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Student Name .....

MATHEMATICAL METHODS

TRIAL EXAMINATION 1

2000

Reading Time: 15 minutes
Writing Time: 90 minutes

Instructions to Students

This exam consists of Part I and Part II. All questions in Part I and Part II should be answered.

Part I consists of 28 multiple-choice questions, which should be answered on the detachable answer sheet which can be found on page 20 of this exam.
Part II consists of 8 short-answer questions which should be answered in the spaces provided.
Part I begins on page 2 of this exam.
Part II begins on page 15 of this exam.
All questions in Part I and in Part II should be attempted.
Part I is worth **28 marks**.
Part II is worth **22 marks**.
Students may bring up to two A4 pages of pre-written notes into the exam.

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PART I Multiple-choice questions

Question 1

A random variable X has the probability distribution shown in the table below.

x	0	1	2	3
$\Pr(X=x)$	0.4	0.3	0.2	0.1

The mean and variance respectively of x are

A. 1 and 1
B. 1 and 1.4
C. 1.4 and 1
D. 1.4 and 1.4
E. 1.5 and 2

Question 2

Which one of the following random variables is not discrete?

A. the number of goals scored in a basketball match
B. the scores received by students in a class for a maths test
C. the weight of a footballer during a season
D. the population of a country town over a decade
E. the price of "Moggy" 1kg packets of cat food over a period of a week at a certain supermarket

Question 3

The weights of packets of 6 Easter buns are normally distributed with a mean of 420 grams and a standard deviation of 8 grams.
The proportion of packets which weigh between 400 and 430 grams is closest to

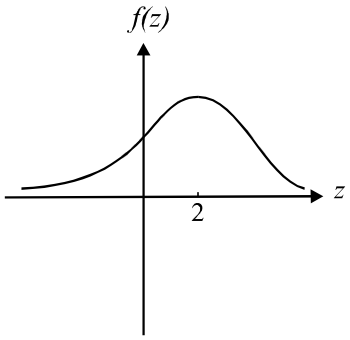
A. 1%
B. 50%
C. 89%
D. 93%
E. 99%

Question 4

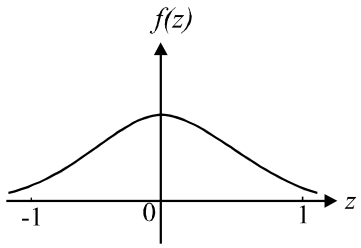
The random variable Z has a standard normal distribution.

Which one of the following diagrams could show the distribution of Z?

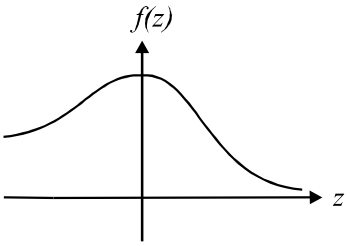
A.



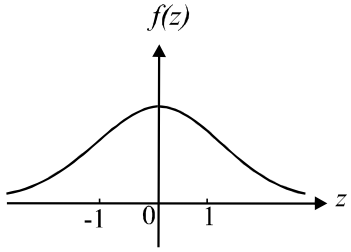
B.



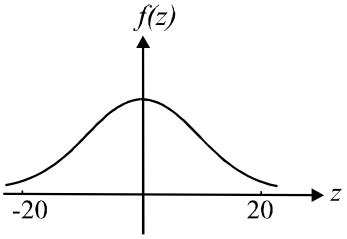
C.



D.



E.



Question 5

A large jar contains "traffic light" lollies. There are 80 red, 50 green and 70 orange lollies in the jar.
A child is allowed to choose 10 lollies from the jar and these lollies are placed in a bag to be taken home. The probability that there will be 6 red lollies in the bag is given by

A. ${}^{10}C_6(0.4)^6(0.6)^4$

B. ${}^{200}C_{10}(0.4)^6(0.6)^4$

C. ${}^{200}C_{10}(6)^{0.4}(4)^{0.6}$

D. $\dfrac{{}^{80}C_6\ {}^{120}C_4}{{}^{200}C_{10}}$

E. $\dfrac{{}^{0.4}C_{0.6}\ {}^{0.6}C_{0.4}}{{}^{200}C_6}$

Question 6

At Seaview Secondary College, a weekly detention class will contain Year 12 students on 15% of occasions. The probability that there will be Year 12s present in at least 1 of the next 3 detention classes held is

A. ${}^3C_1(0.15)^1(0.85)^2$

B. $1-{}^3C_0(0.15)^0(0.85)^3$

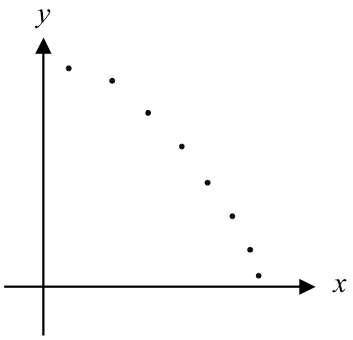
C. ${}^3C_1(0.85)^1(0.15)^2+{}^3C_2(0.85)^2(0.15)^1+{}^3C_3(0.85)^3(0.15)^0$

D. $\dfrac{{}^{0.15}C_1\ {}^{0.85}C_2}{{}^{100}C_3}$

E. $\dfrac{{}^{15}C_1\ {}^{85}C_2}{{}^{100}C_3}$

Question 7

A graph showing the relationship between the variables x and y is shown below.
Given that a is a positive constant, the equation relating x and y could be of the form



A. $y=\dfrac{a}{x^2}$

B. $y=-ax^2$

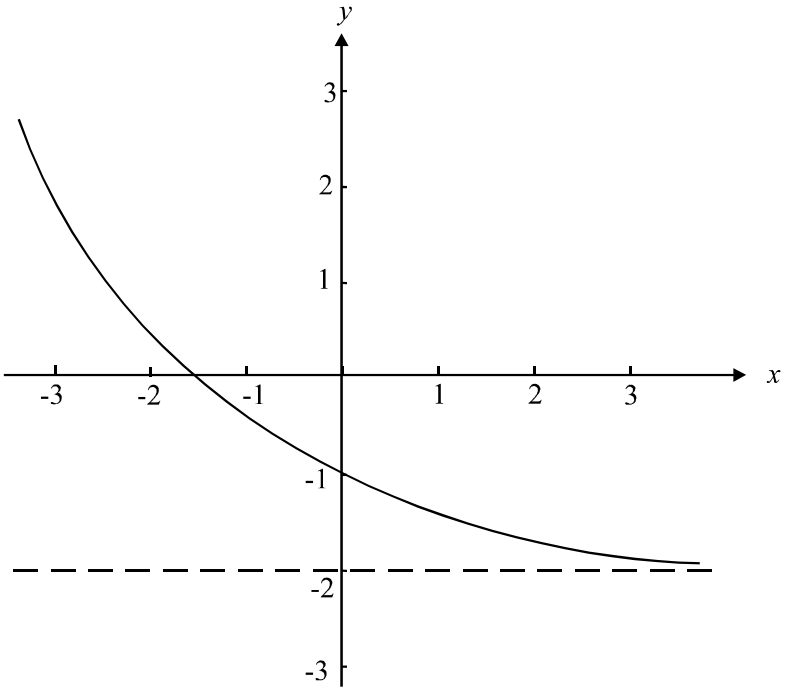
C. $y=ax^{\frac{1}{2}}$

D. $y=\cos(ax)$

E. $y=e^{-ax}$

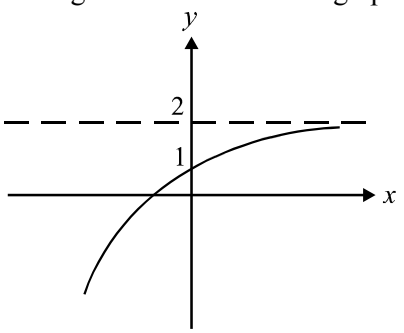
Question 8

The graph of a function is shown below.

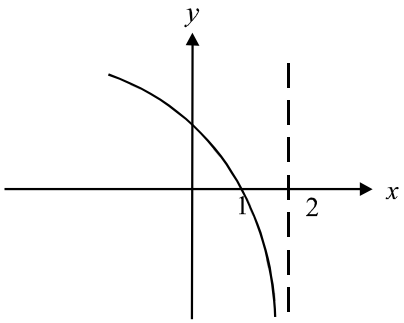


Which one of the following functions shows the graph of its inverse function?

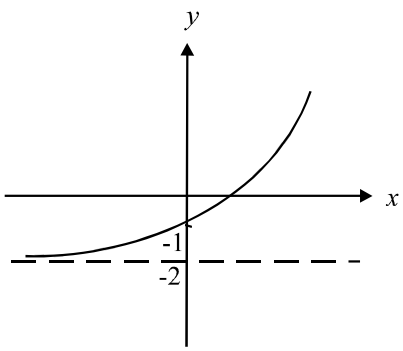
A.



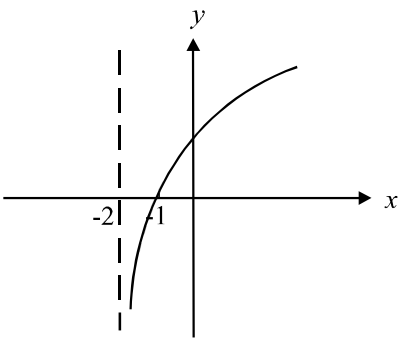
B.



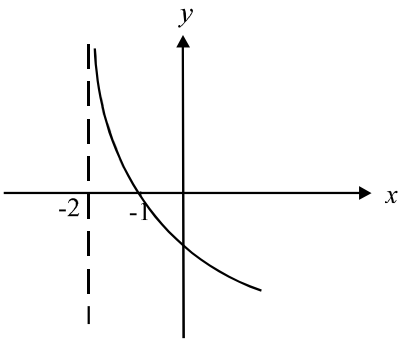
C.



D.



E.



Question 9

The equations of the vertical and horizontal asymptotes of the graph with equation $y = \frac{A}{x+b} + B$ are respectively

A. $x = A$ and $y = B$

B. $x = b$ and $y = A$

C. $x = -b$ and $y = B$

D. $x = A$ and $y = b$

E. $x = B$ and $= -b$

Question 10

The domain and range of the function $y = 2x^{-1} + 4$ are given respectively by

A. $(-\infty, 0) \cup (0, \infty)$ and $(-\infty, 4) \cup (4, \infty)$

B. $(-\infty, 2) \cup (2, \infty)$ and $(-\infty, 0) \cup (0, \infty)$

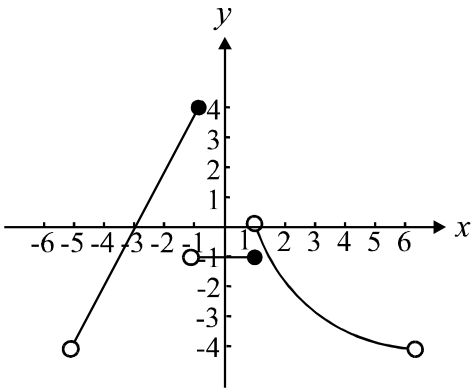
C. $[-\infty, 0) \cup (0, \infty]$ and $[-\infty, 4) \cup (4, \infty]$

D. $(-\infty, 4) \cup (4, \infty)$ and $(-\infty, 0) \cup (0, \infty)$

E. $(-\infty, 0.5) \cup (0.5, \infty)$ and $(-\infty, 0) \cup (0, \infty)$

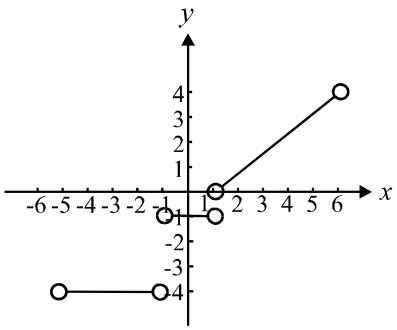
Question 11

The graph of the function g is shown below.

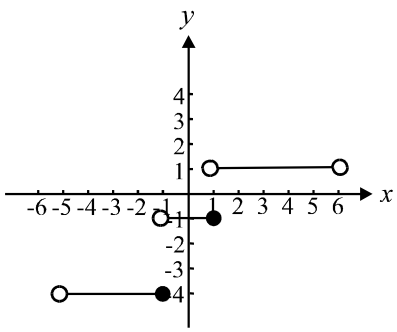


The gradient function of g is closest to

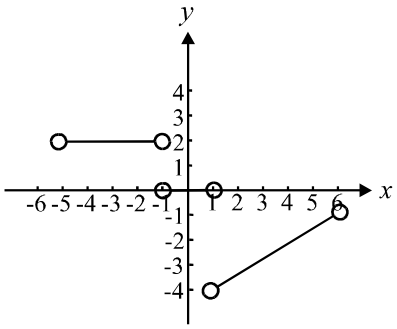
A.



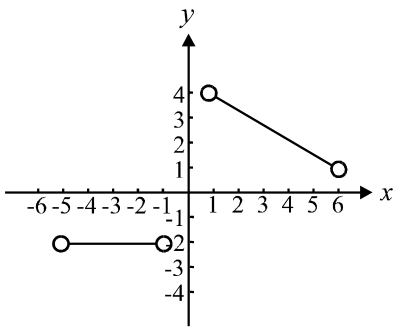
B.



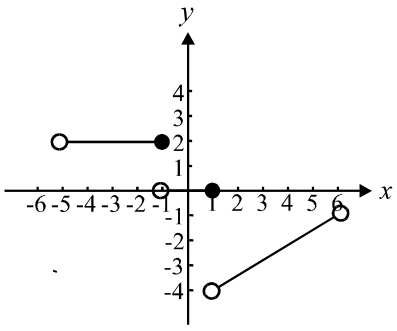
C.



D.



E.



Question 12

If $f(x) = x^2 \tan(4x)$ then $f'(x)$ is equal to

A. $2x\sec^2(4x)$
B. $8x\sec^2(4x)$
C. $2x\tan(4) + x^2\sec^2(4x)$
D. $2x\tan(4x) + 4x^2\sec^2(4x)$
E. $8x\tan(4x) + 2x^2\sec^2(4x)$

Question 13

If $y = \dfrac{e^{\sin(3x)}}{2}$ then $\dfrac{dy}{dx}$ is equal to

A. $\dfrac{\sin(3x)}{2}e^{\sin(3x)}$
B. $\dfrac{\cos(3x)}{2}e^{\sin(3x)}$
C. $\dfrac{3}{2}\sin(3x)e^{\sin(3x)}$
D. $\dfrac{3}{2}\cos(3x)e^{\sin(3x)}$
E. $\dfrac{3}{2}\sin(3x)\cos(3x)e^{\sin(3x)}$

Question 14

The derivative of $\dfrac{\log_e(x^2 - 4x)}{x}$ is given by

A. $\dfrac{1}{x-4}$
B. $\dfrac{2x-4}{x(x-4)}$
C. $\dfrac{2x-4}{x-4} - \log_e(x^2 - 4x)$
D. $\dfrac{2x-4}{x^2(x-4)} - \log_e(x^2 - 4x)$
E. $\dfrac{2x-4}{x^2(x-4)} - \dfrac{1}{x^2}\log_e(x^2 - 4x)$

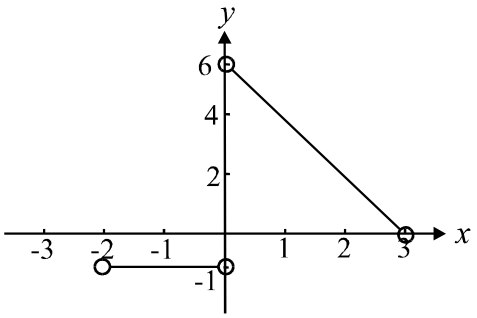
Question 15

The stationary point/s of the curve with equation $y = x^4 + 8x^3 + 5x^2 + x$ is/are closest to

A. (0, 0)
B. (0, 1)
C. (0, 0) and (-5.558, -270.378)
D. (-0.170, -0.027) and (-3.744, 90.055)
E. (0, 0) and (-7.319, 0)

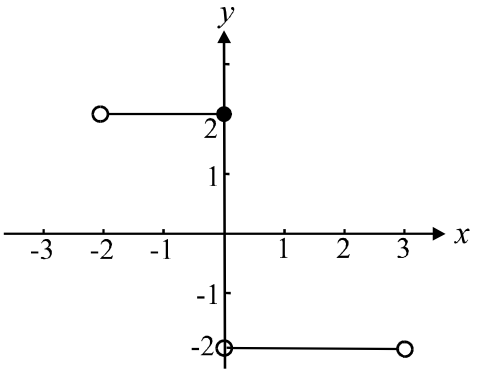
Question 16

The graph of the function *g* over the domain $x \in (-2, 3)$ is shown below.

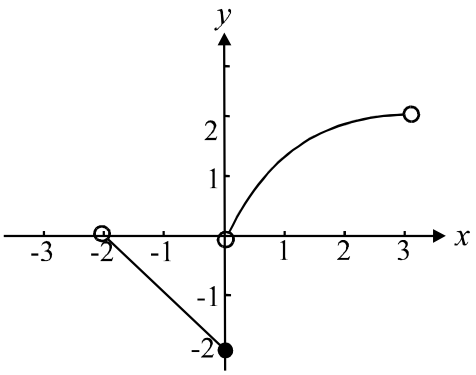


Which one of the following graphs could represent the antiderivative function of *g*?

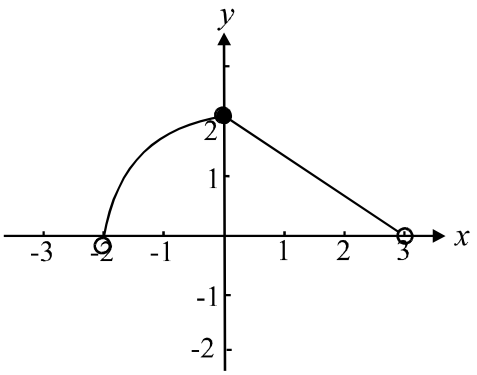
A.



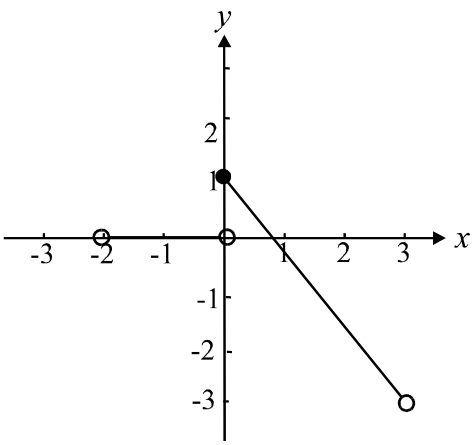
B.



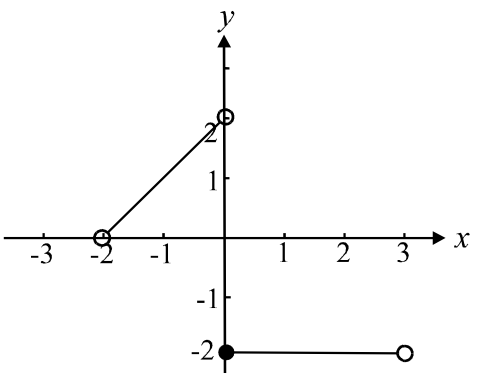
C.



D.



E.



$\int (\sin\frac{x}{2} + \frac{1}{(2x+1)^2})dx$ is equal to

A. $\cos\frac{x}{2} + \frac{1}{2x+1} + c$

B. $-\cos\frac{x}{2} - \frac{1}{(2x+1)^3} + c$

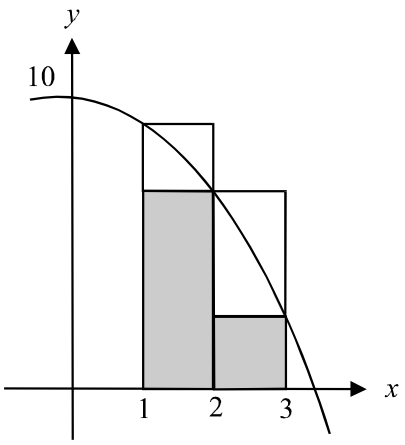
C. $-2\cos\frac{x}{2} - \frac{1}{2(2x+1)} + c$

D. $-\frac{1}{2}\cos\frac{x}{2} + \frac{2}{(2x+1)^3} + c$

E. $-\cos\frac{x}{2} + \frac{2}{2x+1} + c$

Question 18

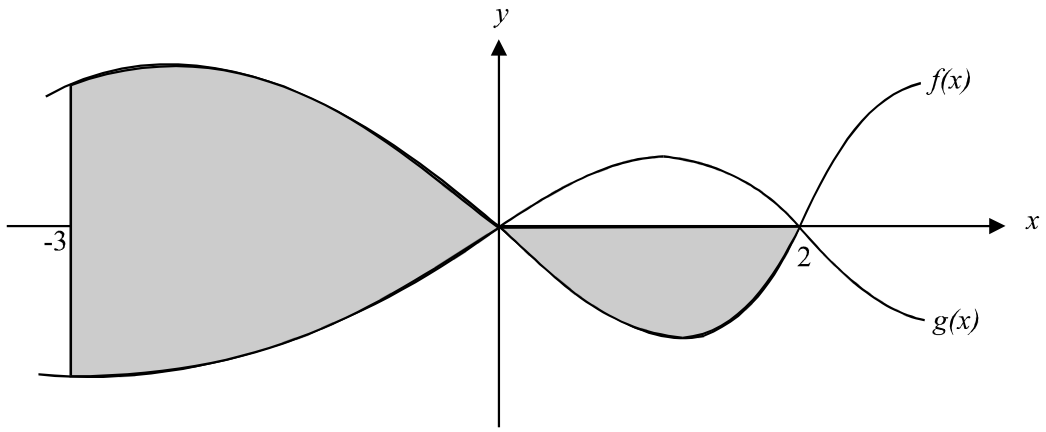
An approximate value of $\int_1^3 (-x^2 + 10)dx$ can be calculated using the rectangles shown in the diagram below.



That approximate value is

A. 6
B. 11
C. 22
D. 25
E. 60

Question 19



The total shaded area shown in the diagram above is given by

A. $\int_{-3}^{0}(f(x)-g(x))dx-\int_{0}^{2}f(x)dx$

B. $\int_{0}^{-3}(f(x)-g(x))dx+\int_{0}^{2}f(x)dx$

C. $\int_{0}^{-3}(f(x)+g(x))dx-\int_{0}^{2}g(x)dx$

D. $\int_{-3}^{0}(g(x)-f(x))dx-\int_{2}^{0}f(x)dx$

E. $\int_{-3}^{0}(g(x)+f(x))dx-\int_{2}^{0}g(x)dx$

Question 20

A trigonometric function is given by $f(x)=3\sin(4x-2\pi)+1$

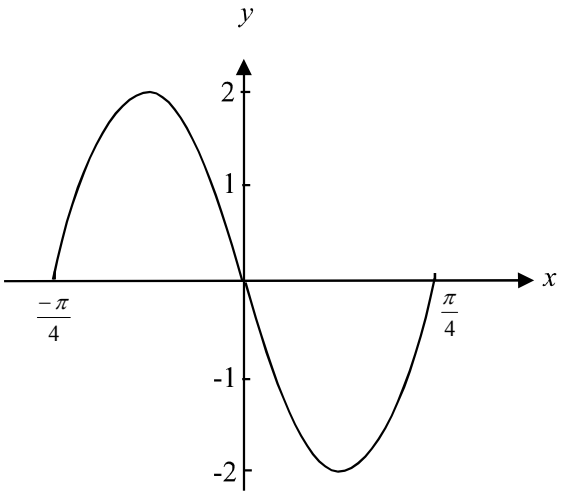
The "translation" of this function *f* is the number of units the basic graph of $y=\sin x$ has been translated horizontally to obtain *f*.

The amplitude, period and translation of *f* are given respectively by

	Amplitude	Period	Translation
A.	3	$\frac{\pi}{2}$	2π right
B.	3	$\frac{\pi}{2}$	$\frac{\pi}{2}$ right
C.	3	2π	2π right
D.	6	$\frac{\pi}{2}$	$\frac{\pi}{2}$ right
E.	6	2π	2π right

Question 21

One cycle of a trigonometric function *f* is shown below.



The rule for this function *f* could be

A. $y = -2\sin(x - \frac{\pi}{4})$

B. $y = 2\cos 2(x + \frac{\pi}{4})$

C. $y = 2\sin 4(x + \frac{\pi}{4})$

D. $y = 4\sin 2(x - \frac{\pi}{4})$

E. $y = 4\cos 4(x - \frac{\pi}{4})$

Question 22

The sum of the solutions to the equation $10\cos(3x) = 5$ over the domain $[0, \pi]$ is

A. $\frac{\pi}{9}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{13\pi}{18}$

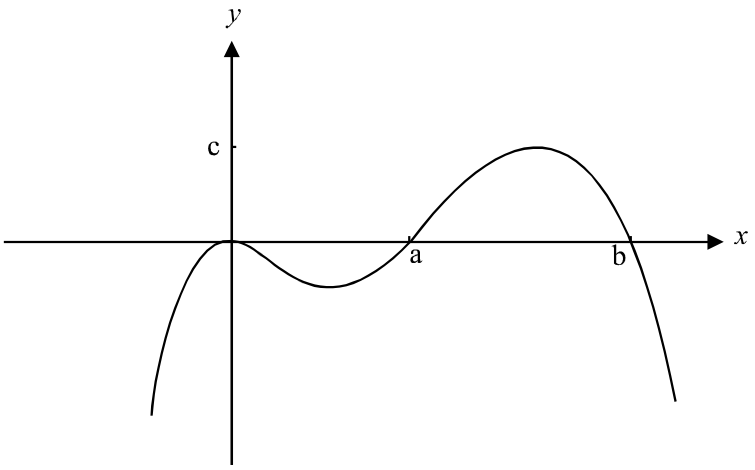
E. $\frac{13\pi}{9}$

Question 23

Let $f(x) = a\sin(x-b)$ where a and b are positive constants. The equation $f(x) = c,\ 0 < c < a$, will certainly have no solutions over the interval $[0,\ \pi]$ if b equals

A. 0
B. $\frac{\pi}{4}$
C. $\frac{\pi}{2}$
D. π
E. 2π

Question 24



The equation of the graph shown above is

A. $y = -x(x-a)(x-b)$
B. $y = x^2(ax-b)^2 + c$
C. $y = x(x-a)(x-b)$
D. $y = -x^2(x-a)(x-b)$
E. $y = -x(x^2-a)(x-b)$

Question 25

In the expansion of $(3x-a)^6$, $a > 0,$ the coefficient of x^4 is 4860.
The value of a is

A. -2
B. 2
C. -2 or 2
D. 4
E. $\sqrt{60}$

Question 26

The function $f:[a, b] \to R$ where $f(x) = x^4 - x^2$ has an inverse function f^{-1} if

A. $a = -1$ and $b = 0$
B. $a = -1$ and $b = 1$
C. $a = 0$ and $b = 1$
D. $a = -\frac{1}{\sqrt{2}}$ and $b = 0$
E. $a = -\frac{1}{\sqrt{2}}$ and $b = \frac{1}{\sqrt{2}}$

Question 27

The function $g:[1, \infty) \to R,$ where $g(x) = 2(x-1)^2 + 3$ has an inverse function g^{-1} which is defined by

A. $g^{-1}:(-\infty,1] \to R,$ where $g^{-1}(x) = 2(x-1)^2 + 3$
B. $g^{-1}:[1, \infty) \to R,$ where $g^{-1}(x) = \sqrt{\frac{x-3}{2}} + 1$
C. $g^{-1}:[3, \infty) \to R,$ where $g^{-1}(x) = \sqrt{\frac{x-1}{2}} + 3$
D. $g^{-1}:[3, \infty) \to R,$ where $g^{-1}(x) = \sqrt{\frac{x-3}{2}} + 1$
E. $g^{-1}:[1, \infty) \to R,$ where $g^{-1}(x) = \sqrt{\frac{x-1}{2}} + 3$

Question 28

If $\frac{1}{2}\log_2 2 - \log_2 \sqrt{8} - \log_2 x = 1$, then x is equal to

A. 0
B. $\frac{1}{4}$
C. $\sqrt{2}$
D. 2
E. 4

PART II Short answer questions

Question 1

Factorise $x^4 - 6x^3 - x^2 + 6x$

2 marks

Question 2

The number of trains, X, travelling from a country town to the city in a day, has a probability distribution shown in the table below.

X	0	1	2	3	4
p(x)	$\frac{2}{q}$	$\frac{7}{2q}$	0.2	$\frac{3}{2q}$	$\frac{1}{q}$

a. Find the value of q.

1 mark

b. Find the expected number of trains, correct to 2 decimal places, travelling from this country town to the city in a day.

1 mark

c. Find the probability, correct to 2 decimal places, that at least 1 train will travel from this country town to the city in a day.

1 mark

Question 3

The time taken for Kerry to complete a mini-triathlon varies according to a normal distribution with a mean of 72 minutes and a standard deviation of 5 minutes.
Kerry's coach notices that at 30% of the many races in which she has competed, she has taken less than n minutes to complete the course.
Calculate the value of n to 2 decimal places.

2 marks

Question 4

a. Find the gradient of the curve with equation $y = 10^{\frac{x}{2}}$ at the point where $x = 1.2$
Express your answer correct to 3 decimal places.

1 mark

b. Since $10^{\frac{1.2}{2}} = 10^{0.6}$ and $10^{0.6} \approx 3.981,$ use an appropriate formula to find an approximate value for $10^{0.65}$. Express your answer correct to 2 decimal places.

2 marks

Question 5

The velocity ,v (m/secs), of a billycart at any time t (secs), during a race is given by

$$v = 0.5\log_e(t^2 + 1) + 2, \quad t \geq 0$$

a. Calculate the average rate of change of velocity of the billycart during the first 3 seconds of the race correct to 1 decimal place.

_____

_____

_____

1 mark

b. Find the instantaneous rate of change of velocity at time t = 3 seconds.

_____

_____

_____

_____

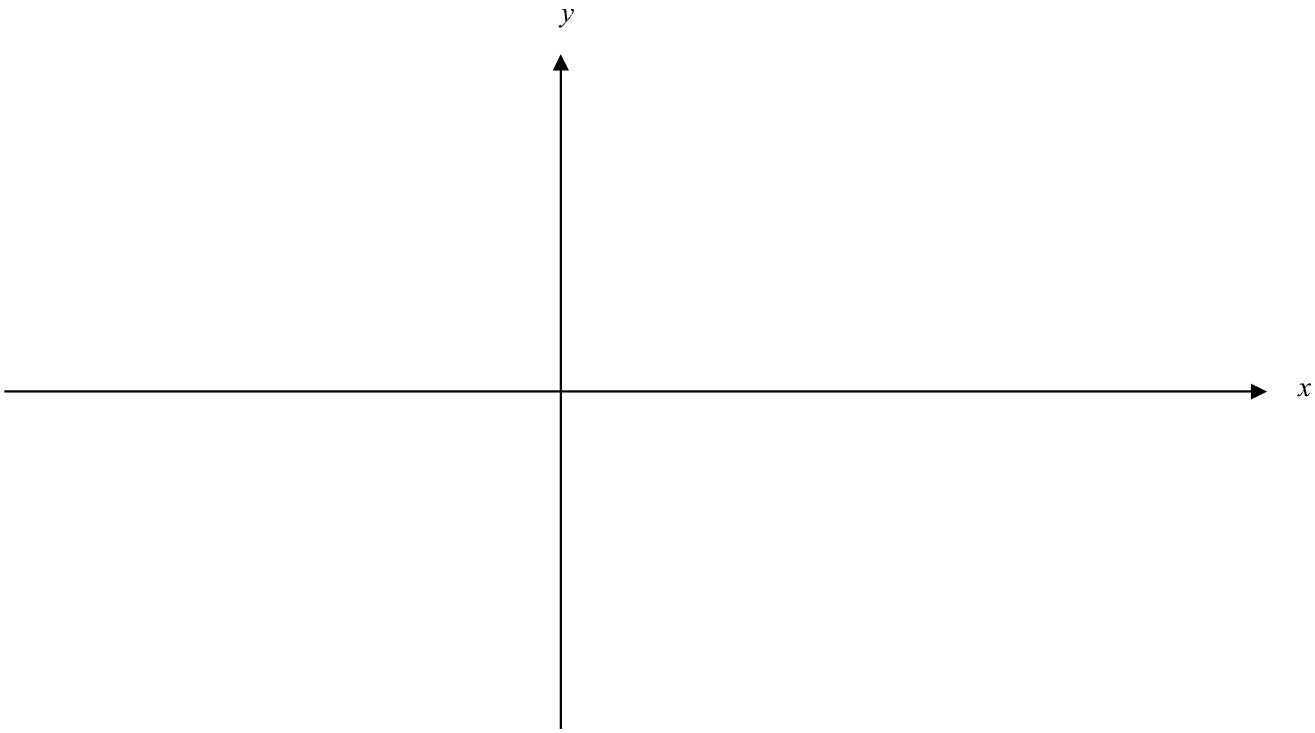
_____

_____

2 marks

Question 6

a. On the set of axes below sketch the graph of $f : (\frac{-3\pi}{4}, \frac{3\pi}{4}) \to R$ where $f(x) = \tan(2x)$ showing clearly the important features of the graph.



2 marks

b. Using your graph in part **a.** or otherwise, find the solutions to the equation

$$\sqrt{3}\sin 2x = \frac{3}{\sqrt{3}}\cos 2x \text{ over the domain } (\frac{-3\pi}{4}, \frac{3\pi}{4})$$

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1 mark

Question 7

The tangent to the curve with equation $y = 2x^2 + 1$ at the point where $x = 2$ intersects with the normal to the curve with equation $y = \sqrt{x - 2}$ at the point where $x = 3$.
Find the coordinates of this point of intersection.

3 marks

Question 8

Find the area bounded by the curves with equations $y = \frac{1}{2}\sin 2x$ and $y = -\cos\frac{x}{3}$, the line $x = \frac{\pi}{2}$ and the y axis.

3 marks
Total 22 marks

MATHEMATICAL METHODS

TRIAL EXAM 1

2000

PART I

MULTIPLE-CHOICE ANSWER SHEET

STUDENT NAME: .....

INSTRUCTIONS
Fill-in the letter that corresponds to your choice. Example: (A) (B) (C) (D) (E)
The answer selected is B. Only one answer should be selected.

1. (A) (B) (C) (D) (E)	11. (A) (B) (C) (D) (E)	20. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)	12. (A) (B) (C) (D) (E)	21. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)	13. (A) (B) (C) (D) (E)	22. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)	14. (A) (B) (C) (D) (E)	23. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)	15. (A) (B) (C) (D) (E)	24. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)	16. (A) (B) (C) (D) (E)	25. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)	17. (A) (B) (C) (D) (E)	26. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)	18. (A) (B) (C) (D) (E)	27. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)	19. (A) (B) (C) (D) (E)	28. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)		

