1997

MATHEMATICAL METHODS

TRIAL CAT 3

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ã **CHEMISTRY ASSOCIATES 1998**

STUDENT NUMBER									LETTER
figures									
words									

Victorian
Mathematics 1997

MATHEMATICAL METHODS
1997 TRIAL CAT 3
Analysis Task

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

(not to be used before Monday, October 13, 1997)

QUESTION AND ANSWER BOOKLET

Directions to students

Materials
Question and answer booklet of 11 pages.
Working space is provided throughout the booklet.
There is a detachable sheet of miscellaneous formulas.
You may bring to the CATup to four pages (two A4 sheets) of pre-written notes
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from this booklet during reading time.
Ensure that you write your **student number** in the space provide on the cover of this booklet.
Answer **all** questions.
The marks allotted to each part of each question are indicated at the end of the part.
There is a total of 60 marks available for the task.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses should be in English.

At the end of the task.
Hand in this question and answer booklet.

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Question 1

a. The number of typing errors per minute, X, on a Word Processor, has the following probability distribution:

x	0	1	2	3	4	5
$Pr(X = x)$	0.20	0.20	0.20	0.15	0.15	0.10

i. Find the probability that in a random minute of typing there will be more than 2 errors.

ii. Calculate the mean and variance for the number of typing errors.

5 marks

b. As an incentive to accurate typing, an employer decides to charge20 cents per typing error up to and including a total of 5 errors

Calculate the expected charge for a minute's typing according to the probability distribution in **part a**.

1 mark

c. **i.** State the name of the distribution *Z*, giving the number of minutes of typing which contain more than 2 errors in a sample of 20 minutes of typing.

ii. Calculate the probability, correct to three decimal places, that from this sample of 20 minutes of typing, that no more than 2 minutes of typing will have more than 2 errors.

3 marks

(d) Research shows that a child who comes into contact with chickenpox has a probability of 0.4 of contracting the disease. If a child who is known to have chickenpox comes into contact with a family of six children, determine the probability that at least two of the children will contract chickenpox.

3 marks

(e) A netball club is conducting two raffles. In the first raffle 700 tickets at 20c each will be sold.In the second raffle 120 tickets at $1 each will be sold. If I spend $2 on each raffle, in which raffle do I have the best chance of winning? (Give a reason for your answer)

3 marks

Total: 15 marks

Question 2

(a) Differentiate:

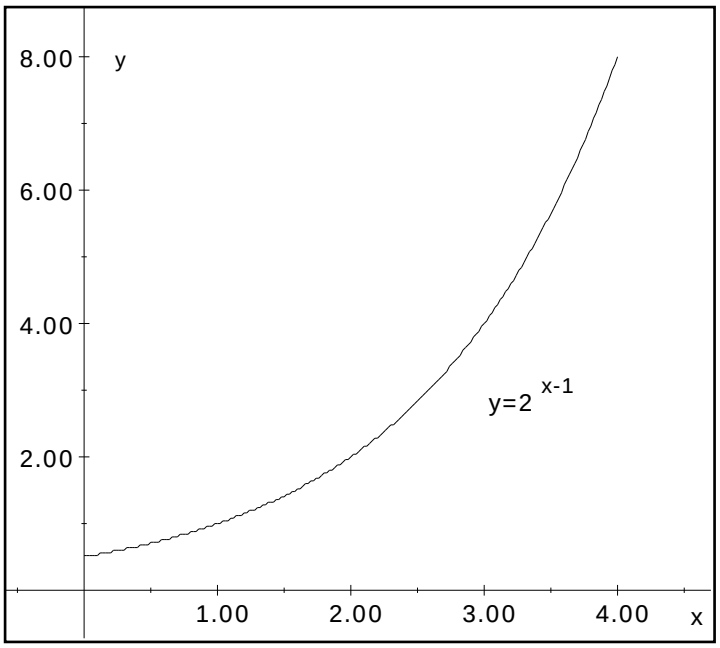
(i) $4x^2 - \frac{5}{x^3} + 3\sqrt{x}$

4 marks

(ii) $(4-5x)^4$

4 marks

(b) Use the trapezoidal rule to find an approximation for $\int_0^4 2^{x-1}\,dx$ using four subintervals.

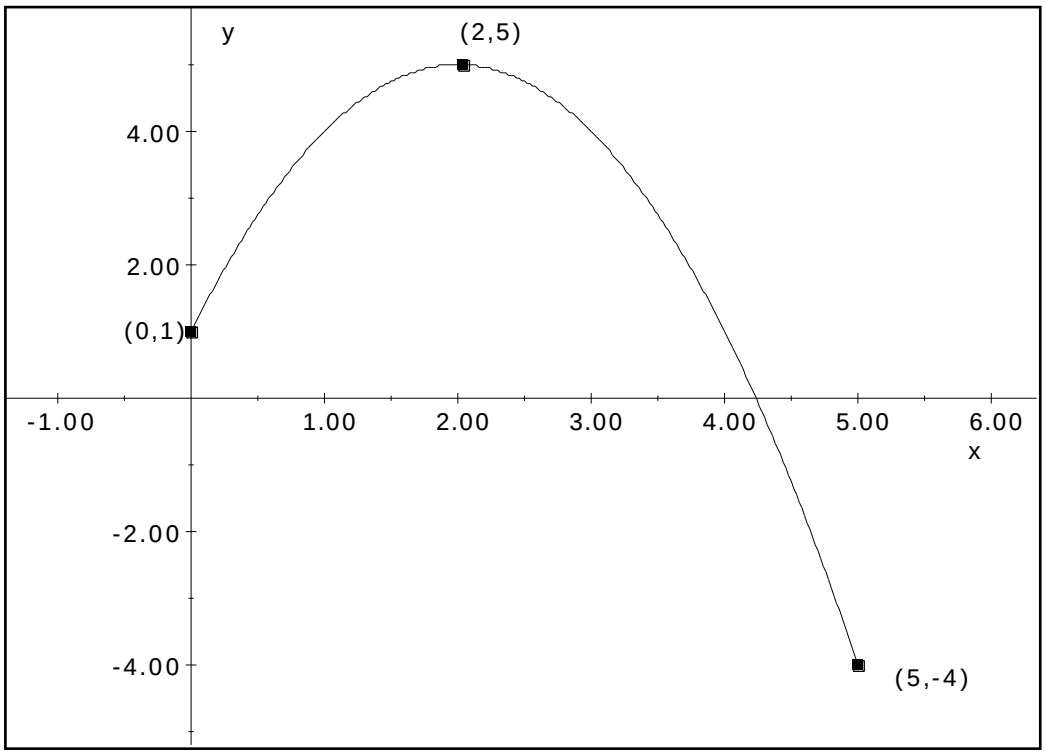


7 marks

Total: 15 marks

Question 3

(a) The graph $y = 1 + 4x - x^2$ over the domain $0 \leq x \leq 5$ is shown below.



(i) State, with a reason, whether the graph represents a function or not.

1 mark

(ii) What is the range of y over this domain.

2 marks

(b) The population in a town is growing such that the number, N, at time t years after 1995 is given by $N = N_0 e^{0.12t}$.

(i) The population of the town in 1995 is 5000. Find the value of the constant N_0.

3 marks

(ii) On the basis of this model estimate the population of the town in the year 2000.

3 marks

(iii) Find the rate at which the population is increasing in the year 2000.

3 marks

(c) A ball is thrown up vertically from ground level. The height of the ball, h metres, at any time t seconds after release is given by the formula $h = 40t - 10t^2$.

Use differential calculus to find the maximum height of the ball.

3 marks

Total: 15 marks

Question 4

A function is defined by $f(x) = 3x^2 - 2x^3$.

(a) Find the coordinates of any turning points and determine their nature.

3 marks

(b) Sketch the curve, indicating all intercepts and turning points.

3 marks

(c) State the domain over which both $f(x) > 0$ and $f'(x) > 0$.

3 marks

(d) On the same set of axes sketch the line $f(x) = \frac{1}{2}$.

3 marks

(e) Use your answer to part (d) to find the number of solutions to the equation $6x^2 - 4x^3 = 1$.

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3 marks

Total: 15 marks

END OF QUESTIONS 1997 MATHEMATICAL METHODS TRIAL CAT 3

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Suggested Solutions to 1997 Mathematical Methods Trial CAT 3

Question 1

a. **i.** $\Pr(X > 2) = \Pr(X = 3) + \Pr(X = 4) + \Pr(X = 5)$

$= 0.15 + 0.15 + 0.10$

$= 0.4$

ii.

x	$p(x)$	$xp(x)$	x^2	$x^2p(x)$
0	0.20	0	0	0
1	0.20	0.20	1	0.20
2	0.20	0.40	4	0.80
3	0.15	0.45	9	1.35
4	0.15	0.60	16	2.40
5	0.10	0.50	25	2.50
sum		2.25		7.25

mean of $X = \mu = \sum xp(x) = 2.25$

$\text{var}(X) = \sum x^2 p(x) - \mu^2$

$= 7.25 - 2.25^2 = 2.19$

b. Let C denote the charge by the Employer.

$C = 0.2X$

$E(C) = 0.2E(X)$

$= 0.2 \times 2.25 = \$0.45$

c. **i.** Z is Binomial

ii. For Z, $n = 20$, $p = 0.4$

$\Pr(Z \le 2) = \Pr(Z = 0) + \Pr(Z = 1) + \Pr(Z = 2)$

$= {}^{20}C_0(0.4)^0(0.6)^{20} + {}^{20}C_1(0.4)^1(0.6)^{19} + {}^{20}C_2(0.4)^2(0.6)^{18}$

$= (1)(1)(0.000036561) + (20)(0.4)(0.000060935) + (190)(0.16)(0.000101559)$

$= 0.000036 + 0.000487 + 0.003087$

$= 0.00361$

$= 0.004$

d.

Let X = number of children who contract chickenpox
X is Binomial with $n = 6$ and p = 0.4

$$p(X > 2) = 1 - p(X = 0) - p(X = 1)$$
$$= 1 - \binom{6}{0}(0.4)^0(0.6)^6 - \binom{6}{1}(0.4)^1(0.6)^5$$
$$= 0.767 \qquad \text{(correct to 3 decimal places)}$$

(e) In the first raffle, 10 tickets can be bought for \$2
$p(\text{win}) = \frac{10}{700} \approx 0.014$

In the second raffle, 2 tickets can be bought for \$2
$p(\text{win}) = \frac{2}{120} \approx 0.017$

The best chance of winning is with the second raffle.

Question 2

(a) (i) Let $f(x) = 4x^2 - \frac{5}{x^3} + 3\sqrt{x}$

$$= 4x^2 - 5x^{-3} + 3x^{\frac{1}{2}}$$
$$f'(x) = 8x + 15x^{-4} + \frac{3}{2}x^{-\frac{1}{2}}$$
$$= 8x + \frac{15}{x^4} + \frac{3}{2\sqrt{x}}$$

(ii) Let $y = (4 - 5x)^4$

If $u = 4 - 5x$, then $y = u^4$

$\frac{du}{dx} = -5$ and $\frac{dy}{du} = 4u^3$

$= 4(4 - 5x)^3$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = -20(4 - 5x)^3$

(b)

x	0	1	2	3	4
2^{x-1}	0.5	1	2	4	8

$$\int_0^4 2^{x-1}\,dx \approx \tfrac{1}{2}[0.5 + 8 + 2(1 + 2 + 4)]$$
$$= \tfrac{1}{2}\left(\tfrac{45}{2}\right)$$
$$= \tfrac{45}{4}$$

Question 3

(a) (i) The graph represents a function because every value of x has only one corresponding value of y.

(ii) The range of y is: $-4 \leq y \leq 5$

(b) (i) For the year 1995, let $t = 0$

$5000 = N_0 e^0$

$N_0 = 5000$

(ii) For the year 2000, $t = 5$

$N = 5000e^{0.12(5)}$

≈ 9111

Population in 2000 will be about 9111.

(iii) Rate of increase $= \frac{dN}{dt}$

$= 0.12 \times 5000e^{0.12t}$

$= 600e^{0.12t}$

If $t = 5$

then $\frac{dN}{dt} = 600e^{0.12(5)}$

≈ 1093

The population will be increasing at a rate of 1093 people/year.

(c) For maximum height, $\frac{dh}{dt} = 0$

$40 - 20t = 0$

$t = 2$

When $t = 2$, $h = 40(2) - 10(2)^2$

$= 40$

The maximum height of the ball is 40 metres.

Question 4

(a) For stationary points, let $f'(x) = 0$

$6x - 6x^2 = 0$

$6x(1 - x) = 0$

$x = 0$ or $x = 1$

Nature of stationary points, look at sign of $f''(x)$

$f''(x) = 6 - 12x$

$f''(0) = 6 > 0$ local minimum

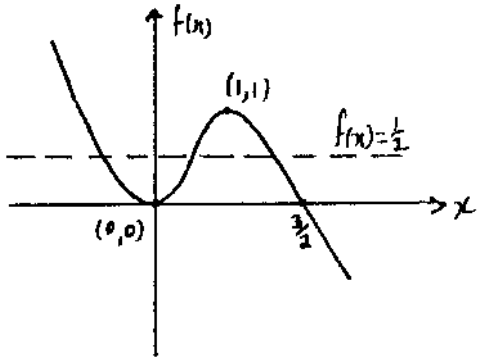
$f''(1) = 6 - 12 = -6 < 0$ local maximum

$f(0) = 0$ $f(1) = 3 - 2 = 1$

(0,0) is a local minimum

(1,1) is a local maximum

(b)



$f(x)$ intercept : $f(0) = 0$

x intercept : $0 = 3x^2 - 2x^3$

$0 = x^2(3 - 2x)$

$x = 0$ or $x = \frac{3}{2}$

(c) $f(x) > 0$ when the graph is above the x axis

$f'(x) > 0$ when the graph is increasing

$f(x) > 0$ and $f'(x) > 0$ when $0 < x < 1$

(d) See the graph for the horizontal line $f(x) = \frac{1}{2}$.

(e) If $6x^2 - 4x^3 > 1$ then $3x^2 - 2x^3 > \frac{1}{2}$

Since the cubic and linear function intersect three times, there are three solutions to the equation $6x^2 - 4x^3 > 1$.

END OF SUGGESTED SOLUTIONS 1997 MATHEMATICAL METHODS TRIAL CAT 3

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