Trial CAT 2 Answers & Solutions

Part I (Multiple-choice) Answers

1.	D	2.	D	3.	C	4.	C	5.	C
6.	D	7.	C	8.	A	9.	A	10.	E
11.	E	12.	D	13.	E	14.	A	15.	C
16.	B	17.	E	18.	C	19.	B	20.	B
21.	B	22.	C	23.	A	24.	B	25.	C
26.	C	27.	B	28.	E	29.	D	30.	A
31.	D	32.	D	33.	C				

Part I (Multiple-choice) Solutions

Question 1 [D]
The point (4, 0) is substituted

A. $3(0) \neq -4(4) + 3$
$0 \neq -13$

B. $4(0) \neq -3(4) + 16$
$0 \neq 4$

C. $3(0) \neq -4(4) + 12$
$0 \neq -4$

D. $4(0) = -3(4) + 12$ Only possible solution
$0 = 0$

E. $3(0) \neq -4(4) + 7$
$0 \neq -9$

Question 2 [D]
Interchange x and y to obtain the inverse.

$$f(x) = -2 + 3e^{2x-1}$$
$$x = -2 + 3e^{2f(x)-1}$$
$$x + 2 = 3e^{2f(x)-1}$$
$$\frac{x+2}{3} = e^{2f(x)-1}$$
$$\log_e(\frac{x+2}{3}) = 2f(x) - 1$$
$$1 + \log_e(\frac{x+2}{3}) = 2f(x)$$
$$\frac{1}{2} + \frac{1}{2}\log_e(\frac{x+2}{3}) = f(x)^{-1}$$

Since the domain of the inverse is the range of the function, the range of $f(x) = -2 + 3e^{2x-1}$ is $(-2, \infty)$ and the domain of the inverse is $(-2, \infty)$.

Question 3 [C]
The period is halved and the amplitude is doubled.

Question 4 [C]

$$f'(x) = -\frac{1}{2}a\sin\frac{x}{2}$$
$$f'\left(\frac{\pi}{3}\right) = -\frac{1}{2}a\sin\frac{\pi}{6}$$
$$= -\frac{1}{2}a \times \frac{1}{2}$$
$$= -\frac{a}{4}$$
$$-\frac{a}{4} = -\sqrt{3}$$
$$a = 4\sqrt{3}$$

Question 5 [C]
x-value of turning point is given by $\frac{-a+b}{2}$.
Minimum value of f then is

$$f\left(\frac{-a+b}{2}\right) = f\left(\frac{b-a}{2}\right)$$

Question 6 [D]
At $(b, 0)$ the graph has a turning point, which suggests that $(x - b)^2$ must be part of the equation. Therefore **A** and **E** can not be considered. $(c, 0)$ creates another factor $(x - c)$. Therefore **C** is not the solution. The graph is a negative quartic suggesting that $a < 0$. Hence **D** is the answer.

Question 7 [C]
When $x = 0$, $f(x) = e^b + c$. Therefore only **C** and **D** can be considered. If positive values of x are used then e^{-x+b} will approach zero therefore $f(x)$ will approach c and since $c < 0$, the graph of $f(x)$ will be below the x-axis.

Question 8 [A]
$y = -(x - 3)^2 + 4$ is transformed from $y = x^2$ by:

1. + 4 suggests a translation of 4 units parallel to the y-axis
2. $x - 3$ suggests a translation of 3 units along the x-axis
3. The negative sign produces a reflection in the x-axis.

Only response **A** has all these characteristics.

Question 9 [A]
$f(x) = 2(x - 2)^2 + 2$, $x \in [0, 7)$
At $x = 0$ $f(x) = 10$
At $x = 7$ $f(x) = 52$
The function is a quadratic and the minimum turning point occurs at $x = 2$. This point is within the domain and therefore will be the minimum point for the range.
At $x = 2$ $f(x) = 2$
Therefore the range is [2, 52)

Question 10 [E]
"Undefined" in the middle of the data table does not suggest a quadratic, exponential or logarithmic function. Therefore only the two reciprocal functions need be considered. Since the data is symmetrical on either side of $x = 1$, the answer must be **E**.

Question 11 [E]
The shaded area can be calculated by integrating $f(x) - g(x)$ between the values of b and a. This would be represented by the following:

$$\int_b^a f(x) - g(x)dx$$

Question 12 [D]

$$\int_{-2}^{1} (3x+2)^3 dx$$

$$= \left[\frac{1}{3\times 4}(3x+2)^4\right]_{-2}^{1} = \left[\frac{1}{12}(3x+2)^4\right]_{-2}^{1}$$

$$= \left[\frac{1}{12}(3+2)^4\right] - \left[\frac{1}{12}(-6+2)^4\right]$$

$$= \left[\frac{1}{12}(5)^4\right] - \left[\frac{1}{12}(-4)^4\right]$$

$$= \left[\frac{625}{12}\right] - \left[\frac{256}{12}\right] = \frac{369}{12} = 30\frac{3}{4}$$

Question 13 [E]

$$f(x) = \frac{1}{2}(x-3)(x+2) = \frac{1}{2}x^2 - \frac{1}{2}x - 3$$

$$f'(x) = x - \frac{1}{2} \text{ set } f'(x) = -1$$

$$-1 = x - \frac{1}{2}, x = -\frac{1}{2}$$

Substitute into the original equation:

$$f(x) = \frac{1}{2}(-\frac{7}{2})(\frac{3}{2}) = -\frac{21}{8}$$

Therefore the coordinate is $(-\frac{1}{2}, -\frac{21}{8})$

Question 14 [A]

At $x = 0$, $y = 2\sin(0)$

$\therefore y = 0$

$$\frac{dy}{dx} = 6\cos(3x)$$

at $x = 0$ $\quad \frac{dy}{dx} = 6\cos(0)$

$$\frac{dy}{dx} = 6 \text{ (gradient of the tangent)}$$

The gradient of the normal to the curve at this point will be $-\frac{1}{6}$. Since the normal goes through the point (0, 0), the equation becomes $y = -\frac{1}{6}x$

Question 15 [C]
The area of each of the rectangles is:
Large rectangle, $2 \times 1 = 2$
Small rectangle, $1.5 \times 1 = 1.5$
Using symmetry there are 2 of each of the rectangles.
Therefore, the total area is: $2 + 2 + 1.5 + 1.5 = 7$ square units.

Question 16 [B]
Minimum value is $-3 - 1 = -4$

Period is $\frac{2\pi}{\frac{1}{4}} = 8\pi$

Question 17 [E]

$$\cos 2x + \sqrt{3}\sin 2x = 0$$

$$\sqrt{3}\sin 2x = -\cos 2x$$

$$\tan 2x = -\frac{1}{\sqrt{3}}$$

$$2x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, \ldots$$

$$x = \frac{5\pi}{12}, \frac{11\pi}{12}, \ldots$$

Question 18 [C]

$\sin(3x) = 1 \quad 0 \le x \le \pi$

$3x = \text{Sin}^{-1}(1)$

$$3x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2} \ldots\ldots\ldots$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6} \ldots\ldots\ldots$$

The only solutions that are required are the ones in the above domain. These are $\frac{\pi}{6}, \frac{5\pi}{6}$ the sum of these solutions is π.

Question 19 [B]

The period of this graph is $\frac{2\pi}{b}$. Since the period is π, $b = 2$. Only **B** and **C** can be considered. The correct option is determined by the amplitude.

From the graph the amplitude is $\frac{3}{2}$

Question 20 [B]

$$3x(1 + \log_3 x^3 - 2\log_3 x) = 3x(1 + 3\log_3 x - 2\log_3 x)$$

$$= 3x(1 + \log_3 x)$$

$$= 3x(\log_3 3 + \log_3 x)$$

$$= 3x\log_3 3x$$

Question 21 [B]

$$(2x-3)^9 = (2x)^9 - 9(2x)^8(3) + {}^9C_2(2x)^7 3^2 - {}^9C_3(2x)^6 3^3$$

$$T_4 = -{}^9C_3 \times 2^6 3^3 \times x^6$$

$$= -84 \times 1728 \times x^6$$

$$= -145152x^6$$

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Question 22 [C]
The resulting function must be one-to-one. **C**, **D**, **E** all fit this scenario, but both **D** and **E** are outside the domain, since -1 and 3 are excluded.

Question 23 [A]

$$\frac{d\log_e(2-x)}{dx} = \frac{1}{2-x}(-1)$$

$$= \frac{1}{2-x}$$

$$\frac{dy}{dx} = x\left(\frac{1}{x-2}\right) + \log_e(2-x)$$

$$= \frac{x}{x-2} + \log_e(2-x)$$

$$= \log_e(2-x) - \frac{x}{2-x}$$

Question 24 [B]

$$\int \sin(3x) + (3x-2)^4 dx = -\frac{1}{3}\cos(3x) + \frac{1}{3\times 5}(3x-5)^5 + c$$

$$= \frac{1}{3}\left[\frac{(3x-2)^5}{5} - \cos(3x)\right] + c$$

Question 25 [C]
Using the quotient rule,

$$\frac{dy}{dx} = \frac{2x\cos(x^2)\times 2x - 2\sin(x^2)}{4x^2}$$

$$= \frac{4x^2\cos(x^2) - 2\sin(x^2)}{4x^2}$$

$$= \frac{2x^2\cos(x^2) - \sin(x^2)}{2x^2}$$

Question 26 [C]

$$f(x) = \int\left(\frac{1}{2x-1} - 2x^3\right)dx$$

$$= \frac{1}{2}\int\left(\frac{2}{2x-1} - 4x^3\right)dx$$

$$= \frac{1}{2}\left[\log_e(2x-1) - x^4\right] + c$$

$$f(1) = 0 \Rightarrow \frac{1}{2}\left[\log_e(2\times 1 - 1) - 1^4\right] + c = 0$$

$$\frac{1}{2}(-1) + c = 0$$

$$c = \frac{1}{2}$$

$$f(x) = \frac{1}{2}\log_e(2x-1) - \frac{1}{2}x^4 + \frac{1}{2}$$

Question 27 [B]
$\Pr(X > 2) = 0.2 + 0.2 = 0.4$

Question 28[E]
Let X be the number of days she is called for work.

$X \sim \text{Bi}(7, 0.3)$

$$\Pr(X = 2) = {}^7C_2 0.3^2 0.7^5$$

$$= 21 \times 0.3^2 \times 0.7^5$$

$$\approx 0.318$$

Question 29 [D]

$$\Pr(X > 10) = \Pr(X^* > 10.5)$$

$$= \Pr\left(Z > \frac{X^* - \mu}{\sigma}\right)$$

$$= \Pr\left(Z > \frac{10.5 - 20}{4}\right)$$

$$= \Pr\left(Z > \frac{-9.5}{4}\right)$$

$$= \Pr\left(Z < \frac{9.5}{4}\right)$$

Question 30 [A]
The 67% confidence interval is regarded as being two standard deviations on either side of the mean.

$a = 20 - 4$ and $b = 20 + 4$

$a = 16$ and $b = 24$

Question 31 [D]

$$\hat{p} = \frac{124}{300} = 0.4133$$

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4133 \times (0.5866)}{300}}$$

$\sigma = 0.0284$

$2\sigma = 0.0569$

95% confidence interval is given by
(0.4133 - 0.0569, 0.4133 - 0.0569) = (0.356, 0.470)

Question 32 [D]
The probability that Sam scores no more than 2 holes in one is equivalent to the sum of the probabilities of zero holes in one, one hole in one and two holes in one.
The sum of these three probabilities is answer **D**.

Question 33 [C]

$$\bar{x} = \frac{\sum xf}{\sum f}$$

$$= \frac{(1\times 5) + (2\times 6) + (3\times 10) + (4\times 8) + (5\times 11) + (6\times 4) + (7\times 6)}{50}$$

$$= 4$$

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Part II (Short Answer Questions) Solutions

Question 1

When $x = \pi$,

$\sqrt{3}\cos(ax) = \sin(ax)$

$\sqrt{3}\cos(a\pi) = \sin(a\pi)$

$\sqrt{3}\tan(a\pi) = 1$

$\tan(a\pi) = \sqrt{3}$

$a\pi = \frac{\pi}{3}, \frac{4\pi}{3}, \ldots$

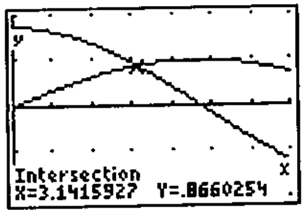
$a = \frac{1}{3}, \frac{4}{3}, \ldots$ [1A]

When $x = \pi$,

$\sin(ax) = \sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2}$

$a = \frac{1}{3}$ and $b = \frac{\sqrt{3}}{2}$ [1A]



Question 2

Using a combination of the chain and product rules,

$f'(x) = 3(2x-3)^2 + (3x+1)\times 3(2x-3)^2 \times 2$ [1M]

$= 3(2x-3)^2[2x-3+2(3x+1)]$

$= 3(2x-3)^2(8x-1)$ [1A]

Question 3

The midpoint for PQ $= \left(\frac{-6+2}{2}, \frac{7+3}{2}\right)$

$= (-2, 5)$ [1A]

The midpoint for RQ $= \left(\frac{-2+2}{2}, \frac{3-5}{2}\right)$

$= (0, -1)$ [1A]

The distance between the midpoints is:

$= \sqrt{(-2-0)^2 + (5--1)^2}$

$= \sqrt{(2)^2 + (6)^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$

$= 6.32$ [1A]

Question 4

a. $h(25) = 30\cos\left(\frac{25\pi}{100}\right) + 40$

$= 30\cos\left(\frac{\pi}{4}\right) + 40$

$= \frac{30}{\sqrt{2}} + 40$

≈ 61.21 metres [1A]

b. $30\cos\left(\frac{\pi x}{100}\right) + 40 = 40$ [1M]

$30\cos\left(\frac{\pi x}{100}\right) = 0$

$\cos\left(\frac{\pi x}{100}\right) = 0$

$\frac{\pi x}{100} = \frac{\pi}{2}, \frac{3\pi}{2}$

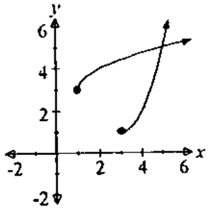
$\frac{x}{100} = \frac{1}{2}, \frac{3}{2}$

$x = 50,\ 150$

150 km is outside the domain, so $x = 50$ km. [1A]

Question 5

a.



[1A]

b. Let $y = 3 + \sqrt{x-1}$

Interchange x and y:

$x = 3 + \sqrt{y-1}$

$\sqrt{y-1} = x - 3$

$y - 1 = (x-3)^2$

$y = (x-3)^2 + 1$ [1A]

$f^{-1} : [3, \infty) \to R$, where $f^{-1}(x) = (x-3)^2 + 1$

Question 6

a. $\text{Var}(x) = npq$

$n = 15$, $\mu = np = 12$

$p = \frac{12}{15} = \frac{4}{5}$

$q = 1 - p = 1 - \frac{4}{5} = \frac{1}{5}$

$npq = 15 \times \frac{4}{5} \times \frac{1}{5} = \frac{12}{5}$ [1A]

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b. $\Pr(X < 14)$

$= 1 - [\Pr(X = 14) + \Pr(X = 15)]$

$= 1 - [{}^{15}C_{14}(0.8)^{14}(0.2)^1 + (0.8)^{15}]$ [1M]

$= 1 - [0.13194 + 0.03518]$

$\approx 1 - 0.16712$

$= 0.83$ [1A]

Question 7

$z = \frac{x-\mu}{\sigma}$

$z = \frac{b-30}{2a}$

$\Pr(X > b) = 0.3 \Rightarrow \Pr(Z > \frac{b-30}{2a}) = 0.3$

By symmetry, $1 - \Pr(Z < \frac{b-30}{2a}) = 0.3$

$\Pr(Z < \frac{b-30}{2a}) = 0.7$ [1M]

From the tables, $z = 0.524$

$0.524 = \frac{b-30}{2a}$

$1.048a = b - 30$

$b = 30 + 1.048a$ [1A]

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