1998 IARTU CAT ③

Question 1. **(11 marks)**

Consider the function $f: R \rightarrow R$ where $f(x) = -5e^{-x} + 3$.

1) State the range of f

(1 mark)

2) Find the exact values of the coordinates of the axes intercepts.

(3 marks)

3) Find the gradient of the normal to the curve at the point where $x = 1$.

(2 marks)

4) Give the rule for f^{-1}.

(3 marks)

5) Find when the inverse function has a value of $2\log_e 5$

(2 marks)

Question 2. **(15 marks)**

A sports coach for the HIGHFLYERS wishes to examine his team's performance during the season. The coach is concerned about the influence of the local venue (home fixture) or a distant venue(away fixture). He has the following statistics for the 1997 season.

	Matches played	Number of wins
Home fixture	15	10
Away fixture	12	6

1) During the 1997 season what was the probability that the Highflyers would win a randomly chosen match?

(1 mark)

2) Midway through the season the team had played 9 home and 6 away games. What would be the expected number of wins for this sample?

(2 marks)

3) Find the 95% (ie the 2 standard deviation) confidence interval for the proportion of home wins.

(3 marks)

4) After playing 9 home matches in the 1998 season, the team had 4 wins. If we assume that the 1998 home performance is the same as the 1997, are this year's results significant at the 2 standard deviation level?

(2 marks)

A sports commentator suggests that the 1998 performance has fallen below the 1997 level and that now the Highflyers only had a 60% chance of winning a home fixture and a 30% chance of winning an away fixture.

Using these new predictions find the following:

5) The probability that the team will win exactly 4 home matches from their first 9 home games.

(2 marks)

6) The probability that the team will win fewer than 3 of their first 9 home matches.

(2 marks)

7) If the team plays 7 matches, 4 home and 3 away, find the probability that it will win 2 home and 1 away match.

(3 marks)

Question 3 (20 marks)

Here we shall approximate the trigonometric function $f(x) = \sin x$ by the polynomial function $g(x) = a + bx + cx^2 + dx^3$.

1) Find the value of 'a' that will make $f(0) = g(0)$.

(1 mark)

2) Find the value of 'b' that will allow $f'(0) = g'(0)$.

(1 mark)

3) Give the value of 'c' which ensures that $f''(0) = g''(0)$.

(1 mark)

4) Continue this process to find the value of 'd'.

(1 mark)

5) What features are common to f(x) and g(x) due to the conditions 1) and 2)?

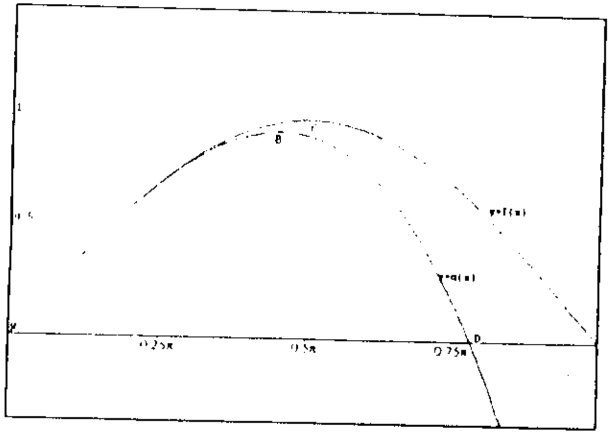
(2 marks)

6) State the difference between the function and its approximation when $x = 1.1$.

(1 mark)

7) Using the approximation you have found state the corresponding approximation for $h(x) = \sin 2x$.

(2 marks)



The graphs of f(x) and g(x) are shown on the left. B and C are the turning points and D is the intercept of y = g(x) and the x-axis.

8) Find the coordinates of B and D.

(3 marks)

9) Calculate the area of the region bounded by the two curves and $x = \frac{\pi}{2}$.

(3 marks)

10) Use the approximation for $f(x) = \sin x$ to find an estimate to $\int_{0.5}^{1} x \sin x\,dx$.

(3 marks)

11) Differentiate $x\cos x$ and hence find $\int x \sin x\,dx$.

(2 marks)

Question 4. (14 marks)

An aircraft leaves Melbourne to fly directly to Sydney which is a total distance of 700 km. A mathematical student models this situation using the following variables and assumptions:

x = (horizontal) distance from Melbourne in km.

h = height of the plane above sea level in km.

Assume that both Melbourne and Sydney are 0 km above sea level.

Initially the student suggests that the journey has three stages:

Ascending along a parabolic path;

Level flight at a height of 10km;

Descending along a parabolic path.

Assume that the change from ascending to level to descending must be smooth.

The equations of the model are of the form:

$$h = \begin{cases} -k(x-100)^2 + 10 & 0 \le x \le 100 \\ m & 100 \le x \le 600 \\ -l(x-600)^2 + 10 & 600 \le x \le 700 \end{cases}$$

1) Use the given information to find the values of k, m and l.

(3 marks)

2) Find the rate of change of height with respect to distance of the plane when it is 50km from Melbourne.

(2 marks)

3) Calculate the angle at which the aircraft would leave the ground according to this model.

(2 marks)

After further thought the student refines the model so that the final stage, the descent, is linear.

4) Give the equation for the straight line for this final stage.

(2 marks)

5) Now the flight plan is changed so that the final descent is delayed till as late in the flight as possible (although the descent must still be linear). Another restriction is that the plane must approach the ground at an angle of 7^{0} or less. What is the furthest distance travelled before descent?

(2 marks)

Another alternative to this model was a trigonometric path where the plane had just two stages: a gradual ascent followed by a gradual descent. Here the model is given by:

$$h = 5\cos(n(x - 350)) + m$$

6) Find suitable values of n and m which fit the given information.

(3 marks)

END OF BOOKLET