THE HEFFERNAN GROUP

P.O. Box 1180
Surrey Hills North VIC 3127
ABN 20 607 374 020
Phone 9836 5021
Fax 9836 5025

Student Name.....

MATHEMATICAL METHODS

TRIAL EXAMINATION 2

2000

Reading Time: 15 minutes
Writing Time: 90 minutes

Instructions to Students

This exam consists of 4 questions.
All questions should be attempted.
The marks allocated to each of the four questions are indicated throughout.
There is a total of **60** marks available.
Students may bring up to two A4 pages of pre-written notes into the exam.

This paper has been prepared independently of the Board of Studies to provide additional exam preparation for students. Although references have been reproduced with permission of the Board of Studies, the publication is in no way connected with or endorsed by the Board of Studies.

Question 1

Consider the function $f : R \to R$ where $f(x) = ae^x - 1$

a. Given $f(0) = 1,$ show that the value of a equals 2.

_____

_____

_____

1 mark

b. Write down the range of f.

_____

_____

1 mark

c. Given that the *x* intercept of the graph is $(\log_e b,\ 0)$, find the value of *b*.

_____

_____

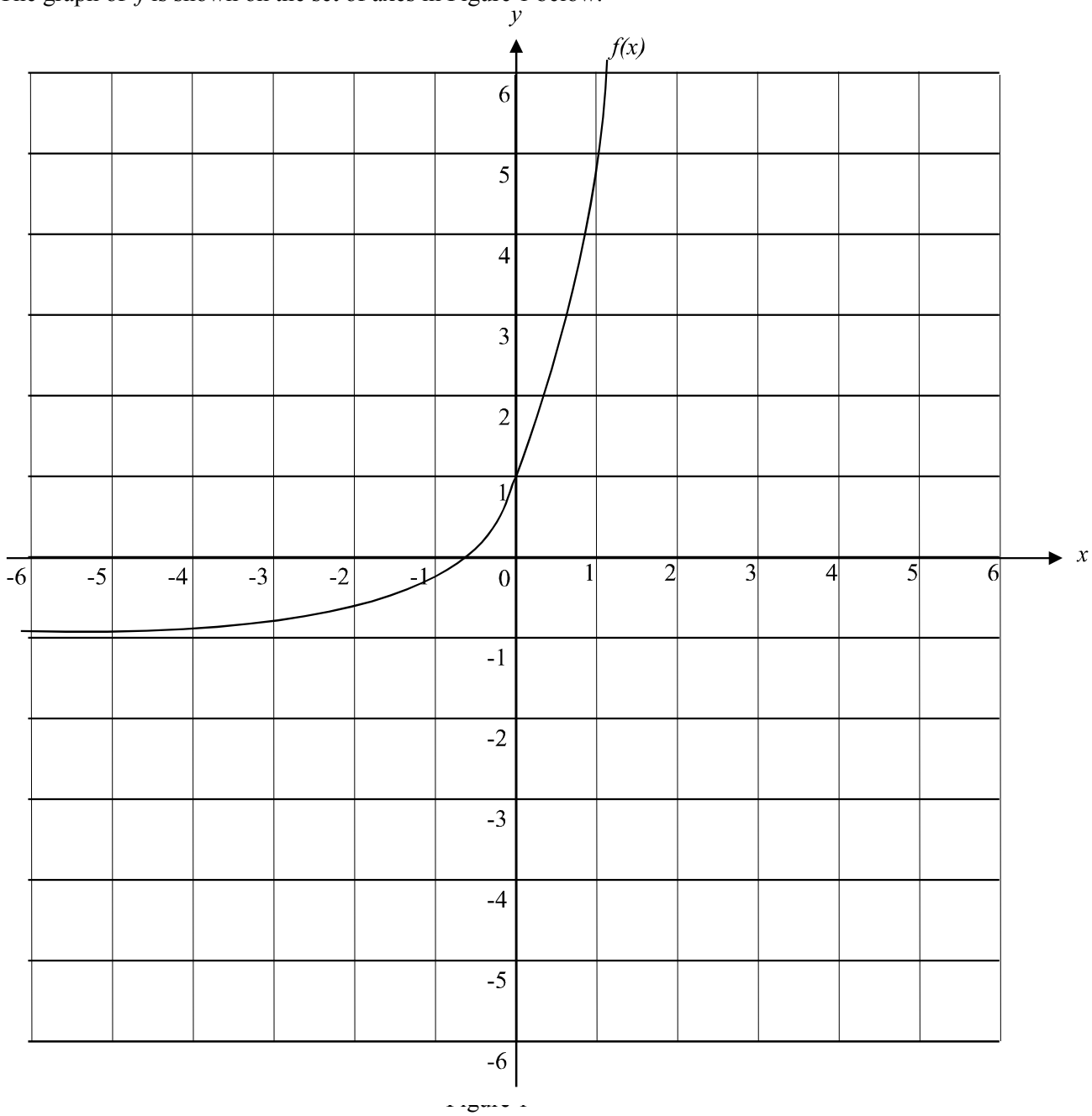
_____

_____

_____

2 marks

The graph of f is shown on the set of axes in Figure 1 below.



d. On the same set of axes sketch the inverse function f^{-1} and label clearly the x and y intercepts of this inverse function.

3 marks

e. Write down the rule and domain of f^{-1}.

_____

_____

_____

_____

2 marks

The graph of f is shown on the set of axes in Figure 2 below.

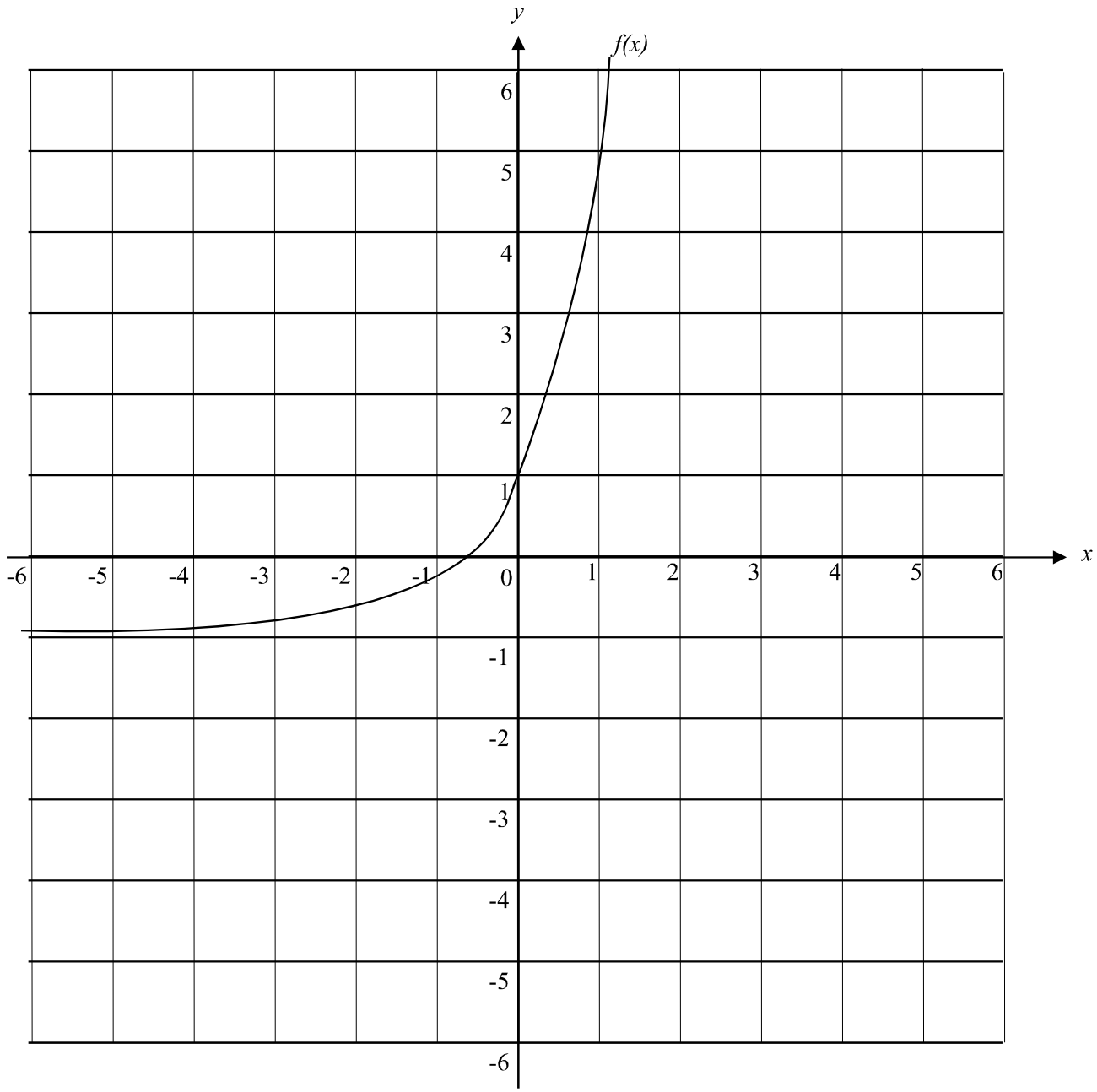


Figure 2

f. **i.** The function g is obtained by translating the function f, 1 unit upwards.
Sketch function g on the set of axes shown in Figure 2 above. Label your graph clearly.

1 mark

ii. The function h is obtained by dilating function f by a factor of 2 parallel to the x axis.
Sketch function h on the set of axes shown in Figure 2 above. Label your graph clearly.

1 mark

The function t is obtained by reflecting the function f in the y axis.

g. **i.** The x intercept of the function $t(x)$ is $\log_e m$. Find m.

_____

_____

_____

1 mark

ii. Find the area enclosed by the function f, the function t and the x axis. Express your answer as an exact value.

_____

_____

_____

_____

_____

_____

_____

_____

_____

4 marks
Total 16 marks

Question 2

Mario and Joe play a game on holidays which involves a large container that holds seven yellow table tennis balls and forty-three white table tennis balls. One of the boys randomly chooses a ball from the container, notes its colour and returns it to the container. He does this 3 times and this whole process is referred to as a "set". If, in a set, all 3 balls are white, then the boy wins a point and has another set. If, in a set, one or more yellow balls appear, the boy loses his turn and it becomes the turn of the other boy.

a. **i.** What is the expected number of yellow balls in a draw?

_____

_____

_____

1 mark

ii. What is the probability that one yellow ball will turn up out of the next three? (Express your answer correct to 4 decimal places.)

_____

_____

_____

2 marks

iii. What is the probability that one of the boys wins a point? (Express your answer correct to 4 decimal places.)

_____

_____

_____

_____

2 marks

iv. What is the probability that in one turn, a boy can score 5 points? (Express your answer correct to 4 decimal places.)

_____

_____

_____

_____

1 mark

b. One holidays when Mario had been playing the game with his father, he noticed that his father seemed to win a lot. Over time, Mario worked out that the probability of his father winning a point was about 0.719 Over time he also noticed that his father would remove the same number of yellow balls from the container each time just before having his turn and return them just before Mario had his turn.
How many yellow balls did Mario's father remove?

_____

_____

_____

_____

_____

_____

_____

_____

3 marks

c. Mario and Joe decide to change the rules of their game a little. They decide that they will no longer replace the balls in a set.

i. What is the expected number of yellow balls in a set now?

_____

_____

1 mark

ii. What is the probability of scoring a point under the new rules? (Express your answer correct to 4 decimal places.)

_____

_____

_____

_____

2 marks

iii. State with reasons whether or not these new rules make the game easier or harder to score a point.

_____

_____

_____

_____

1 mark

iv. What is the probability under these new rules that in a set at least 2 yellow balls will appear? (Express your answer correct to 4 decimal places.)

_____

_____

_____

_____

_____

_____

2 marks

Total 15 marks

Question 3

A wire mesh fence is constructed and part of it is shown in Figure 1 below.

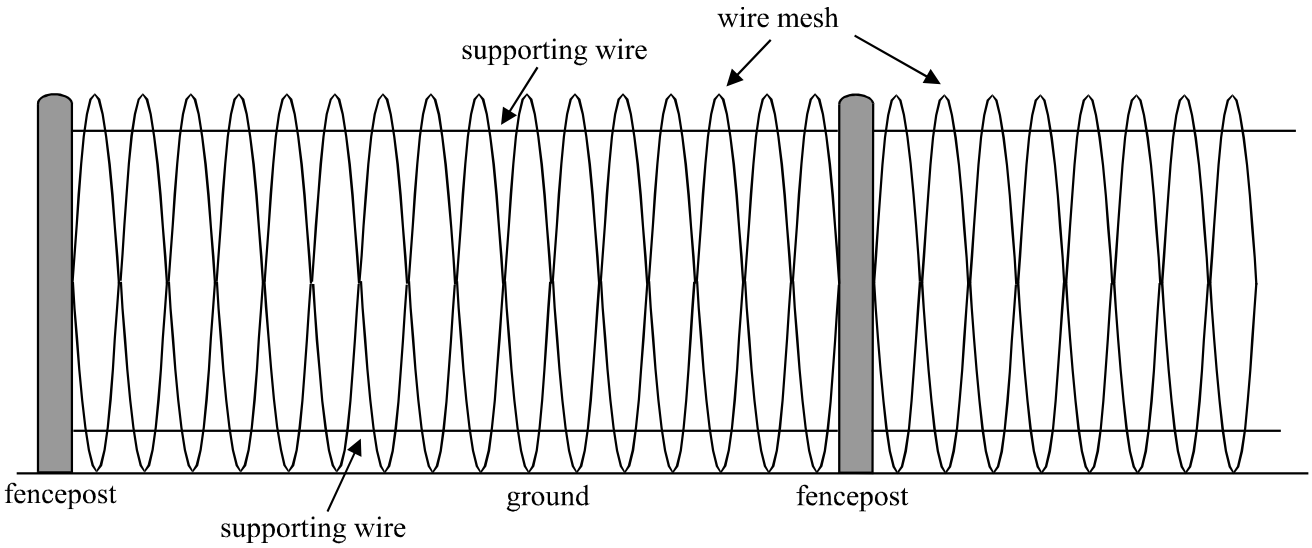


Figure 1

The wire mesh rests on the ground and is the same height as the fence posts. The wire mesh is made by soldering together two lengths of wire. One of the lengths of wire has the shape of the curve with equation A given by $y_A = 60\sin\frac{\pi x}{20} + 60$

The other length of wire has the shape of the curve with equation B given by $y_B = -a\sin\frac{\pi x}{20} + b$.

The graph of these two functions is shown in Figure 2 below.

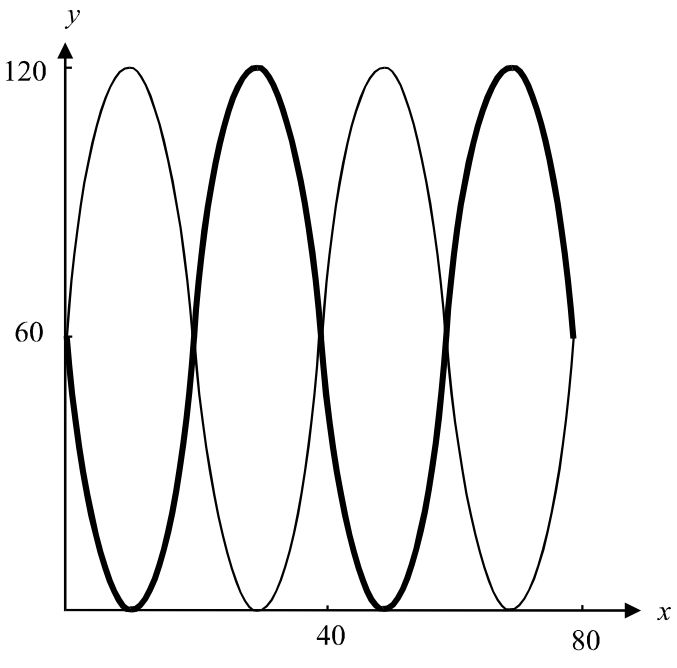


Figure 2

The x axis represents the ground and the y axis represents the edge of the left-hand fence post. All measurements are in centimetres.

a. Write down the values of a and b.

_____

_____

_____

_____

2 marks

b. Write down the equation of the curve drawn in Figure 2 which is indicated by the heavier line.

_____

_____

1 mark

c. Write down the period of the graph with equation A.

_____

_____

1 mark

d. Using Figure 1 and your answer to part **c.**, calculate the distance in metres between the two posts shown in Figure 1.

_____

_____

_____

_____

_____

_____

2 marks

Two horizontal supporting wires are soldered to the wire to add extra strength.
The top supporting wire is 90 cm above the ground.

e. Find the distance from the left-hand fence post to the first two points of intersection between the supporting wire and the curved wire which has the equation

$$y_A = 60\sin\frac{\pi x}{20} + 60$$

Express your answer in exact form.

3 marks

f. Two little boys are playing with sticks. The first little boy wedges a straight stick of length 10 cm, horizontally between 2 wires of the fence as shown in Figure 3.

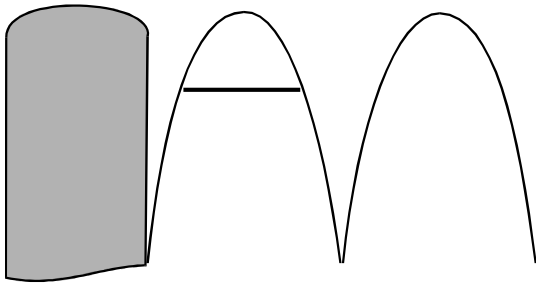


Figure 3

i. How far vertically below the top of the fence is the stick wedged? (Express your answer in centimetres correct to 2 decimal places.)

_____

_____

_____

_____

_____

_____

_____

3 marks

ii. The second little boy manages to wedge his straight stick horizontally between the wires as shown in Figure 4 below. What is the length of his stick?

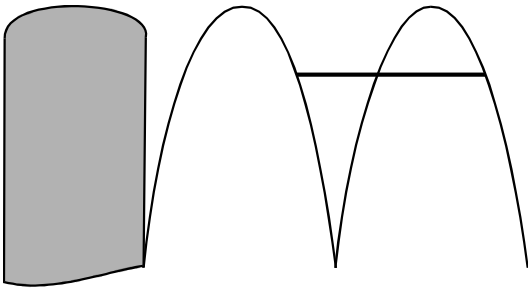


Figure 4

_____

_____

_____

1 mark

Total 13 marks

Question 4

A rubbish tip is closed, then levelled flat and covered with clean fill in order to create public parkland. A straight path is constructed and runs from one of the entry points to the park to a seat at point A which is a horizontal distance of 50 metres from the entry point and is the highest point in the park. A cross-sectional view of this path is shown on the graph in Figure 1 below.

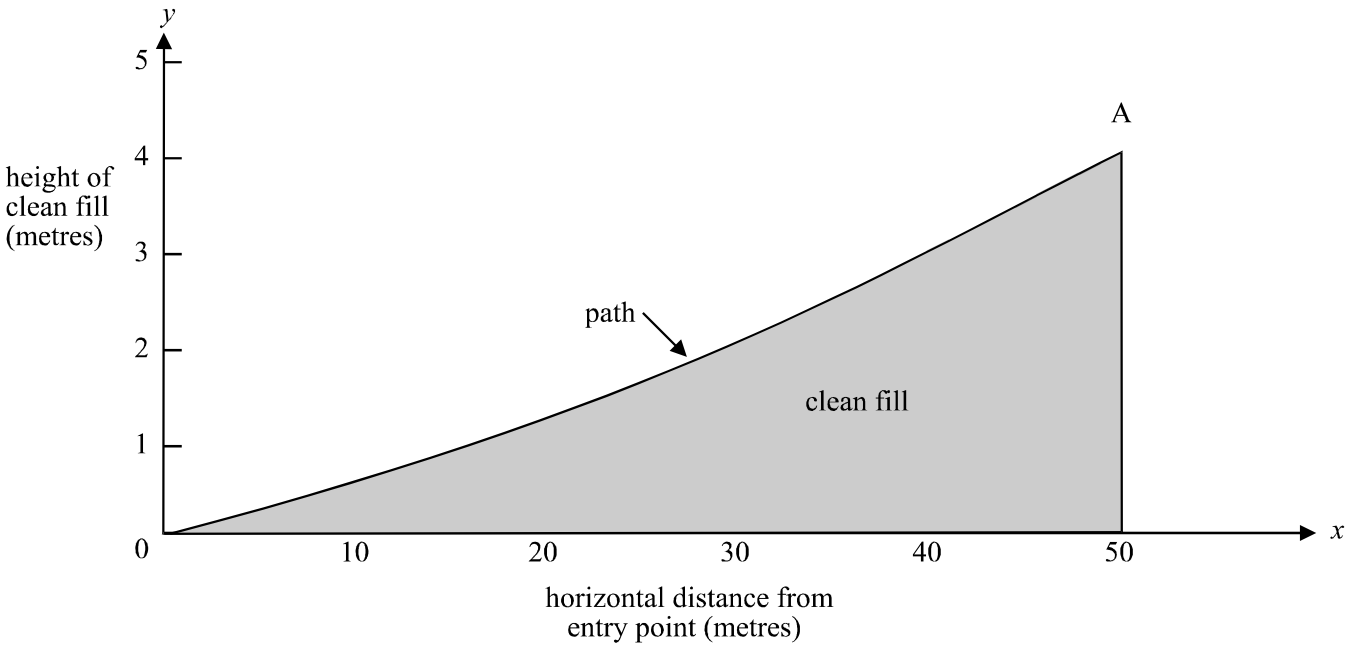


Figure 1

With respect to the axes shown, the path follows a curve with equation $y = \frac{x+1}{50}\log_e(x+1)$

a. i. Write down the coordinates of point A and express the y coordinate correct to 2 decimal places.

_____

_____

_____

_____

_____

1 mark

ii. What is the height above the levelled flat rubbish tip of the highest point in the park? (Answer correct to 2 decimal places.)

_____

_____

_____

1 mark

b. Show algebraically that the graph passes through the origin.

1 mark

c. Find the gradient of the path at point B which is at a horizontal distance of 25 metres from the entry point to the park. Express your answer as a decimal correct to 3 decimal places.

2 marks

d. Using your graphics calculator, explain whether the gradient of the path ever exceeds 0.1

2 marks

e. **i.** Show that the derivative of the function $y = \frac{(x+1)^2}{100}\log_e(x+1)$

is given by $\frac{dy}{dx} = \frac{x+1}{50}\log_e(x+1) + \frac{x+1}{100}$

2 marks

ii. Use your answer to **part i.** to find $\int \frac{x+1}{50}\log_e(x+1)dx$

3 marks

iii. Use your result to part **ii.** to find the area under the graph shown in Figure 1 between $x = 0$ and $x = 50$, and use this to find the volume of clean fill lying directly beneath the 2 metre wide path. Assume that there is no slope on the path from one side to the other for the length of the path. Express your answer correct to 2 decimal places.

4 marks

Total 16 marks