1997

MATHEMATICAL METHODS

TRIAL CAT 2

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Victorian
Mathematics 1997

MATHEMATICAL METHODS
1997 TRIAL CAT 2
Facts, Skills and Applications

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

(not to be used before Monday, October 13, 1997)

Part I

MULTIPLE-CHOICE QUESTION BOOKLET

Directions to students

This task has two parts: Part I (multiple-choice questions) and Part II (short answer questions).
Part I consists of this question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of a separate question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part, proceed immediately to the other part.
A detachable formula sheet for use in both parts is included with this booklet.

At the end of the task.
Place the answer sheet for multiple-choice questions (Part I) inside the back cover of the question and answer booklet (Part II) and hand them in.
You may retain this question booklet.

Directions to students

Materials
Question booklet of 11 pages.
Answer sheet for multiple-choice questions.
Working space is provided throughout the booklet.
You may bring to the CATup to four pages (two A4 sheets) of pre-written notes
An approved scientific and/or graphics calculator may be used

You should have at least one pencil and an eraser.

The task
Detach the formula sheet from this booklet during reading time.
Ensure that you write your **name and student number** on the answer sheet for multiple-choice questions.
Answer **all** questions.
There is a total of 33 marks available for Part I.
All questions should be answered on the answer sheet provided for multiple-choice questions .
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

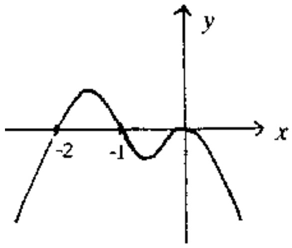
At the end of the task.
Place the answer sheet for multiple-choice questions (Part I) inside the back cover of the question and answer booklet (Part II) and hand them in.
You may retain this question booklet.

MATHEMATICAL METHODS PART 1
MULTIPLE-CHOICE QUESTION BOOKLET

Specific Instructions for Section A

This part consists of 33 questions.
Answer all questions in this section on the answer sheet provided for multiple-choice questions.
A correct answer scores 1, an incorrect answer scores 0.
Marks will not be deducted for incorrect answers. You should attempt every question.
No credit will be given if two or more letters are marked for that question.

Question 1
Which one of the following equations has the graph



A. $y = -x^2(x+2)(x+1)$

B. $y = -x^2(x+1)(x+2)$

C. $y = -x^2(x-2)(x+1)$

D. $y = -x^2(x-2)(x-1)$

E. $y = -x^2(x-1)(x-2)$

Question 2
For the function $f:[-3,2] \to R,\ \ f(x) = (x+1)^2 - 4$ the range is

A. $[-3,5]$

B. $[-4,5]$

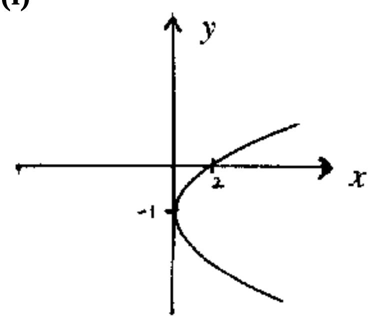
C. $[-2,5]$

D. $[0,5]$

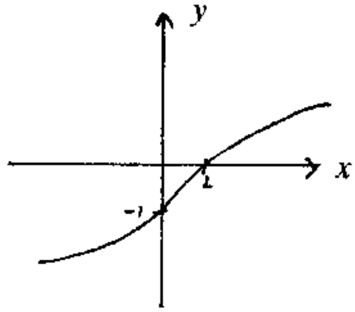
E. $[-1,2]$

Question 3
Which one or more of the following (represented by the graphs below) are **not** functions?

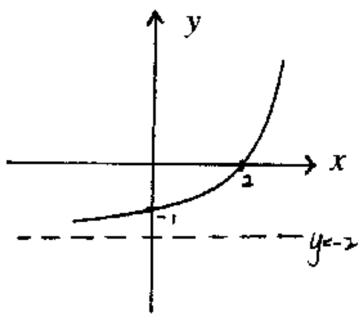
(i)



(ii)



(iii)



A. (i) only

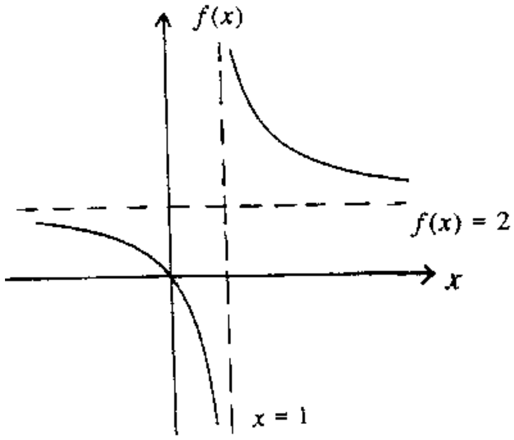
B. (ii) only

C. (i) and (iii) only

D. (ii) and (iii) only

E. all of (i), (ii) and (iii)

Question 4
If A is a negative integer, a possible form of equation for the graph shown is



A. $f(x) = \dfrac{A}{-x+2} + 1$

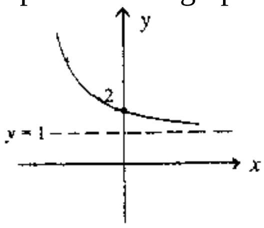
B. $f(x) = \dfrac{A}{x+1} - 2$

C. $f(x) = \dfrac{A}{-x+1} + 2$

D. $f(x) = \dfrac{A}{-x-2} - 1$

E. $f(x) = \dfrac{A}{x-1} - 2$

Question 5
Which one of the following relations represents the graph



A. $y = -1 - e^{-x}$

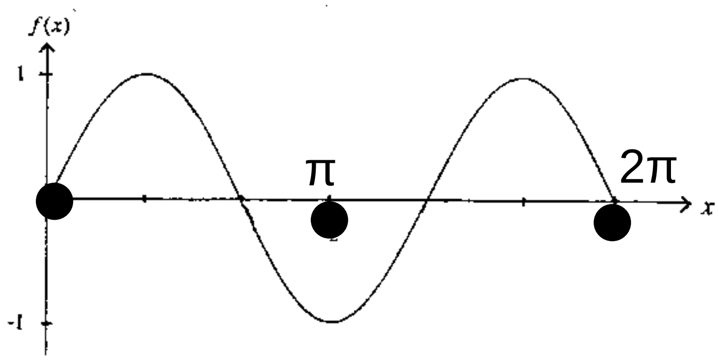
B. $y = -1 + e^{-x}$

C. $y = 1 + e^{x}$

D. $y = 1 + e^{-x}$

E. $y = 1 - e^{-x}$

Question 6
A possible equation for the graph shown is



A. $f(x) = \sin x$

B. $f(x) = \sin(\frac{3x}{2} + \pi)$

C. $f(x) = \cos(x - \frac{\pi}{2})$

D. $f(x) = \cos(\frac{3x + \pi}{2})$

E. $f(x) = \cos(\frac{3x - \pi}{2})$

Question 7
The solutions between 0 and π for which $\sqrt{2}\sin 3x = 1$ are

A. $\frac{5\pi}{12}$

B. $\frac{\pi}{4}, \frac{5\pi}{12}$

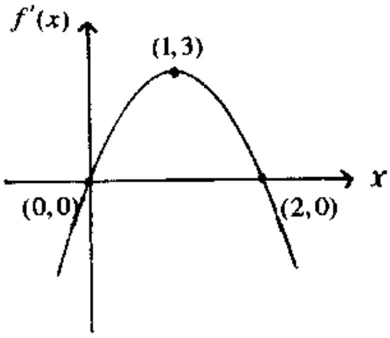
C. $\frac{\pi}{12}, \frac{5\pi}{12}$

D. $\frac{\pi}{12}, \frac{\pi}{4}$

E. $\frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}$

Question 8

The graph of the derived function $f'(x)$ is shown.
Which one of the following statements relating to the function, $f(x)$, is **false**?



A. $f(x)$, is a polynomial of degree three.

B. $f(x)$ has exactly one stationary point.

C. $f(x)$ is decreasing over the domain $(2, \infty)$.

D. $f(x)$ has a maximum turning point at $x = 2$.

E. The gradient of $f(x)$ is negative over the domain $(-\infty, 1)$.

Question 9

The derivative of $\frac{x^2 + 4}{x^2}$ is equal to

A. 8

B. $-\frac{8}{x}$

C. $-\frac{1}{8x^3}$

D. $-\frac{1}{x^3}$

E. $-\frac{8}{x^3}$

Question 10

If $y = 3xe^x$ then $\frac{dy}{dx}$ is

A. $3xe^{3x}$

B. $3xe^{2x}$

C. $3xe^x$

D. $3(x+1)e^x$

E. $3xe^x + e^{2x}$

Question 11

If $f(x) = \sqrt{x^2 + 5}$ then $f'(x)$ is equal to

A. $x\sqrt{x^2 + 5}$

B. $\dfrac{1}{2\sqrt{x^2 + 5}}$

C. $\dfrac{x}{\sqrt{x^2 + 5}}$

D. $\dfrac{x}{x + 2}$

E. $\dfrac{1}{2(x + 2)}$

Question 12

The derivative of $\dfrac{2t + 1}{t + 3}$ is equal to

A. $\dfrac{5}{(t + 3)^2}$

B. $\dfrac{7}{(t + 3)^2}$

C. $\dfrac{-5}{(t + 3)^2}$

D. $\dfrac{-7}{(2t + 1)^2}$

E. 5

Question 13

The maximum value of $-4x^2 + 8x - 5$ is

A. -1 at x = 1

B. -8 at x = 1

C. 1 at x = -1

D. 8 at x = -1

E. 8 at x = -8

Question 14

The gradient of the normal to the curve $f(y) = e^{-y}$ at the point where $y = 1$ is equal to

A. $-e$

B. $\frac{e}{2}$

C. $-\frac{2}{e}$

D. e

E. $\frac{2}{e}$

Question 15

The volume of a gas, G, after t seconds is given by $G(t) = 2t^2(15-t),\ \ 0 \le t \le 10$.

After how many seconds is the volume increasing at the greatest rate?

A. 2

B. 3

C. 4

D. 5

E. 6

Question 16

If x satisfies the equation $(1-e^x)(9+e^{2x}) = 0$ then x is equal to

A. 1 or $\log_e 3$

B. 1 or $\log_e 9$

C. 0 only

D. 0 or $\log_e 9$

E. 0 or $\log_e 27$

Question 17

The coefficient of x^4 in the expansion of $(3-2x)^5$ is equal to

A. +1080

B. -810

C. +240

D. -180

E. +90

Question 18

The function $f : [1, \infty) \to R,\ f(x) = (x-1)^2 - 5$ has an inverse function f^{-1} defined by

A. $f^{-1} : [-1, \infty) \to R,\ f^{-1}(x) = 5 + \sqrt{x+1}$

B. $f^{-1} : [1, \infty) \to R,\ f^{-1}(x) = 1 + \sqrt{x+5}$

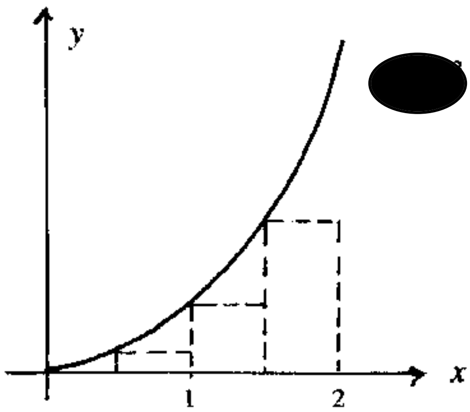
C. $f^{-1} : [-5, \infty) \to R,\ f^{-1}(x) = \sqrt{x+5}$

D. $f^{-1} : [1, \infty) \to R,\ f^{-1}(x) = 5 + \sqrt{x+1}$

E. $f^{-1} : [-5, \infty) \to R,\ f^{-1}(x) = 1 + \sqrt{x+5}$

Question 19

The area under the curve $y = 2x^3$ between $x = 0$ and $x = 2$ is approximated by dividing the interval into four sections equal in width and calculating the area of the lower rectangles.



The **difference** between the exact area under the curve and the approximate area calculated by this technique is

A. 12.5 square units

B. 12.75 square units

C. 8 square units

D. 6 square units

E. 3.5 square units

Question 20

Given that $\int_1^4 f(x)\,dx = 3$ and $g(x) = 1 - 2f(x)$ then $\int_4^1 g(x)\,dx$ is equal to

A. 3

B. - 3

C. - 9

D. 5

E. 9

Question 21

Evaluate $\int_0^{\pi} \sin 2x \, dx$

A. -2

B. -1

C. 0

D. 1

E. 2

Question 22

If c is an arbitrary constant and $f'(x) = \dfrac{2}{\sqrt{4x-1}}$ then $f(x)$ is equal to

A. $12\sqrt{4x+1} + c$

B. $4\sqrt{4x+1} + c$

C. $\dfrac{4}{\sqrt{4x-1}} + c$

D. $\dfrac{4}{3\sqrt{4x-1}} + c$

E. $\sqrt{4x-1} + c$

Question 23

The area bounded by the curve $f(x) = \dfrac{-2}{7-2x}$ and the x-axis from $x = 1$ to $x = 2$ is equal to

A. $\log_e 2$

B. $\log_e 0.6$

C. $\log_e 0.5$

D. $\log_e 0.3$

E. $\log_e 0.06$

Question 24

Calculate $\Pr(X < 4)$ where X has a probability distribution given by

x	1	2	3	4
$\Pr(X = x)$	$3c^2$	$8c^2$	c^2	$4c^2$

A. $\frac{1}{16}$

B. $\frac{3}{16}$

C. $\frac{5}{16}$

D. $\frac{11}{16}$

E. $\frac{12}{16}$

Question 25

The random variable X represents the number of errors on a production line in a factory per day.

x	0	1	2	3	4	5	>5
$\Pr(X = x)$	0.2	0.3	0.2	0.1	0.05	0.05	0.1

The owner of this factory pays all employees a daily bonus according to the following conditions:

if no errors occur a bonus of \$5 is paid
if one or two errors occur a bonus of \$1 is paid
if three or more errors occur no bonus is paid

The employee can expect to receive a weekly bonus of

A. \$1.00
B. \$1.50
C. \$2.00
D. \$2.50
E. \$3.00

Question 26

X is a discrete random variable with mean 7.0 and standard deviation 1.5
The interval in which 95% of the distribution of X would lie is

A. 1 to 9

B. 2 to 8

C. 1 to 8

D. 2 to 9

E. 4 to 10

The following information relates to questions 27 and 28

A dog breeder has 4 dogs. The probability that a dog will have to be treated for fleas during one month is 0.5.

Question 27

The probability that **no more than one** of these dogs will need to be treated for fleas in the next month is closest to

A. 0.026

B. 0.130

C. 0.154

D. 0.313

E. 0.475

Question 28

Over a six month period, the number of times flea treatment would be expected is

A. 2

B. 3

C. 8

D. 12

E. 30

Question 29

X is a binomial random variable with $p = 0.1$.
If $\Pr(X \geq 1) = 0.6$ the variance of X is equal to

A. 0.45

B. 0.90

C. 1.35

D. 1.80

E. 2.25

Question 30
A factory item has a recommended mass of 500 grams. The mass of items is normally distributed with a mean of 500 g and variance of 4 g.
Items which weigh less than 497 g are rejected. Calculate the probability, correct to 4 decimal places, that a randomly selected item will be rejected.

A. 0.7734

B. 0.2266

C. 0.5000

D. 0.0668

E. 0.9932

Question 31
The diagram below shows two normal distributions, *A* and *B*, with means of μ_A and μ_B respectively and standard deviations of σ_A and σ_B respectively. Which of the following is true?

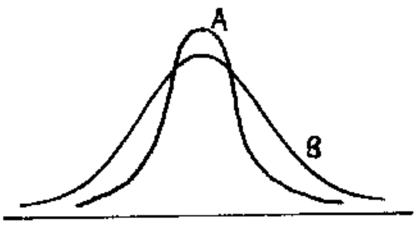
A. $\mu_B < \mu_A$

B. $\mu_B > \mu_A$

C. $\mu_B = \mu_A$

D. $\sigma_B = \sigma_A$

E. $\sigma_B < \sigma_A$



Question 32
X is normally distributed with a mean of 10. Given that $\Pr(X > 14) = 0.4$, the standard deviation of X is closest to
A. 250
B. 37.2
C. 30.4
D. 15.8
E. 6.1

Question 33
From a random sample of 25 people, 4 are left-handed. An approximate 95% confidence interval for the proportion of people who are left-handed is
A. 0.03 — 0.08

B. 0.04 — 0.36

C. 0.05 — 0.16

D. 0.06 — 0.62

E. 0.50— 0.60

STUDENT NUMBER									LETTER
figures									
words									

Victorian Mathematics 1997

MATHEMATICAL METHODS 1997 TRIAL CAT 2 Facts, Skills and Applications

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

Part II

QUESTION AND ANSWER BOOKLET

This task has two parts: Part I (multiple-choice questions) and Part II (short answer questions).
Part I consists of a separate question booklet and must be answered on the answer sheet provided for multiple-choice questions.
Part II consists of this question and answer booklet.
You must complete **both** parts in the time allotted. When you have completed one part, continue immediately to the other part.
A detachable formula sheet for use in both parts is included in the Part I question booklet.

At the end of the task.
Place the answer sheet for multiple-choice questions (Part I) inside the back cover of this question and answer booklet (Part II) and hand them in.

Directions to students

Materials
Question and answer booklet of 4 pages.
Working space is provided throughout the booklet.
You may bring to the CATup to four pages (two A4 sheets) of pre-written notes
You may use an approved scientific and/or graphics calculator, ruler, protractor, set-square and aids for curve-sketching.

The task
Detach the formula sheet from the Part I booklet during reading time.
Ensure that you write your **student number** in the space provide on the cover of this booklet.
The marks allotted to each question are indicated at the end of the question.
There is a total of 17 marks available for part II.
You need not give numerical answers as decimals unless instructed to do so.
Alternative forms may involve, for example, π,*e*, surds or fractions.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
All written responses should be in English.

At the end of the task.
Place the answer sheet for multiple-choice questions (part I) inside the back cover of this question and answer booklet (part II) and hand them in.

MATHEMATICAL METHODS
QUESTION AND ANSWER BOOKLET

Specific instructions to students

Answer **all** questions in this section in the spaces provided.

Question 1

Determine the largest possible domain and range for the function $f(x) = \sqrt{9x - x^2}$

2 marks

Question 2

Find the rule for the inverse function for $y = e^{x-1} + 3, x > 3$

3 marks

Question 3

The gradient at all points on a curve is given by $f'(x) = 3x^2 - 6x + 5$.
Find the equation of the curve given that $f(2) = 4$.

3 marks

Question 4

Find the area bounded by the x axis and the curve $f(x) = \cos x$ in the interval $\frac{\pi}{2} \le x \le \frac{4\pi}{3}$

3 marks

Question 5

Find: (i) $\int \left(2x - \frac{2}{x}\right) dx$ (ii) $\int_0^2 \left(1 - e^{2x}\right) dx$.

3 marks

Question 6

A box contains twenty chocolates of identical shape and size, but twelve have dark chocolate and eight have milk chocolate. If two chocolates are selected at random, what is the probability that they are a different type?

3 marks

END OF QUESTIONS 1997 MATHEMATICAL METHODS TRIAL CAT 2

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Suggested solutions to 1997 Mathematical Methods CAT 2 - part I

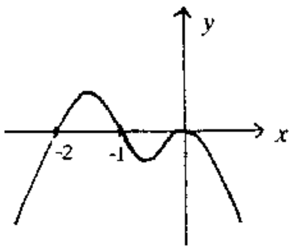
Question 1 **A**

x intercept : let $y = 0$

$0 = -x^2(x+2)(x+1)$

$x = 0, \ -2, \ -1$

$x = 0$ is a turning point



y intercept : let $x = 0$

$y = 0 \times 2 \times 1 = 0$

General shape is a negative quartic

Question 2 **B**

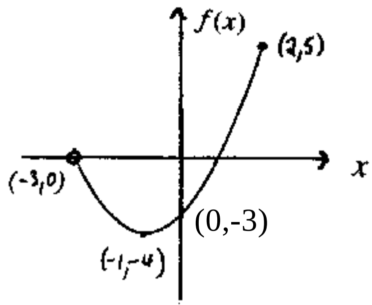
$f(x) = (x+1)^2 - 4$

From translations, turning point at $(-1,-4)$

$f(-3) = (-2)^2 - 4 = -0$

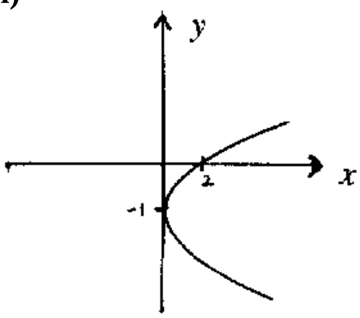
$f(2) = 3^2 - 4 = 5$

The minimum y value is -4 and the maximum y value is 5, therefore the range is $[-4,5]$

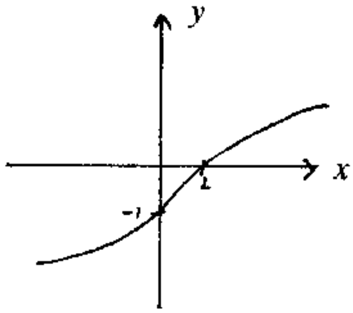


Question 3 **A**

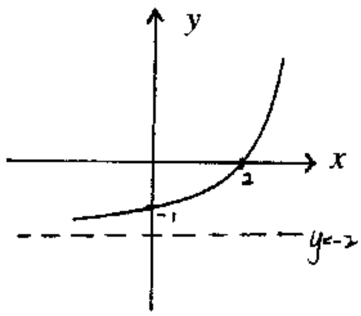
(i)



(ii)



(iii)



(i) is **not** a function, but (ii) and (iii) are both functions.

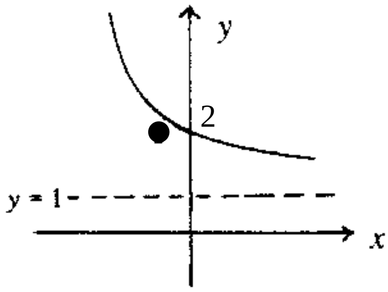
Question 4 **C**

Using the asymptotes given, and making A a positive integer, the equation is of the form:

$$f(x) = \frac{A}{x-1} + 2 \text{ since as } \begin{array}{l} x \to -1, f(x) \to \pm\infty \\ x \to \pm\infty, f(x) \to 2 \end{array}$$

Question 5 **D**

$y = 1 + e^{-x}$



Horizontal asymptote: $y = 1$
Basic shape is reflected in the y axis.
y intercept : $y = 1 + e^0 = 2$

Question 6 **E**

Let the model be of the form $y = A\cos n(x + b)$

amplitude = 1, $A = 1$ period $= \frac{4\pi}{3}$, $\frac{2\pi}{n} = \frac{4\pi}{3}$ $n = \frac{3}{2}$

$$y = \cos\frac{3}{2}(x + b)$$

The cosine curve is translated $\frac{\pi}{3}$ units to the right, $b = -\frac{\pi}{3}$

The equation of the curve is $y = \cos\frac{3}{2}(x - \frac{\pi}{3}) = \cos(\frac{3x-\pi}{2})$

Question 7. **E**

$\sqrt{2}\sin 3x = 1$

$\sin 3x = \frac{1}{\sqrt{2}}$

Sine is positive, angles in 1st & 2nd quadrants
Basic angle is $\frac{\pi}{4}$ as $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$

$x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}$

Question 8. **B**

f(x) has two stationary points (where f'(x) = 0). Hence, **B** is false.

Question 9. **E**

Let $f(x) = \dfrac{x^2 + 4}{x^2} = 1 + 4x^{-2}$

$f'(x) = -8x^{-3} = -\dfrac{8}{x^3}$

Question 10. D
Using the Product rule:
$\frac{dy}{dx} = 3x(e^x) + e^x(3)$

$= 3e^x(x+1)$

$= 3(x+1)e^x$

Question 11. C
Using the Chain rule:
Let $u(x) = x^2 + 5$ $\quad f(u) = u^{\frac{1}{2}}$

$u'(x) = 2x$ $\quad f'(u) = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}} = \frac{1}{2\sqrt{x^2+5}}$

$$f'(x) = \frac{2x}{2\sqrt{x^2+5}} = \frac{x}{\sqrt{x^2+5}}$$

Question 12. A
Using the Quotient rule:
Let $f(t) = \frac{2t+1}{t+3}$
then $f'(t) = \frac{(t+3)(2)-(2t+1)(1)}{(t+3)^2} = \frac{2t+6-2t-1}{(t+3)^2} = \frac{5}{(t+3)^2}$

Question 13. A
Let $f(x) = -4x^2 + 8x - 5$
For local maximum or minimum solve $f'(x) = 0$
$-8x + 8 = 0$
$x = 1$

Test for local maximum:

$f'(0) = +8 > 0$
$f'(2) = -16 < 0$

x	<1	1	>1
$f'(x)$	>0	0	<0
	/	_	\

$x = 1$ gives maximum value
maximum value $= -4(1)^2 + 8(1) - 5 = -1$

Question 14. D
Gradient of tangent $f'(y) = -e^{-y}$
At $y = 1$ gradient of tangent $= -e^{-1} = -\frac{1}{e}$
At $y = 1$ gradient of normal $= -1 \div -\frac{1}{e} = e$

Question 15. D

$B(t) = 2(15t^2 - t^3)$

Rate of change $= 2(30t - 3t^2)$

For maximum rate of change let $G'(t) = 0$

$2(30 - 6t) = 0$

$30 - 6t = 0$

$t = 5$

Test for maximum:

$G''(4) = 2\big(30 - 6(4)\big) > 0$

$G''(6) = 2\big(30 - 6(6)\big) < 0$

t	< 5	5	> 5
$G''(t)$	> 0	0	< 0
	/	–	\

Volume is changing at the greatest rate after 5 seconds.

Question 16. C

$(1 - e^x)(9 + e^{2x}) = 0$

either $e^x = 1$ or $e^{2x} = -9$

$x = 0$ or *N*O SOLUTION

Question 17. B

$(3 - 2x)^5 = (3)^5 - 5(3)^4(2x) + 10(3)^3(2x)^2 - 10(3)^2(2x)^3 + 5(3)(2x)^4 - (2x)^5$

coefficient of $x^4 = -5 \times 2 \times 3^4 = -810$

Question 18. E

Let $y = (x - 1)^2 - 5$

Interchanging x and y gives

$x = (y - 1)^2 - 5$

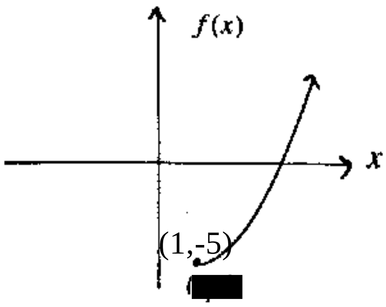
$x + 5 = (y - 1)^2$

$\sqrt{x + 5} = y - 1$

$y = 1 + \sqrt{x + 5}$

$f^{-1}(x) = 1 + \sqrt{x + 5}$

Domain of f^{-1} = range of $f = [-5, \infty)$



Inverse of $f = [-5, \infty) \rightarrow R$, $f^{-1}(x) = 1 + \sqrt{x + 5}$

Question 19. E

If $x = 0.5$, $y = 2(0.5)^3 = 0.25$

If $x = 1$, $y = 2(1)^3 = 2$

If $x = 1.5$, $y = 2(1.5)^3 = 6.75$

Approximate area $= 0.5 \times 0.25 + 0.5 \times 2 + 0.5 \times 6.75 = 4.5$ square units.

Exact area = $\int_0^2 2x^3 dx = [\frac{1}{2}x^4]_0^2 = 8$. Hence, the difference = 8 - 4.5 = 3.5 square units.

Question 20. A

$$\int_4^1 g(x)\,dx = \int_4^1 -2f(x)+1\,dx$$
$$= -2\int_4^1 f(x)\,dx + \int_4^1 1\,dx$$
$$= +2\int_1^4 f(x)\,dx + [x]_4^1$$
$$= +2(3)+(1-4)$$
$$= 3$$

Question 21. C

$$\int_0^{\pi} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi}$$
$$= -\frac{1}{2}\cos 2\pi + \frac{1}{2}\cos 0$$
$$= -\frac{1}{2}+\frac{1}{2}$$
$$= 0$$

Question 22. E

$$f'(x) = \frac{2}{\sqrt{4x-1}} = 2(4x-1)^{-\frac{1}{2}}$$
$$f(x) = \frac{2}{\frac{1}{2}\times 4}(4x-1)^{+\frac{1}{2}} + c$$
$$= \sqrt{4x-1}+c$$

Question 23. B

$$\text{Area} = \int_1^2 \tfrac{-2}{7-2x}\,dx$$
$$= \left[\log_e(7-2x)\right]_1^2$$
$$= \left(\log_e 3 - \log_e 5\right)$$
$$= \log_e 0.6$$

Question 24. E

$$\sum \Pr(X=x) = 1$$
$$3c^2 + 8c^2 + c^2 + 4c^2 = 1$$
$$16c^2 = 1$$
$$c^2 = \tfrac{1}{16}$$

$$\Pr(X<4) = \Pr(X=3)+\Pr(X=2)+\Pr(X=1)$$
$$= \tfrac{1}{16}+\tfrac{8}{16}+\tfrac{3}{16}$$
$$= \tfrac{12}{16} = \frac{3}{4}$$

Question 25. B
Let B denote the bonus paid

b	$\Pr(B=b)$	$b\Pr(B=b)$
5	0.2	1
1	0.3	0.3
1	0.2	0.2
0	0.3	0

The expected daily bonus is \$1 + \$0.3 + \$0.2 = \$1.50

Question 26. B
$\mu = 7.0, \quad \sigma = 1.5$

$\mu + 2\sigma = 7.0 + 2(1.5) = 10.0 \qquad \mu - 2\sigma = 7.0 - 2(1.5) = 4.0$

Since X is a discrete random variable, the 95% confidence interval is 4 to 10.

Question 27. D
Let X denote the number of dogs which need to be treated for fleas in one month.
$n = 4, \quad p = 0.5$

$$\Pr(X \le 1) = \Pr(X = 0) + \Pr(X = 1)$$
$$= \binom{4}{0}(0.5)^0(0.5)^4 + \binom{4}{1}(0.5)^1(0.5)^3$$
$$= 0.3125$$

Question 28. D
$E(X) = np = 4 \times 0.5 = 2.0$

On average 2.0 treatments per month would needed. Therefore it would be expected that 6 x 2.0 = 12 treatments would be needed over a six month period.

Question 29. A

$\Pr(X \geq 1) = 0.7941$

$\Pr(X = 0) = 1 - 0.6 = 0.4$

$$\binom{n}{0}(0.1)^0(0.9)^n = 0.6$$

$$(0.9)^n = 0.6$$

$$\log_{10}(0.9)^n = \log_{10} 0.6$$

$$n = \frac{\log_{10} 0.6}{\log_{10} 0.9}$$

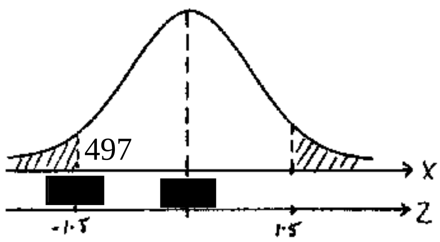
$$= 5$$

$n = 5, \quad p = 0.1, \quad \sigma^2 = np(1-p) = 5 \times 0.1 \times 0.9 = 0.45$

Question 30. D

$\mu = 500, \qquad \sigma = \sqrt{4} = 2$

$$\Pr(X < 497) = \Pr(Z < \tfrac{497-500}{2})$$
$$= \Pr(Z < -1.5)$$
$$= \Pr(Z > 1.5)$$
$$= 1 - \Pr(Z < 1.5)$$
$$= 1 - 0.9332$$
$$= 0.0668$$



Question 31. C

Both normal distributions are centred about the same value, $\mu_B = \mu_A$
Distribution *B* has a greater spread than distribution *A* $\mu_B > \mu_A$

Question 32. D

$\mu = 10$

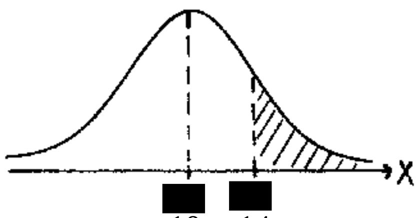
$\Pr(X > 14) = 0.4$

$\Pr(X < 14) = 0.6$

$\frac{14-10}{\sigma} = 0.253$

$\sigma = 15.8$

$\sigma^2 = 250$



Question 33. B

$\hat{p} = \frac{5}{25} = 0.2$

$se(\hat{p}) = \sqrt{\frac{0.8 \times 0.2}{25}} = 0.08$

lower limit = 0.2-2(0.08) = 0.04

Upper limit = 0.2+2(0.08) = 0.36

95% confidence interval is 0.04 to 0.36

Suggested solutions to 1997 Mathematical Methods CAT 2 - part II

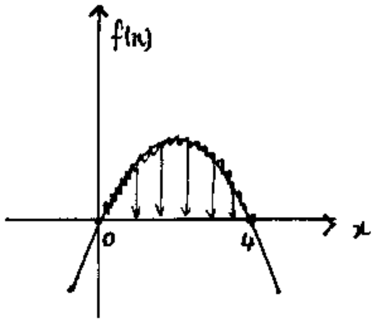
Question 1

$f(x)$ is defined when $9x - x^2 \geq 0$

$x(9 - x) \geq 0$

$0 \leq x \leq 9$

this is the largest possible domain
When $x = 4.5$, $f(x) = 20.25$
The largest possible range is [0,20.25]



Question 2

Interchanging x and y gives:

$x = e^{y-1} + 3$

$x - 3 = e^{y-1}$

$y - 1 = \log_e(x - 3)$

$y = 1 + \log_e(x - 3)$

The inverse of the function is
$y = 1 + \log_e(x - 3)$

Question 3

$f'(x) = 3x^2 - 6x + 5$

$f(x) = x^3 - 3x^2 + 5x + c$

$f(2) = 4 \qquad 4 = 2^3 - 3(2)^2 + 5(2) + c$

$4 = 6 + c$

$c = -2$

$f(x) = x^3 - 3x^2 + 5x - 2$

Question 4

$f(x) = \cos x$

$$\text{Area} = \int_{\frac{\pi}{2}}^{\pi} (\cos x)\, dx + \left| \int_{\pi}^{\frac{4\pi}{3}} (\cos x)\, dx \right|$$

$$= [\sin x]_{\frac{\pi}{2}}^{\pi} + \left| [\sin x]_{\pi}^{\frac{\pi}{3}} \right|$$

$$= \sin \pi - (\sin \frac{\pi}{2}) + |\, (\sin \frac{4\pi}{3} - (\sin \pi) \,|$$

$$= 0 - 1 + \sqrt{\frac{3}{2}} - 0$$

$$= -1 + \sqrt{\frac{3}{2}} \text{ square units}$$

Question 5

(i) $\int \left(2x - \frac{2}{x}\right) dx = x^2 - 2\ln x + c$

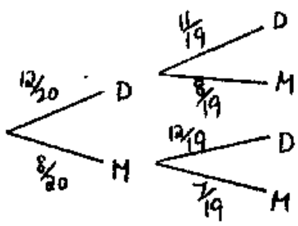
(ii) $\int_0^2 \left(1 - e^{2x}\right) dx = \left[x - \frac{1}{2}e^{2x}\right]_0^2$

$= 2 - \frac{1}{2}e^4 - \left(0 - \frac{1}{2}e^0\right)$

$= 2 - \frac{1}{2}e^4 + \frac{1}{2}$

$= \frac{5}{2} - \frac{1}{2}e^4$

Question 6



$p(\text{different type}) = \frac{12}{20} \times \frac{8}{19} + \frac{8}{20} \times \frac{12}{19}$

$= \frac{96}{380} + \frac{96}{380}$

$= \frac{192}{380}$

$= \frac{48}{95}$

END OF SUGGESTED SOLUTIONS 1997 MATHEMATICAL METHODS TRIAL CAT 2

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