Victorian Certificate of Education
1998

MATHEMATICAL METHODS

Common Assessment Task 2: Written examination
(Facts, skills and applications task)

Thursday 5 November 1998: 9.00 am to 10.45 am
Reading time: 9.00 am to 9.15 am
Writing time: 9.15 am to 10.45 am
Total writing time: 1 hour 30 minutes

PART I

MULTIPLE-CHOICE QUESTION BOOK

Directions to students

This task has two parts: Part I (multiple-choice questions) and Part II (short-answer questions).
Part I consists of this question book and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer book.
You must complete **both** parts in the time allotted. When you have completed one part continue immediately to the other part.
A detachable formula sheet for use in both parts is in the centrefold of this book.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
You may retain this question book.

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
33	33	33

Directions to students

Materials
Question book of 17 pages, including one blank page for rough working.
Answer sheet for multiple-choice questions.
You may bring to the CAT up to four pages (two A4 sheets) of pre-written notes.
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.
You should have at least one pencil and an eraser.

The task
Detach the formula sheet from the centre of this book during reading time.
Ensure that you write your **name and student number** on the answer sheet for multiple-choice questions.
Answer **all** questions.
There is a total of 33 marks available for Part I.
All questions should be answered on the answer sheet provided for multiple-choice questions.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the task
Place the answer sheet for multiple-choice questions (Part I) inside the front cover of the question and answer book (Part II).
You may retain this question book.

Specific instructions to students

This part consists of 33 questions.

Answer **all** questions in this part on the answer sheet provided for multiple-choice questions. A correct answer scores 1, an incorrect answer scores 0.

Marks will not be deducted for incorrect answers. You should attempt every question.

No credit will be given for a question if two or more letters are marked for that question.

The following information relates to Questions 1 and 2.

The depth of water in a channel changes with the tides according to the rule $y = 5 - 2\cos\left(\frac{\pi t}{8}\right)$, where t is the time in hours after the low tide and y is the depth of water in the channel in metres. On a particular day a low tide occurs at 9 am.

Question 1

The next **high** tide occurs at

A. 3 pm.

B. 5 pm.

C. 7 pm.

D. 8 pm.

E. 1 am the next day.

Question 2

At 9 pm on the same day, the height of the water in the channel is

A. 2 metres.

B. 3 metres.

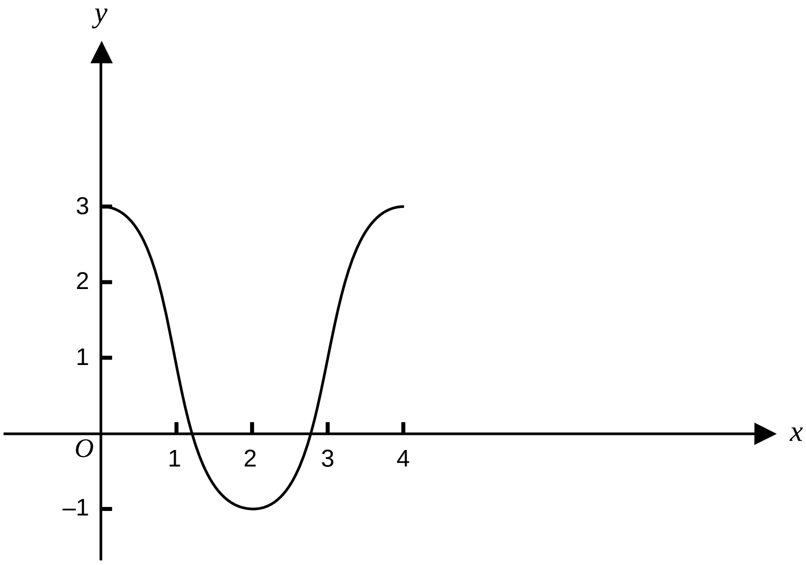
C. 3.5 metres.

D. 5 metres.

E. 7 metres.

Question 3

The graph below shows one cycle of a trigonometric function.



The rule for this function could be

A. $y = 2 + \cos\left(\frac{\pi x}{2}\right)$

B. $y = 1 + 2\cos(2\pi x)$

C. $y = 1 + 2\cos\left(\frac{\pi x}{2}\right)$

D. $y = 1 + 2\sin\left(\frac{\pi x}{2}\right)$

E. $y = 2 + \sin(2\pi x)$

Question 4

A solution of the equation $\sin(2x) - a\cos(2x) = 0$ is $\frac{\pi}{6}$. The value of a is

A. $-\sqrt{3}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{3}}$

E. $\sqrt{3}$

Question 5

For the equation $2 \sin x + \sqrt{3} = 0$ the **sum** of the solutions on the interval $[0, 2\pi]$ is

A. π

B. $\frac{5\pi}{3}$

C. 2π

D. $\frac{7\pi}{3}$

E. 3π

Question 6

If $3 \sin(2x) = -\sqrt{3} \cos(2x)$ then

A. $\tan x = \frac{-1}{\sqrt{3}}$

B. $\tan x = \sqrt{3}$

C. $\tan(2x) = \sqrt{3}$

D. $\tan(2x) = \frac{-1}{\sqrt{3}}$

E. $\tan(2x) = -\sqrt{3}$

Question 7

The equations of four lines L_1, L_2, L_3 and L_4 are as follows

$L_1 : 3x + 4y = 5$

$L_2 : 6x + 8y = -7$

$L_3 : 5x + 8y = 10$

$L_4 : 8x - 5y = 16$

Which one of the following statements is correct?

A. All four lines are parallel.

B. L_1 is perpendicular to L_4.

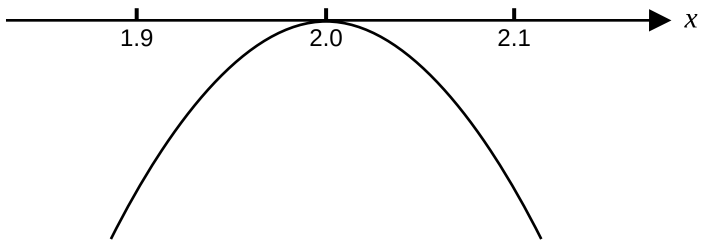
C. L_2 is perpendicular to L_4.

D. L_1 is parallel to L_3.

E. L_3 is perpendicular to L_4.

Question 8

A polynomial p(x) has degree four. The portion of its graph near the value $x = 2$ looks like



Which one of the following might be the fourth degree polynomial p(x)?

A. $p(x) = -(x-2)^2$

B. $p(x) = x(x-1)(x-2)^2$

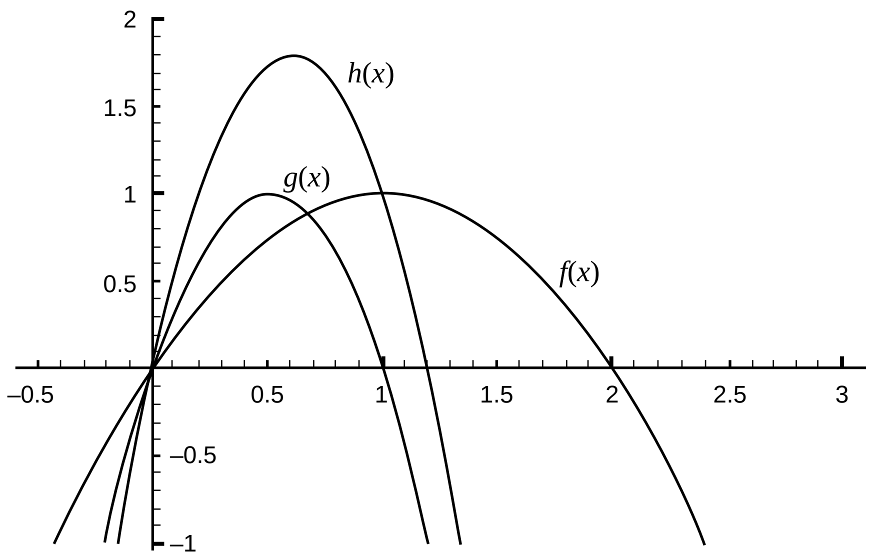
C. $p(x) = x(x-2)^3$

D. $p(x) = x(1-x)(x-2)^2$

E. $p(x) = x(x+1)(x-2)^2$

Question 9

The diagram below shows the graphs of three functions f, g and h.



Which one of the following statements is true?

A. $f(x) = g(x) + h(x)$

B. $f(x) + g(x) = h(x)$

C. $f(x) = 2g(x)$

D. $g(x) = h(x+1)$

E. $h(x) = 2g(x)$

Question 10

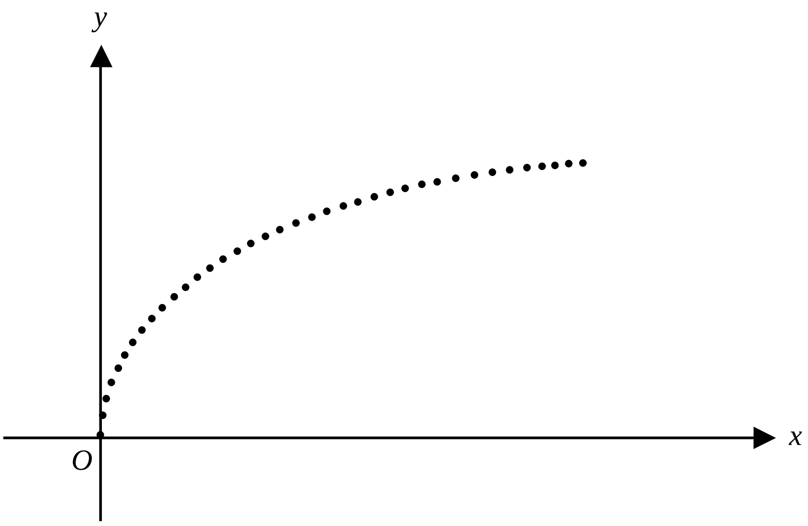
A function with rule $f(x) = x^{-2}$ can be defined on different domains. In the list below the first set represents the domain selected and the second set the corresponding range, but in one case the range is incorrect.

Which pair **does not** have the correct range for the given domain?

	domain	**range**
A.	$[-2, -1]$	$\left[\frac{1}{4}, 1\right]$
B.	$[-2, 0) \cup (0, 1]$	$\left[\frac{1}{4}, \infty\right)$
C.	$[-1, 1] \setminus \{0\}$	$[1, \infty)$
D.	$[-1, 2] \setminus \{0\}$	$\left[\frac{1}{4}, 1\right]$
E.	$[1, 2)$	$\left(\frac{1}{4}, 1\right]$

Question 11

Data about the relationship between two quantities x and y are represented graphically as shown below.



If a is a positive constant, the equation relating x and y is most likely to be of the form

A. $y = \dfrac{a}{x}$

B. $y = \dfrac{a}{x^2}$

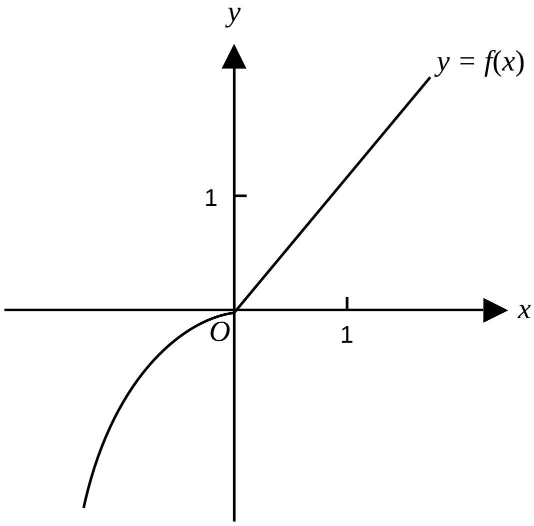
C. $y = ae^x$

D. $y = ax^2$

E. $y = ax^{\frac{1}{2}}$

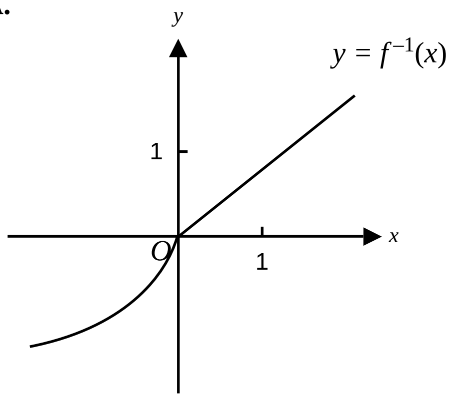
Question 12

The graph of the function f is shown below.

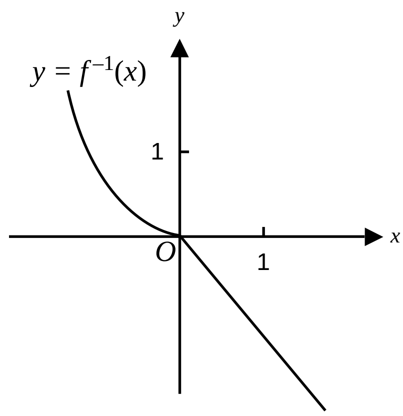


The graph of the inverse function f^{-1} is most likely to be

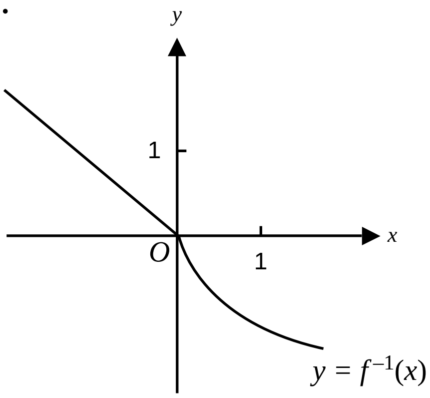
A.



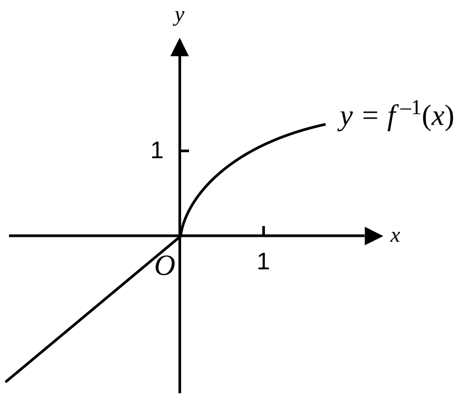
B.



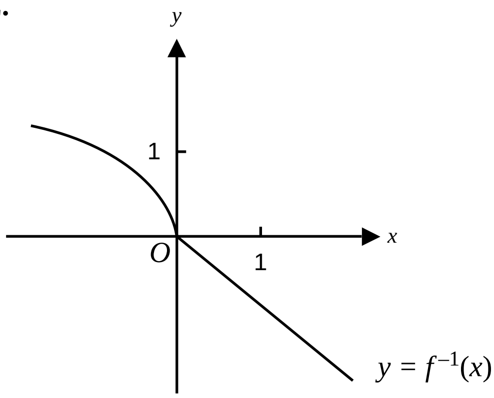
C.



D.



E.



Question 13

The line with equation $y = -x + c$ is a tangent to the curve with equation $y = x^2 + 1$. The value of c is

A. $\frac{-1}{2}$

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. $\frac{5}{4}$

E. 4

Question 14

If $y = e^{(2\cos(3x))}$, then $\frac{dy}{dx}$ is equal to

A. $6\sin(3x)e^{(2\cos(3x))}$

B. $e^{(2\cos(3x))}$

C. $-6\sin(3x)e^{(2\cos(3x))}$

D. $e^{(-6\sin(3x))}$

E. $-6\cos(3x)e^{(-2\sin(3x))}$

Question 15

If $y = e^{(ax)}\cos(bx)$, where a and b are constants, then $\frac{dy}{dx}$ is equal to

A. $ae^{(ax)}\cos(bx) - be^{(ax)}\sin(bx)$

B. $ae^{(ax)}\cos(bx) + be^{(ax)}\sin(bx)$

C. $-bae^{(ax)}\sin(bx)$

D. $e^{(ax)}\cos(bx) - be^{(ax)}\sin(bx)$

E. $\left(\frac{1}{ab}\right)e^{(ax)}\sin(bx)$

Question 16

An anti-derivative of $\dfrac{1}{(2x+3)} - \cos(2x+3),\ x > 0,$ is

A. $\dfrac{-2}{(2x+3)^2} + 2\sin(2x+3)$

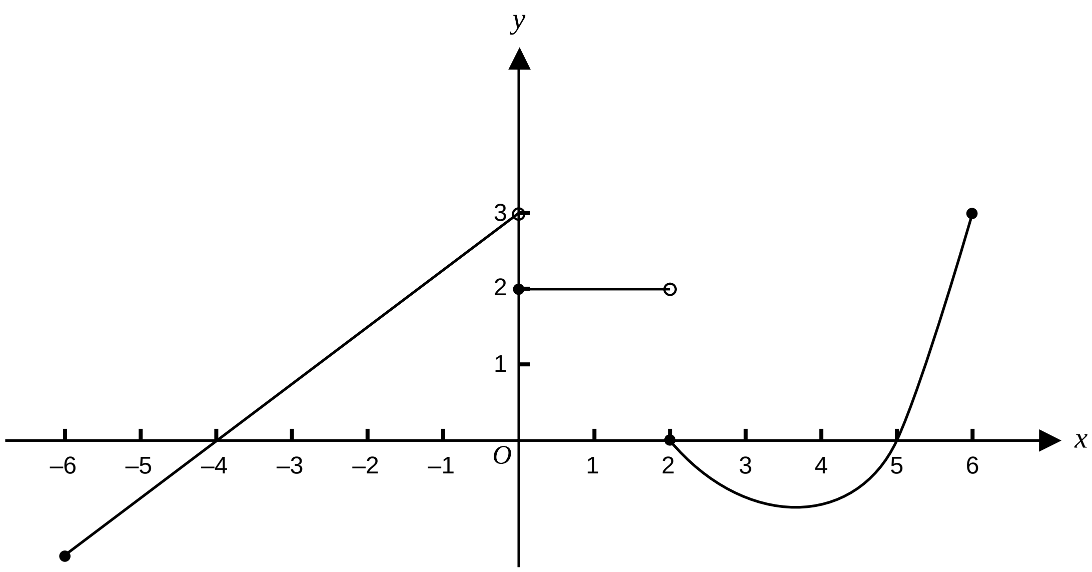
B. $\dfrac{-2}{(2x+3)^2} + \dfrac{1}{2}\sin(2x+3)$

C. $\dfrac{1}{2}\log_e(2x+3) - \dfrac{1}{2}\sin(2x+3)$

D. $\log_e(2x+3) + \sin(2x+3)$

E. $\dfrac{1}{2}\log_e(2x+3) + \dfrac{1}{2}\sin(2x+3)$

Question 17



For the function whose graph is shown above, which one of the following statements is true?

A. The value of the derived function increases as x increases for x ε (2, 6).
B. The value of the derived function is positive only when x is negative.
C. The derived function does not exist for x ε (0, 2).
D. The derived function is continuous.
E. The derived function has a constant value for x ε (–6, 2).

Question 18

Given that $f'(x) = x^2 - \frac{1}{x}$ and $f(1) = 0$, then for $x > 0$, $f(x)$ is equal to

A. $2x + \frac{1}{x^2} - 3$

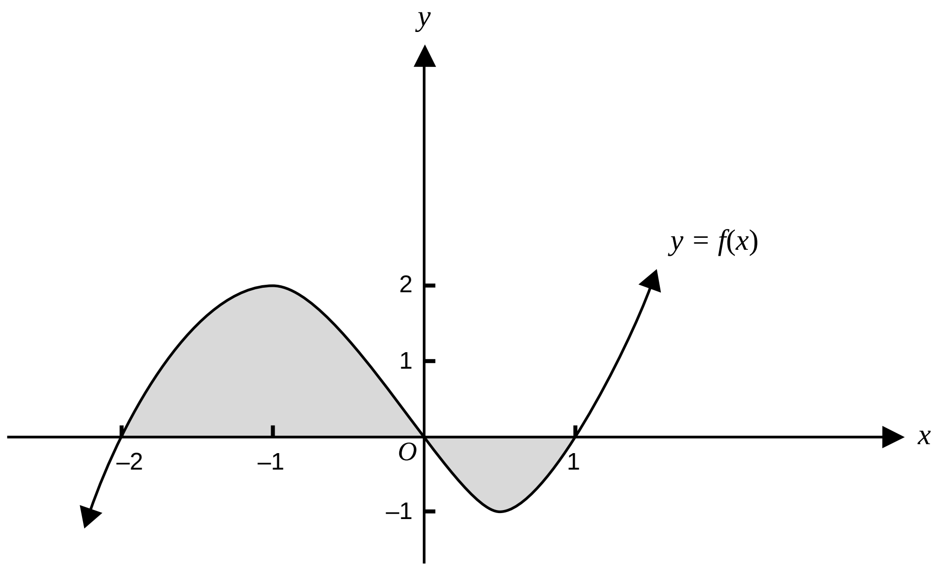
B. $\frac{x^3}{3} - \log_e x - \frac{1}{3}$

C. $\frac{x^3}{3} - \log_e x$

D. $\frac{x^3}{3} + \log_e x - \frac{1}{3}$

E. $2x - \frac{1}{x^2}$

Question 19



The total area of the shaded region shown is given by

A. $\int_{-2}^{1} f(x)dx$

B. $\int_{-2}^{0} f(x)dx + \int_{0}^{1} f(x)dx$

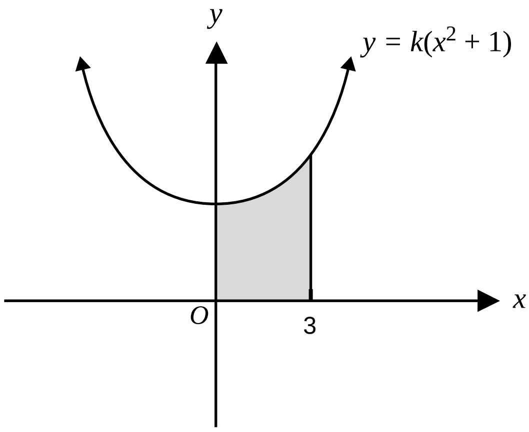
C. $\int_{-2}^{0} f(x)dx - \int_{0}^{1} f(x)dx$

D. $\int_{-1}^{2} f(x)dx$

E. $\int_{0}^{1} f(x)dx - \int_{-2}^{0} f(x)dx$

Question 20

The graph with equation $y = k(x^2 + 1)$, $k \in R$ is shown below. The area of the shaded region is equal to 1.



The value of k is

A. $\frac{2}{33}$

B. $\frac{1}{12}$

C. $\frac{1}{9}$

D. $\frac{1}{6}$

E. $\frac{1}{3}$

Question 21

Given that the derivative of $x \log_e x$ is equal to $1 + \log_e x$, which one of the following statements is true?

A. $\int x \log_e x \, dx = 1 + \log_e x + c$

B. $\int \log_e x \, dx = x \log_e x - x + c$

C. $\int \log_e x \, dx = x \log_e x - 1 + c$

D. $\frac{d}{dx}\left(\frac{1}{x \log_e x}\right) = \frac{1}{1 + \log_e x}$

E. $\frac{d}{dx}\left(1 + \log_e x\right) = x \log_e x$

Question 22

The coefficient of x^3 in $(2x + a)^6$ is 4320.

The value of a is

A. 2

B. 3

C. 6

D. 9

E. 12

Question 23

Which one of the following is a **complete** set of the linear factors of the fourth degree polynomial $ax^4 - bx^2$, where $a > 0$ and $b > 0$?

A. $x, x, ax^2 - b$

B. $x, x, ax - \sqrt{b},\ ax + \sqrt{b}$

C. $x, \sqrt{(ax - b)}$

D. $x, ax–b, ax–b$

E. $x, x, x\sqrt{a} - \sqrt{b}, x\sqrt{a} + \sqrt{b}$

Question 24

If $3 \log_{10} x - \log_{10} (x^2) = 1 + \log_{10} y$, then x is equal to

A. y

B. $y + 10$

C. $\dfrac{y}{10}$

D. $10y$

E. $\dfrac{10}{y}$

Question 25

The function f has rule $f(x) = (x - 1)^2 - 7$. Which one of the following sets is a possible domain for f if f^{-1} exists?

A. R

B. $[0, \infty)$

C. $(-\infty, 0]$

D. $(-1, \infty)$

E. $[-7, \infty)$

Question 26

The function $f\colon R \to R$ where $f(x) = e^{(2x)} + 1$ has an inverse f^{-1}.
The rule and domain of f^{-1} are given by

A. $f^{-1}(x) = \frac{1}{2}\log_e(x-1)$ domain of $f^{-1} = (1, \infty)$

B. $f^{-1}(x) = e^{(2x+1)}$ domain of $f^{-1} = R$

C. $f^{-1}(x) = \frac{x-1}{e^2}$ domain of $f^{-1} = (1, \infty)$

D. $f^{-1}(x) = \frac{x-1}{2}$ domain of $f^{-1} = R$

E. $f^{-1}(x) = 2\log_e(x-1)$ domain of $f^{-1} = (1, \infty)$

Question 27

Melissa constructs a spinner that will fall onto one of the numbers 1 to 5 with the following probabilities.

Number	1	2	3	4	5
Probability	0.3	0.2	0.1	0.1	0.3

The mean and standard deviation of the number that the spinner falls onto are, correct to two decimal places,

	mean	**standard deviation**
A.	3	1
B.	3	1.45
C.	3	2.10
D.	2.9	2.85
E.	2.9	1.64

Question 28

Kimberley, a Year 12 student, drives her father's car to school each day crossing just one intersection controlled by traffic lights. She has recorded that, in the long run, the traffic lights have been green on 40% of occasions.

In a two-week period of ten school days, the probability that the traffic lights have been green on exactly nine occasions is

A. ${}^{10}C_9\,(0.6)(0.4)^9$

B. ${}^{10}C_9\,(0.4)(0.6)^9$

C. ${}^{10}C_9\,(0.6)(0.4)^9 + (0.4)^{10}$

D. ${}^{10}C_9\,(0.4)(0.6)^9 + (0.6)^{10}$

E. $1 - (0.4)^{10}$

Question 29

The random variable X has a normal distribution with mean 10.2 and standard deviation 2.5. If Z has the standard normal distribution, then the probability that X is greater than 15.2 is equal to

A. $\Pr(Z < 2)$

B. $1 - \Pr(Z > 2)$

C. $\Pr(Z > 2)$

D. $\Pr(Z > -2)$

E. $1 - \Pr(Z < -2)$

Question 30

Callum carefully measures the quantity of milk contained in one-litre cartons of a particular brand and he finds that the actual quantity of milk is approximately normally distributed with a mean of 1.05 litres and a standard deviation of 0.03 litres.

The proportion of cartons that contain more than one litre of milk is closest to

A. 0.98

B. 0.95

C. 0.90

D. 0.10

E. 0.05

Question 31

The maximum daily temperature in Melbourne in January is approximately normally distributed with mean 28ºC. If the maximum temperature exceeds 35ºC on 10% of days, then the standard deviation of the maximum daily temperature is closest to

A. 0.70

B. 1.97

C. 2.33

D. 5.46

E. 12.97

Question 32

Two hundred people were given a taste test to find their preference between two brands of cola. 120 preferred Brand X and 80 preferred Brand Y.

If we think of this as a sample of the preference of the population, then the approximate 95% confidence interval for the proportion of the population who prefer Brand X is

A. [0.30, 0.50]

B. [0.33, 0.67]

C. [0.50, 0.70]

D. [0.53, 0.67]

E. [0.33, 0.47]

Question 33

Tran recorded the number of occasions he was late to school in a 100-day semester. The proportion of occasions when he was late was equal to 0.15. The standard error of this proportion is equal to, correct to four decimal places,

A. 0.0013

B. 0.0357

C. 0.0387

D. 0.0922

E. 0.3571

Working space

END OF PART I MULTIPLE-CHOICE QUESTION BOOK