Question 5

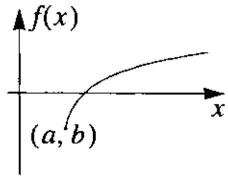
A. $f(x) = \sqrt{x+a} - b$

The translation is left and down to give:



B. $f(x) = \sqrt{x-a} - b$

The translation is right and down to give:

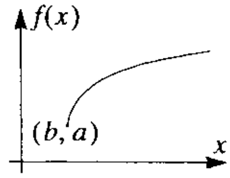


C. $f(x) = \sqrt{x-a} + b$

Correct.

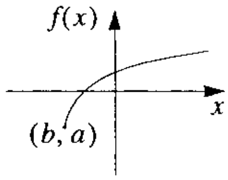
D. $f(x) = \sqrt{x-b} + a$

This is b to the right and a up.



E. $f(x) = \sqrt{x+b} - a$

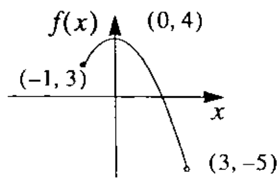
This is b to the left and a down.



Answer C

Question 6

The graph of the function $f(x) = 4 - x^2, x \in [-1, 3)$ is



A. $[-1, 3)$ is the range.

B. R a stock answer, incorrect in this case.

C. $(-\infty, 4]$ ignores the stated domain.

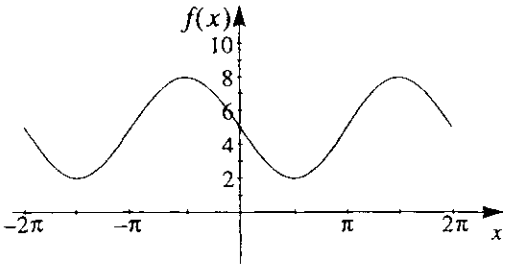
D. $(-5, 4]$ correct.

E. $(-5, 3]$ is the answer obtained by looking only at the extremes of the domain.

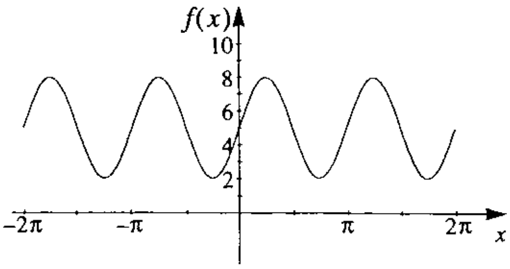
Answer D

Question 7

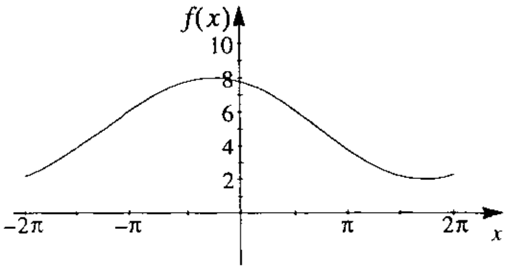
A. $f(x) = 3\cos\left(x + \frac{\pi}{4}\right) + 5$



B. $f(x) = -3\cos\left(2\left(x + \frac{\pi}{4}\right)\right) + 5$



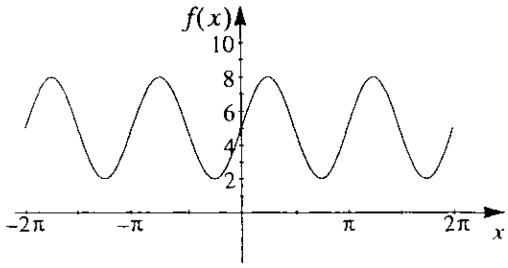
C. $f(x) = 3\cos\left(\frac{1}{2}\left(x + \frac{\pi}{4}\right)\right) + 5$



D. $f(x) = 3\cos\left(2\left(x + \frac{\pi}{4}\right)\right) + 5$

Correct.

E. $f(x) = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right) + 5$



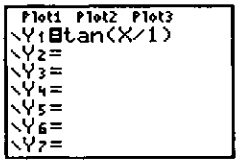
F.

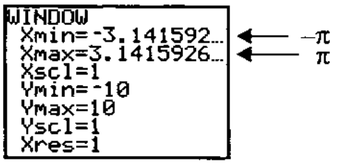
Answer D

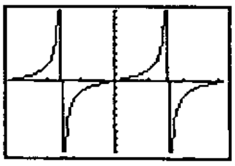
Question 8

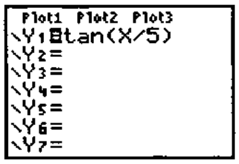
$f(\theta) = A\tan\left(\frac{\theta}{B}\right), \quad -B\pi \le \theta \le B\pi$ has a horizontal dilation of factor B (the vertical dilation does not matter as there are no horizontal asymptotes). Since the same 'dilation' has been applied to the domain, there will always be the same number of asymptotes as there are on the graph of $f(\theta) = \tan\theta, \quad -\pi \le \theta \le \pi$, or 2.

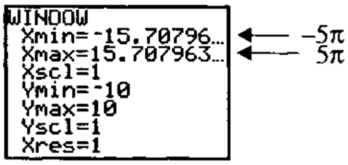
Here are two examples generated on a graphic calculator:

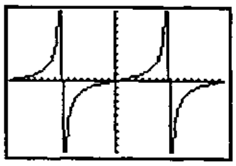












Answer C

Question 9

$\sqrt{3}\sin(2x) = \cos(2x), -\pi \le x \le \pi$

$$\frac{\sin(2x)}{\cos(2x)} = \frac{1}{\sqrt{3}}$$

$$\tan(2x) = \frac{1}{\sqrt{3}}$$

$$2x = \frac{\pi}{6}, \frac{\pi}{6} \pm \pi, \frac{\pi}{6} \pm 2\pi, \frac{\pi}{6} \pm 3\pi \ldots$$

$$x = \frac{\pi}{12}, \frac{\pi}{12} \pm \frac{\pi}{2}, \frac{\pi}{12} \pm \pi, \frac{\pi}{12} \pm \frac{3\pi}{2} \ldots$$

The values in the stated domain are $\frac{\pi}{12} - \pi, \frac{\pi}{12} - \frac{\pi}{2}, \frac{\pi}{12}, \frac{\pi}{12} + \frac{\pi}{2}$, i.e. $-\frac{11\pi}{12}, -\frac{5\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}$

Answer A

Question 10

$$30 + 275\sin\left(\frac{\pi t}{30}\right) = 150$$

$$275\sin\left(\frac{\pi t}{30}\right) = 120$$

$$\sin\left(\frac{\pi t}{30}\right) = \frac{120}{275}$$

$$\frac{\pi t}{30} = \sin^{-1}\left(\frac{120}{275}\right)$$

$$\frac{\pi t}{30} = 0.45155326$$

$$t = \frac{30(0.45155326)}{\pi} = 4.312016 \approx 4.31 \text{ hours}$$

Answer C

Question 11

The transformations are a dilation to change the time measurement from hours to minutes (divide the minutes variable T by 60). There is also a 2 hour translation.

Another way to view this is to see that when $T = 0$ (2 p.m.), $t = 2$ or two hours after noon.

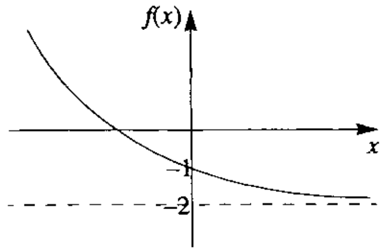
When $T = 180$ (180 minutes after 2 p.m. or 5 p.m.), $t = 5$. The modelling function is a function of t and so T must be transformed to be t before it can be used in f.

The transformation is $t = \frac{T}{60} + 2$ and the function is $P = f\left(\frac{T}{60} + 2\right)$.

Answer A

Question 12

The graph is:



The asymptote is $y = -2$.

Answer B

Question 13

As $f'(x)$ is a measure of gradient, when $x < 0$ the gradient of the line is 1. When $x > 0$ the gradient begins at zero and increases linearly with a value of 2. Consider also the *derivatives* of $y = x + 1$ for $x < 0$ and $y = x^2 - 1$ for $x > 0$.

$$f'(x) = \begin{cases} 1 & x < 0 \\ 2x & x > 0 \end{cases}$$

Answer C

Question 14

At $x = a$ the gradient changes from negative to positive as x increases, hence a local minimum will appear here on $y = f(x)$. At $x = b$ the gradient reaches a local maximum and then decreases to a zero gradient at $x = c$. For $x > c$ the gradient remains positive. Therefore we should see a point of horizontal inflection at $x = c$ and a non-stationary point of inflection at $x = b$.

Answer D

Question 15

Use the chain rule. Let $u = 2\sin 3x$, so that $y = e^u$. $\therefore \frac{du}{dx} = 6\cos 3x$ and $\frac{dy}{du} = e^u$.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u \times 6\cos 3x = 6\cos 3x e^{2\sin 3x}$$

Answer E

Question 16

The rate of change = $\frac{dy}{dx}$. Here one could use the quotient rule or simplify first and use the product rule.

Using the latter approach, $y = xe^x$

$$\frac{dy}{dx} = xe^x + e^x$$

When $x = 1$, $\frac{dy}{dx} = 1 \times e^1 + e^1 = 2e$

Answer A

Question 17

Expressing in index form gives $\int x^1 \times x^{\frac{1}{2}}\, dx = \int x^{\frac{3}{2}}\, dx = \frac{2x^{\frac{5}{2}}}{5} + c$.

If $c = 0$, an antiderivative is alternative C.

Answer C

Question 18

If $f'(x)$ is given, we must find the antiderivative to deduce $f(x)$.

$$\therefore f(x) = \frac{(2x+3)^2}{2 \times 3} + e^{-x} + c$$

To find c we substitute $x = 0$, $f(x) = \frac{7}{2}$ giving

$$\frac{7}{2} = \frac{3^3}{6} + e^0 + c$$

$$\frac{7}{2} = \frac{27}{6} + 1 + c$$

$$\therefore c = \frac{7}{2} - \frac{27}{6} - 1$$

$$\therefore c = -2$$

Answer B

Question 19

The shaded area is found by calculating

$$\int_0^3 (g(x) - f(x))\, dx = \int_0^3 (4x - x^2 - ax)\, dx = \left[\frac{4x^2}{2} - \frac{x^3}{3} - \frac{ax^2}{2}\right]_0^3 = \frac{4 \times 3^2}{2} - \frac{3^3}{3} - \frac{9a}{2} = 18 - 9 - \frac{9a}{2} = 9 - \frac{9a}{2}$$

As this area is equal to 5 units^2, we have $9 - \frac{9a}{2} = 5$

$$\frac{9a}{2} = 4$$

$$\therefore a = \frac{8}{9}$$

Answer E

Question 20

$P(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

A. $(x+2)$, $P(-2) = (-2)^4 - 4(-2)^3 - 7(-2)^2 + 22(-2) + 24 = 0$ (factor).

B. $(x+1)$, $P(-1) = (-1)^4 - 4(-1)^3 - 7(-1)^2 + 22(-1) + 24 = 0$ (factor).

C. $(x-3)$, $P(3) = (3)^4 - 4(3)^3 - 7(3)^2 + 22(3) + 24 = 0$ (factor).

D. $(x-4)$, $P(4) = (4)^4 - 4(4)^3 - 7(4)^2 + 22(4) + 24 = 0$ (factor).

E. $(x-1)$, $P(1) = (1)^4 - 4(1)^3 - 7(1)^2 + 22(1) + 24 = 36$ (not a factor).

Answer E

Question 21

$f(x) = 3e^{-2x} + 5$ or $y = 3e^{-2x} + 5$ has the inverse $x = 3e^{-2y} + 5$.

$$x = 3e^{-2y} + 5$$

$$3e^{-2y} = x - 5$$

$$e^{-2y} = \frac{x-5}{3}$$

$$-2y = \log_e\left(\frac{x-5}{3}\right)$$

$$y = -\frac{1}{2}\log_e\left(\frac{x-5}{3}\right)$$

Answer D

Question 22

$$2\log_{10}(a^2b) - \log_{10}(ab) = \log_{10}(a^2b)^2 - \log_{10}(ab) = \log_{10}\left(\frac{a^4b^2}{ab}\right) = \log_{10}(a^3b)$$

Answer B

Question 23

$$3 \times 10^{2x} = 7$$

$$10^{2x} = \frac{7}{3}$$

$$2x = \log_{10}\left(\frac{7}{3}\right)$$

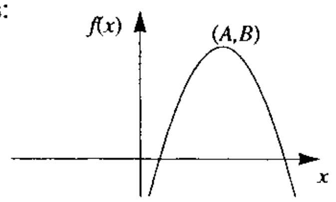
$$x = \frac{1}{2}\log_{10}\left(\frac{7}{3}\right)$$

$$= 0.184$$

Answer B

Question 24

The graph of $f(x) = B - (x - A)^2$ is:



The function is one:one over either $[A, \infty)$ or $(-\infty, A]$.
Answers such as $(-\infty, B]$ or $[B, \infty)$ may be correct but only for some values of A and B. e.g. $[B, \infty)$ is correct if $B \neq A$. The question, however, says: "all values of A and B".

Answer D

Question 25

We must first deduce the value of K.

$\Sigma \Pr(X = x) = 1$

$10K = 1$

$\therefore\ K = 0.1$

So $\Pr(2 \leq X < 4) = \Pr(X = 3) + \Pr(X = 3)$

$= 0.1 + 0.2$

$= 0.3$

Answer C

Question 26

Here one could contract a new table with variable $2X + 1$, or more simply calculate $E(X)$ and use the simplification $E(2X + 1) = 2E(X) + 1$.

$E(X) = \Sigma x \cdot px$

$= 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.3 + 4 \times 0.2 + 5 \times 0.2$

$= 0.1 + 0.4 + 0.9 + 0.8 + 1.0 = 3.2$

Hence $E(2X + 1) = 2 \times 3.2 + 1 = 7.4$

Answer D

Question 27

As X has a binomial distribution, $E(X) = np$ and $\text{VAR}(X) = np(1 - p)$.

So $\text{VAR}(X) = E(X) \times (1 - p)$, i.e. $2.1 = 7(1 - p)$

$0.3 = 1 - p$

$\therefore\ p = 1 - 0.3$

$p = 0.7$

As $np = 7$

$n = \dfrac{7}{0.7}$

$\therefore\ n = 10$

$\Pr(X = 7) = {}^{10}C_7(0.7)^7(0.3)^3 = 0.2668$

Answer E

Question 28

Using the graphics calculator one can generate the probability distribution for $X = Bi(10, 0.7)$ (if required).

Now $\sigma = \sqrt{\text{VAR}(X)} = \sqrt{2.1} = 1.4491$, so $7 + 2\sigma = 7 + 2 \times 1.4491 = 9.898$.

So $\Pr(7 < X < 9.898) = \Pr(X = 8) + \Pr(X = 9) = 0.23347 + 0.12106 = 0.35453$

(These could also be evaluated on the graphics calculator.)

Answer A

Question 29

Selection without replacement suggests a hypergeometric distribution.

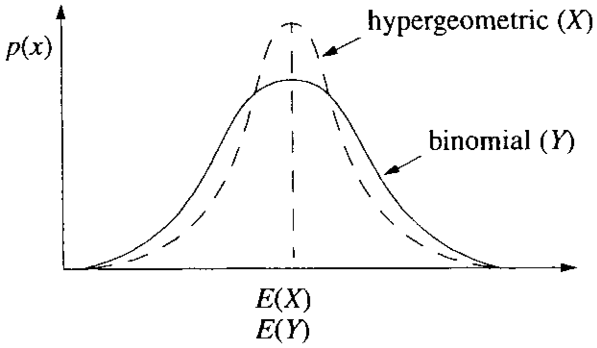
If formulas are used, $N = 9$, $n = 3$, $x = 2$ and $D = 5$.

Hence $\Pr(X = 2) = \dfrac{{}^4C_1 \times {}^5C_2}{{}^9C_3}$.

Answer E

Question 30

Where N is reasonably small, the distributions for binomial and hypergeometric are as follows:



i.e. $E(X) = E(Y)$
$\text{VAR}(X) < \text{VAR}(Y)$

Note also from formulae, since $\dfrac{D}{N} = p$, $E(X) = n\dfrac{D}{N} = E(Y) = np$.

As samples are not being replaced in the hypergeometric situation, the probability of success ($p(x)$) where x is away from the mean will be less than that of the binomial. This makes the variance of the hypergeometric less than that of the binomial. (For large batches this difference is very small.)

Also, by comparing formulas for the variance:

Binomial	**Hypergeometric**
$\text{VAR}(Y) = np(1-p)$	$\text{VAR}(X) = \dfrac{nD}{N}\left(1 - \dfrac{D}{N}\right)\left(\dfrac{N-n}{n-1}\right)$

and as $0 < \dfrac{N-n}{n-1} < 1$, $\text{VAR}(X) < \text{VAR}(Y)$.

Hence as N increases, $\dfrac{N-n}{n-1} \to 1$ so for large values of N, $\text{VAR}(X) \approx \text{VAR}(Y)$.

Answer A

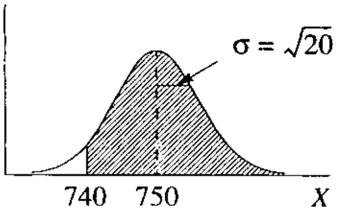
Question 31

If using tables, $z = \dfrac{740 - 750}{\sqrt{20}} = -2.2361$

$\Pr(Z > -2.2361) = \Pr(Z < 2.2361)$

$= 0.9873$

If using a graphics calculator, students must calculate $\sqrt{20}$ and then enter correct limits to obtain correct answer.



Answer E

Question 32

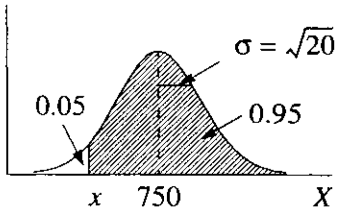
If X = weight of cereal box, $\Pr(X < x) = 0.05$ using either tables or preferably a graphics calculator. Using inverse normal, $X = 742.6$.

If using tables, from $Z = \dfrac{X - \mu}{\sigma}$

$\therefore X = Z\sigma + \mu$

(where $z = -1.645$, $\Pr(Z < z) = 0.95$).

So $X = -1.645 \times \sqrt{20} + 750 = 742.6$



Answer D

SECTION II (SHORT ANSWER)

Question 1

$y = \dfrac{1}{x+2} - 3,\ x \neq -2$

The inverse is: $x = \dfrac{1}{y+2} - 3$

$$\frac{1}{y+2} = x + 3$$

$$y + 2 = \frac{1}{x+3}$$

$$y = \frac{1}{x+3} - 2$$

$$f^{-1}(x) = \frac{1}{x+3} - 2$$ A1

Domain $R\backslash\{-3\}$ A1

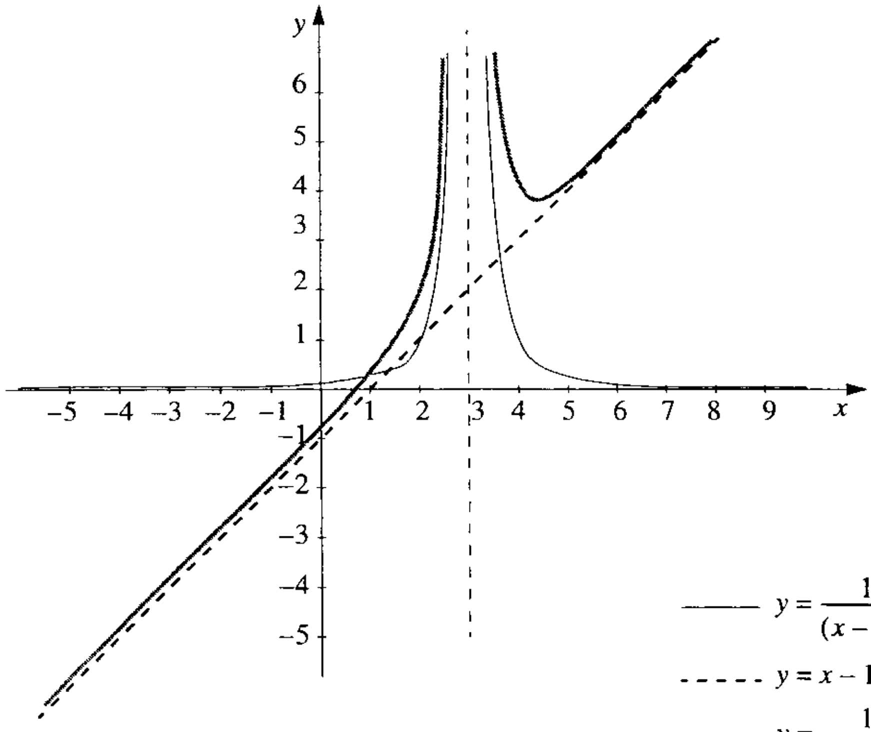
Question 2

The amplitude is about $\dfrac{7-3}{2} = 2$. M1

The period can be estimated by noticing that the maximum is at about 2.5 and the minimum is about 7.5. The period is about $2(7.5 - 2.5) = 10$. M1

The modelling function (using the sine option) might be: Temp $= 5 + 2\sin\left(\dfrac{\pi t}{5}\right)$. A1

Question 3



—— $y = \dfrac{1}{(x-3)^2}, x \neq 3$ A1

- - - - - $y = x - 1$ A1

—— $y = \dfrac{1}{(x-3)^2} + x - 1, x \neq 3$ A1

Question 4

a. $V(5) = 300\sin\left(\dfrac{\pi \times 5}{20}\right)$

$= 300\sin\dfrac{\pi}{4}$

$= \dfrac{300 \times \sqrt{2}}{2}$

≈ 212 litres A1

b. The rate of change $= V'(5)$

$V'(t) = \dfrac{\pi}{20}(300)\cos\left(\dfrac{5\pi}{20}\right)$ M1

When $t = 5$, $V'(5) = 15\pi\cos\dfrac{\pi}{4}$

$= 15\pi \times \dfrac{1}{\sqrt{2}}$

$= \dfrac{15\pi\sqrt{2}}{2}$ L/min A1

Question 5

a. Use the product rule. Let $u = \tan 2x$ and $v = \log_e x^2$.

Then $\frac{du}{dx} = 2\sec^2 2x$ and $\frac{dv}{dx} = \frac{2x}{x^2} = \frac{2}{x}$. M1

Hence $\frac{dy}{dx} = \tan 2x \times \frac{2}{x} + \log_e x^2 \times 2\sec^2 2x = 2\left(\frac{\tan 2x}{x} + \frac{\log_e x^2}{\cos^2 2x}\right)$ A1

b. Gradient $= \frac{dy}{dx}$.

At $x = \pi$, gradient $= 2\left(\frac{\tan 2\pi}{\pi} + \frac{\log_e \pi^2}{\cos^2 2\pi}\right)$ M1

$\tan 2\pi = 0$ and $\cos 2\pi = 1$,

so gradient $= 2\log_e \pi^2$ or $4\log_e \pi$. A1

Question 6

a. **i.** This is hypergeometric with $N = 10$, $D = 4$, $n = 3$ and $x = 2$.

$$\Pr(X = 2) = \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3}$$

$$= 0.300$$ A1

ii. This is binomial with $n = 3$, $p = 0.4$ and $x = 2$.

$$\Pr(X = 2) = {}^3C_2(0.4)^2(0.6)^1$$

$$= 0.288$$ A1

b. As the means for both distributions in **a.** yield equivalent values, either formula could be used.

$\mu = E(X) = np = 3 \times 0.4 = 1.2$ A1